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APPLICATION OF NEW HOMOTOPY PERTURBATION METHOD IN SOLVING A SIMPLE PREDATOR PREY MODEL WITH RICH DYNAMICS

B. SEETHALAKSHMI, V. ANANTHASWAMY* and S. NARMATHA

Research Centre and PG
Department of Mathematics
The Madura College (Autonomous)
Madurai-625011 Tamil Nadu, India
E-mail: seetharajas@gmail.com

Research Centre and PG
Department of Mathematics
The Madura College (Autonomous)
Madurai-625 011 Tamil Nadu, India

Department of Mathematics
Lady Doak College (Autonomous)
Madurai-625 002 Tamil Nadu, India
E-mail: narmatha@ldc.edu.in

Abstract

The New Homotopy perturbation method is applied to solve a previously developed simple predator prey model with rich dynamics. Approximate analytical expressions for the prey and predator species populations are derived. The analytical solutions are compared with the numerical simulations and are found to make a very good fit.

1. Introduction

Mathematical modeling is an attempt to study some part (or form) of the real life problem in mathematical terms. It is an essential tool for

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*Corresponding author; E-mail: ananthu9777@gmail.com
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understanding the world. The process of translation of a real-life problem into a mathematical form can give a better representation and solution of certain problems. This process of translation is called Mathematical Modeling.

A Lotka-Volterra model is the simplest model of predator-prey interactions. The model was developed independently by Lotka (1925) and Volterra (1926). A predator is an organism that hunts, kills and eats other organisms to survive. A prey is an organism which gets hunted and is taken as food by other organisms. The Lotka-Volterra equations, is also known as the predator-prey equations. These equations are a pair of first-order non-linear differential equations, frequently used to describe the dynamics of biological systems in which two species interact, one as a predator and the other as prey. The objective of the present investigation is to find an alternate solution to a given differential equation using new Homotopy perturbation method and to analyze the mathematical model using graphs.

2. Mathematical Formulation of the Problem

Mathematical modeling of Predator-Prey interactions have attracted wide attention since the original work by Lodka-Volterra in 1920's and there have been extensive studies for the Rich dynamics [1-3]. For a class of predator-prey systems, Brauer and Soudack obtained different types of dynamics for which the harvesting was in prey or a predator. Xiao and Jennings further studied a ratio-dependent predator-prey model with a constant harvest on prey.

The mathematical model developed by Bing Li et al. [6] has been considered in this paper. This model was not solved analytically previously. In this work, an approximate analytical solution is derived for the model using the new Homotopy perturbation method.

In this model, if the density of the predator is below a switched value, the harvest has a linear harvesting rate. Otherwise, the harvesting rate is constant. Here, the model exhibits new dynamical features compared to those with a linear harvesting rate or a constant harvesting rate [1-5].

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right) - aNP \tag{1}$$

$$\frac{dP}{dt} = caNP - dP - H(P) \tag{2}$$

where N and P represent the prey and predator species populations, In the absence of the predation, r is intrinsic growth rate and K is carrying capacity. In the presence of the predator, a represents the rate of predation, c denotes the efficiency of predation and d denotes the mortality rate.

$$H(P) = \begin{cases} mP, & 0 \leq P \leq P_0 \\ h, & P_0 < P \end{cases} h = mP_0$$

Initial conditions are

$$t = 0,\ P = P_0,\ N = N_0 \tag{3}$$

3. Approximate Analytical Solution to Equations (1) to (3) Using New Homotopy Perturbation Method

In order to obtain approximate analytic solutions to non-linear differential equations, asymptotic method such as Variational iteration method [7], Adomain decomposition method [8], Homotopy analysis method [9-11], Homotopy perturbation method [12-18] and the new approach to Homotopy perturbation method [19-23] are employed. When compared to all these methods, the new Homotopy perturbation method gives a better simple approximate solution in the zeroth iteration itself. The advantage of this method is that it does not need a small parameter in the system and hence has a wide application in solving non-linear differential equations.

We shall derive the solution of equations (1) to (3) using New Homotopy perturbation method.

