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MICRO-αg-SEPARATION AXIOMS

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Abstract

The objective of this paper is to introduce the new spaces namely, Micro-αg-$D_i (i = 0, 1, 2)$ and Micro-αg-$T_i (i = 0, 1, 2)$ and study their properties in Micro topological space.

1. Introduction

The concept of α-sets, generalized closed sets and α generalized closed sets in topological spaces were introduced respectively by Njastad [9], Levine [7] and Maki et al. [8]. Nano topology was initially proposed by Carmel Richard and Lellis Thivagar [3]. Later as an extension to it, Micro topological spaces were introduced and examined by Sakkraiveeranan Chandrasekar [12] in 2019. He also defined Micro pre-open and Micro semi-open sets in Micro topological spaces. Following this Chandrasekar and Swathi [4] and Rasheed and Jasim [11] investigated the concept of Micro-α-open and Micro-α-closed sets in Micro topological spaces and derived their properties. Ibrahim [5] introduced Micro α-closed sets, Micro $T_{1/2}$-spaces in Micro topological spaces and studied their fundamental properties. Later on, Ibrahim [5] [6]

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introduced respectively Micro $D_i(i = 0, 1, 2)$ spaces and Micro $T_i(i = 0, 1, 2)$ spaces in Micro topological spaces and derived their basic properties. Anandhi and Balamani [1] introduced the concept of Micro α-Generalized closed sets in Micro topological spaces and investigated its fundamental properties. Anandhi and Balamani [2] introduced and studied the properties of Micro α-generalized closure operator and Micro α-generalized interior operator in Micro topological spaces. In this paper, we introduce Micro Difference α-generalized set and Micro α-generalized kernel of a set in Micro topological spaces and derive some of their basic properties. Also, we define new spaces namely Micro-αg-$D_i(i = 0, 1, 2)$ and Micro-αg-$T_i(i = 0, 1, 2)$ using Micro Difference α-generalized set and Micro α-generalized open sets respectively and examine their properties.

2. Preliminaries

Definition 2.1 [10]**.** Let U be a nonempty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Element belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

1. The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \subseteq X\}$, where $R(x)$ denotes the equivalence class determined by x.

2. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $U_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \cap X \neq \phi\}$.

3. The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not-X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$.

Definition 2.2[3]**.** Let U be the Universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$, where $X \subseteq U$. Then $\tau_R(X)$ satisfies the following axioms:

1. U and $\phi \in \tau_R(X)$.

2. The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.

3. The intersection of the elements of any finite sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ is a topology on U called the Nano topology on U with respect to X. We call $(U, \tau_R(X))$ as the Nano topological space. The elements of $\tau_R(X)$ are called as Nano open sets and the complement of a Nano open set is called a Nano closed set.

Definition 2.3[12]**.** Let $(U, \tau_R(X))$ be a Nano topological space. Then $\mu_R(X) = \{N \cup (N' \cap \mu) : N, N' \in \tau_R(X) \text{ and } \mu \notin \tau_R(X)\}$ and $\mu_R(X)$ satisfies the following axioms:

(i) $U, \phi \in \mu_R(X)$.

(ii) The union of the elements of any sub-collection of $\mu_R(X)$ is in $\mu_R(X)$.

(iii) The intersection of the elements of any finite sub-collection of $\mu_R(X)$ is in $\mu_R(X)$.

Then, $\mu_R(X)$ is called the Micro topology on U with respect to X. The triplet $(U, \tau_R(X), \mu_R(X))$ is called Micro topological space and the elements of $\mu_R(X)$ are called Micro open sets and the complement of a Micro open set is called a Micro closed set.

Definition 2.4[1]**.** Let $(U, \tau_R(X), \mu_R(X))$ be a Micro topological space. A subset A of U is said to be Micro-α-generalized closed (briefly Mic- αg -closed) if $Mic\text{-}\alpha\text{cl}(A) \subseteq L$ whenever $A \subseteq L$ and L is Micro open in U.

Definition 2.5[1]**.** A subset A of a Micro topological space $(U, \tau_R(X), \mu_R(X))$ is called Mic- αg -open if its complement A^c is Mic- αg -closed in $(U, \tau_R(X), \mu_R(X))$.

Definition 2.6[2]**.** Let $(U, \tau_R(X), \mu_R(X))$ be a Micro topological space and $A \subseteq U$. Then the Micro- αg -closure of a set A is denoted by

Mic-αg-cl(A) and is defined as

$$Mic\text{-}\alpha g\text{-cl(A)} = \bigcap\{T : T \text{ is } Mic\text{-}\alpha\text{g-closed in } U \text{ and } A \subseteq T\}.$$

Definition 2.7[2]**.** Let $(U, \tau_R(X), \mu_R(X))$ be a Micro topological space and $A \subseteq U$. Then the Micro-αg-interior of a set A is denoted by Mic-αg-int(A) and is defined as

$$Mic\text{-}\alpha g\text{- int(A)} = \bigcap\{T : T \text{ is } Mic\text{-}\alpha\text{g-open in } U \text{ and } A \supseteq T\}.$$

3. Micro Difference α-generalized set and Micro αg kernel of a set

Definition 3.1. Let $(U, \tau_R(X), \mu_R(X))$ be a Micro topological space. Then a subset A of U is called a Micro Difference α-generalized set (briefly Mic-$D\alpha g$ set) if there are two Mic-αg-open sets L, M in U such that $L \neq U$ and $A = L \setminus M$.

Example 3.2. Let $U = \{a, b, c, d\}, U/R = \{\{a\}, \{b\}, \{c, d\}\}$. Let $X = \{b\} \subseteq U$. Then $\tau_R(X) = \{U, \phi, \{b\}\}$. Let $\mu = \{a, c\} \notin \tau_R(X)$. Then $\mu_R(X) = \{\phi, \{b\}, \{a, c\}, \{a, b, c\}, U\}$.

Mic-αg-open sets are $\phi, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}, \{a, b, c\}, U$.

If $L = \{a, b, c\} \neq U$ and $M = \{b\}$ then $L \setminus M = \{a, b, c\} \setminus \{b\} = \{a, c\}$. Thus $A\{a, c\}$ is a Mic-$D\alpha g$ set.

Remark 3.3. Every Mic-αg-open set $A \neq U$ is Mic-$D\alpha g$ set, since $A = L$ and $M = \phi$.

Definition 3.4. Let $(U, \tau_R(X), \mu_R(X))$ be a Micro topological space and $A \subseteq U$. Then the Micro αg kernel of A denoted by Mic-αg-ker(A) is defined as $Mic\text{-}\alpha g\text{-ker(A)} = \bigcap\{L : L \text{ is } Mic\text{-}\alpha\text{g-closed in } U \text{ and } A \subseteq L\}$.

Example 3.5. Let $U = \{a, b, c, d\}, U/R = \{\{a\}, \{b\}, \{c, d\}\}$. Let $X = \{b\} \subseteq U$. Then $\tau_R(X) = \{U, \phi, \{b\}\}$. Let $\mu = \{a, c\} \notin \tau_R(X)$. The $\mu_R(X) = \{\phi, \{b\}, \{a, c\}, U\}$.

Mic-αg-open sets are $\phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, U$. Here Mic-$\alpha g$-ker$(\{c, d\}) = \{b, c, d\}$.

Theorem 3.6. *Let A be a subset of a Micro topological space $(U, \tau_R(X), \mu_R(X))$. Then* $x \in$ Mic-αg-cl(A) *if and only if for every Mic-* αg *-open subset L of U containing* x, $L \cap A \neq \phi$.

Proof. Let $x \in$ Mic-αg-cl(A) and assume that $L \cap A \neq \phi$ for some Mic-αg-open set L which contains x. Then, $(U \setminus L)$ Mic-αg-closed and $A \subseteq (U \setminus L)$, i.e., $U \setminus L$ is a Mic-αg-closed set containing A and $x \notin U \setminus L$. Hence, we have $x \notin$ Mic-αg-cl(A). But $x \in$ Mic-αg-cl(A) $\subseteq U \setminus L$ which is a contradiction. Hence $L \cap A \neq \phi$.