Construct the Homotopy for equation (1) as follows:

$$(1\text{-p})\left(\frac{dN}{dt} - rN\left(1 - \frac{N}{K}\right) + aNP0\right) + p\left(\frac{dN}{dt} - rN\left(1 - \frac{N}{K}\right) + aNP\right) = 0 \tag{4}$$

Construct the Homotopy for equation (2) as follows:

For P less than or equal to P_0

$$(1\text{-p})\left(\frac{dP}{dt} - caN_0P + dP + mP\right) + P\left(\frac{dP}{dt} - caN + dP + mP\right) = 0 \tag{5}$$

For P greater than P_0

$$(1\text{-p})\left(\frac{dP}{dt} - caN_0P + dP + mP0\right) + p\left(\frac{dP}{dt} - caN + dP + mP0\right) = 0 \tag{6}$$

where

$$N = N_0 + N_1 p + N_2 p^2 + \ldots \tag{7}$$

$$P = P_0 + P_1 p + P_2 p^2 + \ldots \tag{8}$$

Solving the above equations using initial approximations we get,

For P less than or equal to P_0

$$N = N_0 e^{\frac{-((-K+N_0)r+P_0aK)t}{K}} \tag{9}$$

$$P = P_0 e^{\frac{-N_0acKe^{\frac{-((-K+N_0)r+P_0aK)t}{K}} + (-t(d+m)r + Kac)N_0 - Kt(d+m)(P_0a - r)}{(P_0a - r)K + rN_0}} \tag{10}$$

For P greater than P_0

$$N = N_0 e^{\frac{-((-K+N_0)r+P_0aK)t}{K}} \tag{11}$$

$$P = \frac{P_0((caN_0 - d - m)e^{(caN_0 - d)t} + m)}{caN_0 - d} \tag{12}$$

4. Numerical Simulation

The equations (1) to (3) are also solved numerically. The initial value problem has been solved numerically by using in MATLAB software in this paper. The obtained analytical results are compared with the numerical simulation.

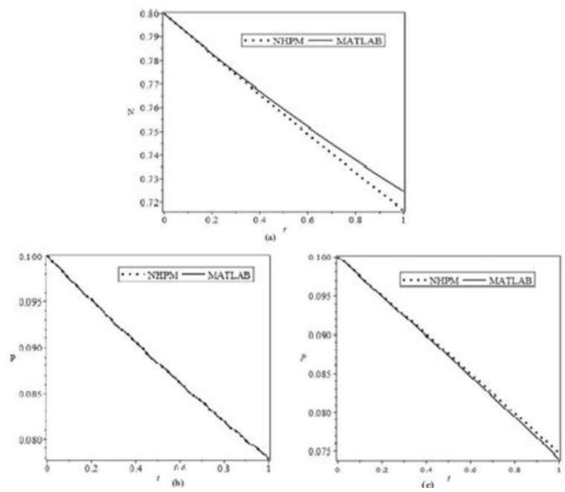

Figure 1(a), (b) and **(c).** Plot of the prey species population (N) and Plot of the predator species population (P) versus time t for cases: P less than or equal to P_0 and P greater than P_0 respectively; for parameter values $a = 0.1$, $c = 0.8$, $d = 0.01$, $r = 0.1$, $K = 0.4$, $N_0 = 0.8$, $P_0 = 0.1$.

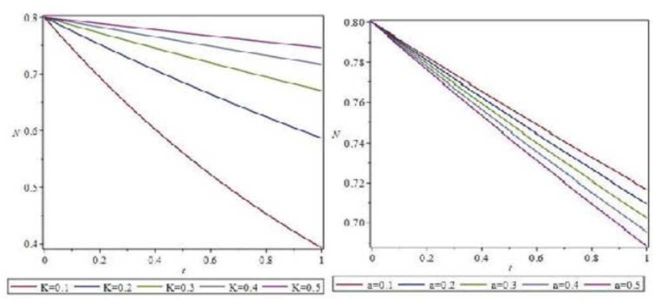

Figure 2(a) and **(b).** Plot of the prey species population (N) versus time t for parameter values $a = 0.1$, $c = 0.8$, $d = 0.01$, $r = 0.1$, $N_0 = 0.8$, $P_0 = 0.1$, $K = 0.4$.