Conversely, assume that A is a subset of U. Suppose that $x \in U$ and for every Mic-αg-open set L containing x, $L \cap A \neq \phi$. If $x \notin$ Mic-αg-cl(A), then there exists a Mic-αg-closed set F such that $A \subseteq F$ and $x \notin F$. Then, $U \setminus F$ is a Mic-αg-open set with $x \in U \setminus F$ and disjoint from A which is a contradiction. Hence $x \in$ Mic-αg-cl(A).

Theorem 3.7. *Let* $(U, \tau_R(X), \mu_R(X))$ *be a Micro topological space and for any distinct points x and y in U. Then* $y \in$ Mic-αg-ker({x}) *if and only if* $x \in$ Mic-αg-*cl*({y}).

Proof. If $y \in$ Mic-αg-ker($\{x\}$), then $y \in \bigcap\{L : L$ is Mic-αg-open in U and $\{x\} \subseteq L\}$. i.e., y belongs to every Mic-αg-open set containing x, which implies that $L \cap \{y\} \neq \phi$ for every Mic-αg-open set L containing x. By Theorem 3.6, we have $x \in$ Mic-αg-cl({y}). Conversely assume that $x \in$ Mic-αg-cl({y}). Then by Theorem 3.6, we get that $L \cap \{y\} \neq \phi$ for every Mic-αg-open set L containing x. Hence y belongs to every Mic-αg-open set containing $\{x\}$. Hence $y \in$ Mic-αg-ker($\{x\}$).

Theorem 3.8. *Let* $(U, \tau_R(X), \mu_R(X))$ *be a Micro topological space and* $A \subseteq U$. *Then* $x \in$ Mic-αg-ker(A) *if and only if* Mic-αg-cl({x}) $\cap A \neq \phi$.

Proof. Let $x \in$ Mic-αg-ker(A). Suppose if Mic-αg-cl({x}) $\cap A \neq \phi$. Then, $x \notin$ U$\setminus$Mic-αg-cl({x}) which is a Mic-αg open set that contains A. Which is a contradiction to $x \in$ Mic-αg-ker(A). Hence Mic-αg-cl($\{x\}$)$\cap A \neq \phi$. Conversely, for $x \notin U \setminus \text{Mic-}\alpha\text{g-cl}(\{x\}) \cap A \neq \phi$. and suppose $x \notin \text{Mic-}\alpha\text{g-ker}(A)$. Hence there exist a Mic-αg-open set L such that $A \subseteq L$ and $x \notin L$. Let $z \in \text{Mic-}\alpha\text{g-cl}(\{x\}) \cap A \subseteq \textit{Mic}\text{-}\alpha\text{g-cl}(\{x\}) \cap L$. Hence L is a Mic-αg-open set such that $z \in L$ and $z \in \text{Mic-}\alpha\text{g-cl}(\{x\})$, which implies that L intersects $\{x\}$. i.e., $x \in L$ which is a contradiction. Therefore $x \in \text{Mic-}\alpha\text{g-ker}(A)$.

Theorem 3.9. *Let* $(U, \tau_R(X), \mu_R(X))$ *be a Micro topological space. Then the following properties hold for the subsets* A, B *of* U

(i) $A \subseteq \textit{Mic}\text{-}\alpha\text{g-ker}(A)$.

(ii) $A \subseteq B$ *implies that* $\textit{Mic}\text{-}\alpha\text{g-ker}(A) \subseteq \textit{Mic}\text{-}\alpha\text{g-ker}(B)$.

(iii) $\textit{Mic}\text{-}\alpha\text{g-ker}(\phi) = \phi$ *and* $\textit{Mic}\text{-}\alpha\text{g-ker}(U) = U$.

(iv) *A is Mic-*αg*-open in U if and only if* $A = \text{Mic-}\alpha\text{g-ker}(A)$.

(v) $\textit{Mic}\text{-}\alpha\text{g-ker}(\textit{Mic}\text{-}\alpha\text{g-ker}(A)) = \textit{Mic}\text{-}\alpha\text{g-ker}(A)$.

Proof. (i), (ii), (iii) and (iv) follows from the Definition 3.4

(v) Suppose that if $x \notin \text{Mic-}\alpha\text{g-ker}(A) = \bigcap\{L : L$ is Mic-αg-open in U and $A \subseteq L\}$, then there exists a Mic-αg-open set L containing A such that $x \notin L$. By (iv), $\text{Mic-}\alpha\text{g-ker}(L) = L$ where L is a Mic-αg-open set. Hence Mic-αg-ker(A) $\subseteq$ Mic-αg-ker$(L) = L$. By (ii), $\text{Mic-}\alpha\text{g-ker}(\text{Mic-}\alpha\text{g-ker}(A)) \subseteq L$ which implies that $x \notin \text{Mic-}\alpha\text{g-ker}(\text{Mic-}\alpha\text{g-ker}(A))$. Therefore $\text{Mic-}\alpha\text{g-ker}(\text{Mic-}\alpha\text{g-ker}(A)) \subseteq \text{Mic-}\alpha\text{g-ker}(A)$. From (i) and (ii), we get $\text{Mic-}\alpha\text{g-ker}(A) \subseteq \text{Mic-}\alpha\text{g-ker}(\text{Mic-}\alpha\text{g-ker}(A))$. Hence $\text{Mic-}\alpha\text{g-ker}(\text{Mic-}\alpha\text{g-ker}(A)) = \text{Mic-}\alpha\text{g-ker}(A)$.

Theorem 3.10. *Let* $(U, \tau_R(X), \mu_R(X))$ *be a Micro topological space. Then for any points x and y in U the following statements are equivalent*

(i) $\text{Mic-}\alpha\text{g-ker}(\{x\}) \neq \text{Mic-}\alpha\text{g-ker}(\{y\})$.

(ii) $\text{Mic-}\alpha\text{g-cl}(\{x\}) \neq \text{Mic-}\alpha\text{g-cl}(\{y\})$.

Proof. (i) $\Rightarrow$ (ii). If $\text{Mic-}\alpha\text{g-ker}(\{x\}) \neq \text{Mic-}\alpha\text{g-ker}(\{y\})$. Then there exists a point z in U such that $z \in$ Mic-αg-ker$(\{x\})$ and $z \notin$ Mic-αg-ker$(\{y\})$. Since $z \in$ Mic-αg-ker$(\{x\})$, by Theorem 3.7, $x \in$ Mic-αg-cl$(\{z\})$. Assume $z \notin$ Mic-αg-ker$(\{y\})$ implies that Mic-αg-ker$(\{z\}) \cap \{y\} = \phi$. i.e., $y \notin$ Mic-αg-cl$(\{z\})$. Since $x \in$ Mic-αg-cl$(\{z\})$, Mic-αg-cl$(\{x\}) \subseteq$ Mic-αg-cl$(\{z\})$ implies that Mic-αg-cl$(\{x\}) \cap \{y\} = \phi$. Hence, we get Mic-$\alpha$g-cl$(\{x\})$ $\neq$ Mic-αg-cl$(\{y\})$.

(ii) $\Rightarrow$ (i) Let Mic-αg-cl$(\{x\}) \neq$ Mic-αg-cl$(\{y\})$. Then there exists a point z in U such that $z \in$ Mic-αg-$cl(\{y\})$ and $z \notin$ Mic-αg-$cl(\{x\})$. Since $z \in$ Mic-αg-$cl(\{x\})$, by Theorem 3.6, there exists a Mic-αg-open set L containing z such that $L \cap \{x\} = \phi$. This implies that L is a Mic-αg-open set which contains z but not x. Similarly, since $z \in$ Mic-αg-cl$(\{y\})$, $L \cap \{x\} = \phi$ which implies L is a Mic-αg-open set which contains y but not x, i.e., $x \notin$ Mic-αg-ker$(\{y\})$. Hence Mic-αg-cl$(\{x\}) \neq$ Mic-αg-cl$(\{y\})$.

Theorem 3.11. *Let* $(U, \tau_R(X), \mu_R(X))$ *be a Micro topological space. If* $\{x\}$ *is a Mic-*$D\alpha g$ *set in U then* *Mic*-αg-ker$(\{x\}) \neq U$ *for a point* $x \in U$.