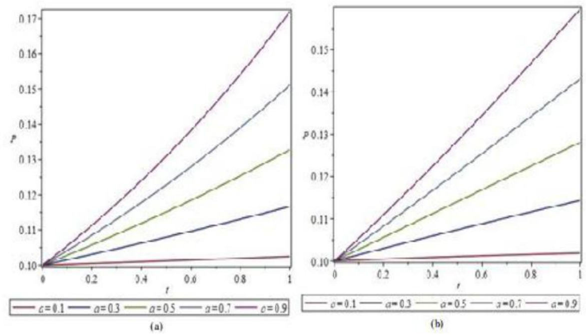

Figure 3(a) and (b). Plot of the predator species population (P) versus time t for cases: P less than or equal to P_0 and P greater than P_0 respectively; for parameter values $K = 0.3$, $d = 0.01$, $c = 0.8$, $r = 0.1$, $n = 0.8$, $p = 0.1$, $m = 0.03$, $N_0 = 0.8$, $P_0 = 0.1$.

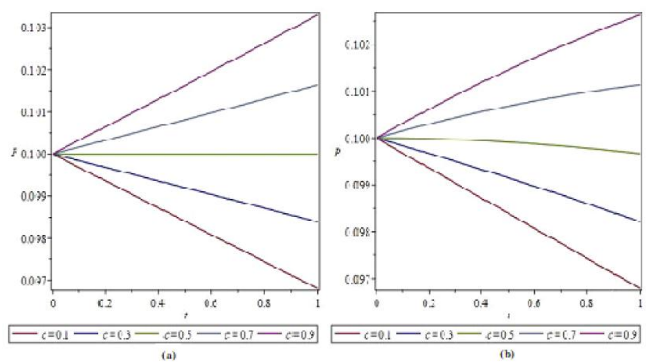

Figure 4(a) and (b). Plot of the predator species population (P) versus time t for cases: P less than or equal to P_0 and P greater than P_0 respectively; for parameter values $K = 0.3$, $a = 0.1$, $d = 0.01$, $r = 0.1$, $n = 0.8$, $p = 0.1$, $m = 0.03$, $N_0 = 0.8$, $P_0 = 0.1$.

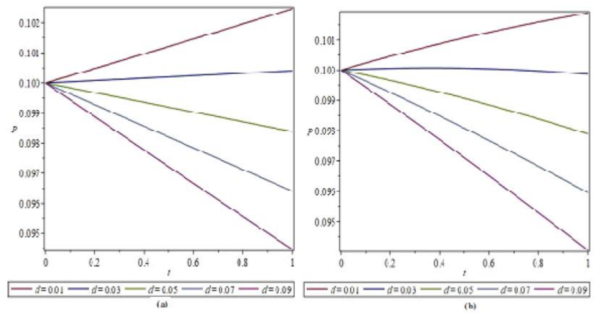

Figure 5(a) and **(b).** Plot of the predator species population (P) versus time t for cases: P less than or equal to P_0 and P greater than P_0 respectively; for parameter values $K = 0.3$, $a = 0.1$, $c = 0.8$, $r = 0.1$, $n = 0.8$, $p = 0.1$, $m = 0.03$, $N_0 = 0.8$, $P_0 = 0.1$.

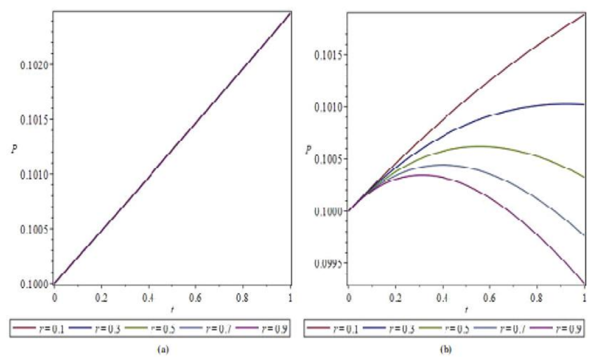

Figure 6(a) and (b). Plot of the predator species population (P) versus time t for cases: P less than or equal to P_0 and P greater than P_0 respectively; for parameter values $K = 0.3$, $a = 0.1$, $c = 0.8$, $d = 0.01$, $n = 0.8$, $p = 0.1$, $m = 0.03$, $N_0 = 0.8$, $P_0 = 0.1$.