Proof. Let $\{x\}$ be a Mic-$D\alpha g$ set in U, then there exist a two Mic-αg-open sets L, M such that $\{x\} = L \setminus M$, $\{x\} \subseteq L$ and $L \neq U$. Therefore Mic-αg-ker$(\{x\}) \subseteq L \neq U$

4. Micro-αg-$D_i (i = 0, 1, 2)$ and Micro-αg-$T_i (i = 0, 1, 2)$ spaces

Definition 4.1. A space $(U, \tau_R(X), \mu_R(X))$ is said to be Micro-αg-D_0 (briefly Mic-αg-D_0) if for any pair of distinct points x and y in U, there exists a Mic-$D\alpha g$ set P of U such that either $x \in P$ and $y \notin P$ or $x \notin P$ and $y \in P$.

Example 4.2. Let $U = \{a, b, c\}$, $U/R = \{\{a, b, c\}\}$. Let $X = \{a, b\} \subseteq U$. Then $\tau_R(X) = \{U, \phi\}$. Let $\mu = \{a\} \notin \tau_R(X)$. Then $\mu_R(X) = \{\phi, \{a\}, U\}$.

Mic-αg-open sets are $\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, U$.

Mic-$D\alpha g$ sets are $\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}$.

Here $(U, \tau_R(X), \mu_R(X))$ is a Mic-αg-D_0 space.

Definition 4.3. A space $(U, \tau_R(X), \mu_R(X))$ is said to be Micro-αg-D_1 (briefly Mic-αg-D_1) if for any pair of distinct points x and y in U, there exist two Mic-$D\alpha g$ sets P and Q of U such that $x \in P$ but $y \notin P$ and $y \in Q$ but $x \notin Q$.

Example 4.4. Let $U = \{a, b, c\}$, $U/R = \{\{a, b, c\}\}$. Let $X = \{c\} \subseteq U$. Then $\tau_R(X) = \{U, \phi\}$. Let $\mu = \{c\} \notin \tau_R(X)$. Then $\mu_R(X) = \{\phi, \{c\}, U\}$.

Mic-αg-open sets are ϕ, $\{a\}$, $\{b\}$, $\{c\}$, $\{a, b\}$, $\{a, c\}$, U.

Here $(U, \tau_R(X), \mu_R(X))$ is a Mic-αg-D_1 space.

Definition 4.5. A space $(U, \tau_R(X), \mu_R(X))$ is said to be Micro-αg-D_2 (briefly Mic-αg-D_2) if for any pair of distinct points x and y of U there exist two disjoint Mic-$D\alpha g$ sets P and Q of U containing x and y, respectively.

Example 4.6. Let $U = \{a, b, c, d\}, U/R = \{\{a\}, \{b\}, \{c\}, \{d\}\}$. Let $X = \{c\} \subseteq U$. Then $\tau_R(X) = \{U, \phi, \{c\}\}$. Let $\mu = \{b\} \notin \tau_R(X)$. Then $\mu_R(X) = \{\phi, \{b\}, \{c\}, \{b, c\}, U\}$.

Mic-αg-open sets are ϕ, $\{a\}$, $\{b\}$, $\{c\}$, $\{d\}$, $\{a, b\}$, $\{a, c\}$, $\{b, c\}$, $\{b, d\}$, $\{c, d\}$, $\{a, b, c\}$, $\{b, c, d\}$, U.

Here $(U, \tau_R(X), \mu_R(X))$ is a Mic-αg-D_2 space.

Proposition 4.7. *If $(U, \tau_R(X), \mu_R(X))$ is a Micro-αg-D_i space then it is a Micro-αg-D_{i-1} space, for $i = 1, 2$.*

Definition 4.8. A space $(U, \tau_R(X), \mu_R(X))$ is said to be Micro-αg-T_0 (briefly Mic-αg-T_0) if for each pair of distinct points x, y in U, there exists a Mic-αg-open set L such that either $x \in L$ and $y \notin L$ or $x \notin L$ and $y \in L$.

Example 4.9. Let $U = \{a, b, c\}$, $U/R = \{\{a, b, c\}\}$. Let $X = \{b, c\} \subseteq U$. Then $\tau_R(X) = \{U, \phi\}$. Let $\mu = \{a\} \notin \tau_R(X)$. Then $\mu_R(X) = \{\phi, \{a\}, U\}$.

Mic-αg-open sets are ϕ, $\{a\}$, $\{b\}$, $\{c\}$, $\{a, b\}$, $\{a, c\}$, U.

Here $(U, \tau_R(X), \mu_R(X))$ is a Mic-αg-T_0 space.

Definition 4.10. A space $(U, \tau_R(X), \mu_R(X))$ is said to be Micro-αg-T_1 (briefly Mic-αg-T_1) if for each pair of distinct points x, y in U, there exist two Mic-αg-open sets L and M such that $x \in L$ but $y \notin L$ and $y \in M$ but $x \notin M$.

Example 4.11. Let $U = \{a, b, c, d\}, U/R = \{\{a\}, \{b\}, \{c, d\}\}$. Let $X = \{a\} \subseteq U$. Then $\tau_R(X) = \{\phi, \{a\}, U\}$. Let $\mu = \{a, b\} \notin \tau_R(X)$. Then $\mu_R(X) = \{\phi, \{b\}, \{a, b\}, U\}$.

Mic-αg-open sets are ϕ, $\{a\}$, $\{b\}$, $\{c\}$, $\{d\}$, $\{a, b\}$, $\{a, c\}$, $\{a, d\}$, $\{b, c\}$, $\{b, d\}$, $\{a, b, c\}$, $\{a, b, d\}$, $\{a, c, d\}$, U.

Here $(U, \tau_R(X), \mu_R(X))$ is a Mic-αg-T_1 space.

Definition 4.12. A space $(U, \tau_R(X), \mu_R(X))$ is said to be Micro-αg-T_2 (briefly Mic-αg-T_2) if for each pair of distinct points x, y in U, there exist a pair of disjoint Mic-αg-open sets L and M containing x and y respectively.

Example 4.13. Let $U = \{a, b, c, d\}, U/R = \{\{a\}, \{b\}, \{c, d\}\}$. Let $X = \{a\} \subseteq U$. Then $\tau_R(X) = \{\phi, \{a\}, U\}$. Let $\mu = \{d\} \notin \tau_R(X)$. Then $\mu_R(X) = \{\phi, \{a\}, \{b\}, \{a, d\}, U\}$.

Mic-αg-open sets are ϕ, $\{a\}$, $\{b\}$, $\{c\}$, $\{d\}$, $\{a, b\}$, $\{a, c\}$, $\{a, d\}$, $\{b, d\}$, $\{c, d\}$, $\{a, b, d\}$, $\{a, c, d\}$, U.

Here $(U, \tau_R(X), \mu_R(X))$ is a Mic-αg-T_2 space.

Theorem 4.14. *A space $(U, \tau_R(X), \mu_R(X))$ is Mic-αg-T_0 if and only if for every pair of distinct points $x, y \in U$, Mic-αg-cl$(\{x\}) \neq$ Mic-αg-cl$(\{y\})$.*

Proof. (Necessity) Let $(U, \tau_R(X), \mu_R(X))$ be a Mic-αg-T_0 space and $x \neq y$ belong to U. By the definition of Mic-αg-T_0, there exists a Mic-αg-open set L such that $x \in L$ and $y \notin L$. Then $y \in U \setminus L$ and $U \setminus L$ is Mic-αg-closed. Thus *Mic*-αgcl$(\{y\}) \neq$ *Mic*-αgcl$(\{U \setminus L\}) = U \setminus L$. That is, *Mic*-$\alpha$gcl$(\{y\}) = U \setminus L$. Now $x \in$ *Mic*-αgcl$(\{x\})$ but $x \notin U \setminus L$ implies $x \notin$ *Mic*-αgcl$(\{y\})$. Therefore *Mic*-αgcl$(\{x\}) \neq$ *Mic*-αgcl$(\{y\})$.