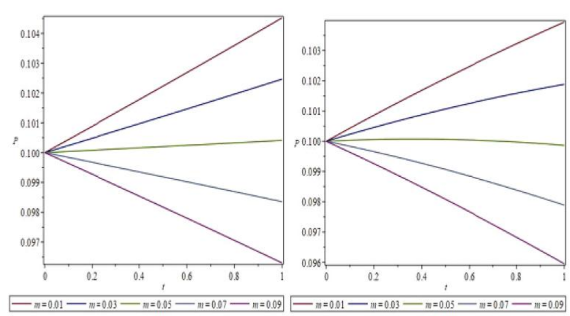

Figure 7(a) and **(b).** Plot of the predator species population (P) versus time t for cases: P less than or equal to P_0 and P greater than P_0 respectively; for parameter values $K = 0.3$, $a = 0.1$, $c = 0.8$, $d = 0.01$, $r = 0.1$, $n = 0.8$, $p = 0.1$, $N_0 = 0.8$, $P_0 = 0.1$.

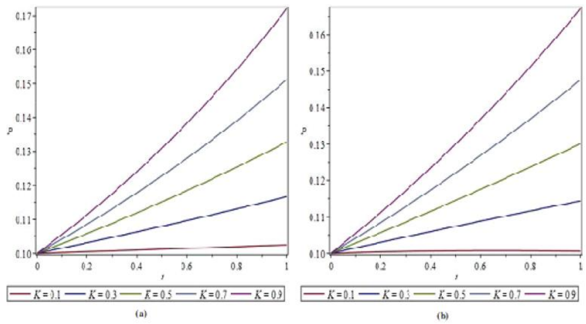

Figure 8(a) and **(b).** Plot of the predator species population (P) versus time t for cases: P less than or equal to P_0 and P greater than P_0 respectively; for parameter values $a = 0.1$, $c = 0.8$, $d = 0.01$, $r = 0.1$, $m = 0.03$, $n = 0.8$, $P = 0.1$, $N_0 = 0.8$, $P_0 = 0.1$.

Table 1. Comparison between analytical values and numerical values in Figure 1.

Value of N			
Value of t	Numerical value	Analytical value	Absolute deviation percentage
0	0.8	0.8	0
0.2	0.7829671	0.7825922	0.047885217
0.4	0.7670012	0.7655632	0.187492839
0.6	0.7520101	0.7489047	0.412952228
0.8	0.7379117	0.7326087	0.718655885
1	0.7246329	0.7166673	1.099252193
Average percentage of deviation			0.41104

Table 2. Comparison between analytical values and numerical values in Figure 2(b).

Value of P			
Value of t	Numerical value	Analytical value	Absolute deviation percentage
0	0.1	0.1	0
0.2	0.09	0.0951	0.097646455
0.4	0.0898	0.0901	0.231713349
0.6	0.08.47	0.0850	0.404587091
0.8	0.0794	0.0799	0.619802427
1	0.0741	0.0747	0.882427994
Average percentage of deviation			0.372696

5. Results and Discussion

The equations (9)-(12) represent the simple approximate analytical solution to the prey predator model with rich dynamics. The derived analytical expressions are compared with the numerical solutions obtained using MATLAB in Figure 1. The percentage error is given in tables 1 and 2. From the tables it is evident that the percentage error is a very minimum, hence we may say that the derived solution is an approximate analytical solution to equations (1) to (3). From Figure 2, we observe that the prey species population varies inversely with the rate of predation and the intrinsic growth rate, but directly with the carrying capacity. Further, from Figures 3 to 8, we may conclude that predator species population varies directly with the rate of predation, efficiency of predation and the carrying capacity, but inversely with the mortality rate and the intrinsic growth rate.

6. Conclusion

In this paper, time dependent approximate analytical expressions for prey species population and predator species population are reported. The New Homotopy perturbation method is used to obtain the solution. The analytical results make a very good fit with the numerical results. The obtained analytical results will help the researchers to interpret the effect of the different parameters over the prey and predator species population.

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