(Sufficiency) Suppose that $x, y \in U, x \neq y$ and *Mic*-αg-cl$(\{x\}) \neq$

Mic-αg-cl$(\{y\})$. Then there exists a point z of U such that $z \in Mic$-αg-cl$(\{x\})$ and $z \notin Mic$-αgcl$(\{y\})$. Suppose $x \in Mic$-αgcl$(\{y\})$ then Mic-$\alpha gcl(\{x\}) \subseteq Mic$-$\alpha$gcl$(\{y\})$ implies Mic-αgcl$(\{x\}) \subseteq Mic$-αgcl$(\{y\})$ and hence $z \in Mic$-αgcl$(\{y\})$, which is a contradiction. Hence $x \notin Mic$-αgcl$(\{y\})$ implies $U \setminus Mic$-αgcl$(\{y\}) = L$ (say) which is a Mic-αg-open set containing x but not y. Hence $(U, \tau_R(X), \mu_R(X))$ is a Mic-αg-T_0 space.

Theorem 4.15. *If every singleton set in* $(U, \tau_R(X), \mu_R(X))$ *is Mic-*αg*-closed, then it is Mic-*αg-T_1.

Proof. Assume that every singleton set is Mic-αg-closed. Let $x \neq y$ in U. Then by hypothesis, we get two Mic-αg-open sets $U \setminus \{y\}$ and $U \setminus \{x\}$ containing x and y respectively. Since $U \setminus \{y\}$ and $U \setminus \{x\}$ are Mic-αg-open, $x \in U \setminus \{y\}$ and $y \in U \setminus \{x\}$ which implies $y \notin U \setminus \{y\}$ and $x \notin U \setminus \{x\}$. Therefore $(U, \tau_R(X), \mu_R(X))$ is Mic-αg-T_1.

Remark 4.16. Converse of Theorem 4.15 need not be true as seen from the following example.

Example 4.17. Let $U = \{a, b, c, d\}$, $U/R = \{\{a\}, \{b\}, \{c, d\}\}$. Let $X = \{a\} \subseteq U$. Then $\tau_R(X) = \{\phi, \{a\}, U\}$. Let $\mu = \{a, b\} \notin \tau_R(X)$. Then $\mu_R(X) = \{\phi, \{a\}, \{a, b\}, U\}$.

Mic-αg-open sets are ϕ, $\{a\}$, $\{b\}$, $\{c\}$, $\{d\}$, $\{a, b\}$, $\{a, c\}$, $\{a, d\}$, $\{b, c\}$, $\{b, d\}$, $\{a, b, c\}$, $\{a, b, d\}$, $\{a, c, d\}$, U.

Here $(U, \tau_R(X), \mu_R(X))$ is Mic-αg-T_1 but the singleton set $\{a\}$ is not Mic-αg-closed.

Theorem 4.18. *In a Micro topological space* $(U, \tau_R(X), \mu_R(X))$ *the following statements are equivalent*:

(i) *U is Mic-*αg-T_2.

(ii) *For each* $x \neq y$ *in* U, *there exists a Mic-*αg*-open set* L *containing* x *such that* $y \notin Mic$-αg-cl(L).

(iii) *For each x in* $U, \bigcap \{Mic\text{-}\alpha\text{g-cl}(L) : x \in L\} = \{x\}$.

Proof. (i) $\Rightarrow$ (ii) If U is Mic- αg-T_2. Then there exist two Mic- αg -open sets L and M containing x and y such that $L \cap M = \phi$. Which implies that $L \subseteq U \setminus M$. Hence $Mic\text{-}\alpha\text{g-cl}(\{y\}) \subseteq Mic\text{-}\alpha\text{g-cl}(\{U \setminus M\}) = U \setminus M$. Since $y \notin U \setminus M$, we have $y \notin Mic\text{-}\alpha\text{g-cl(L)}$.

(ii) $\Rightarrow$ (iii) Assume (ii). Suppose if for each x in $U, \bigcap \{Mic\text{-}\alpha\text{g-cl}(L)$ $: x \in L\} \neq \{x\}$, then there exists some $y \in \bigcap \{Mic\text{-}\alpha\text{g-cl}(L) : x \in L\}$. Consequently, $y \in Mic\text{-}\alpha\text{g-cl(L)}$ for every Mic- αg -open set L containing x, which leads to a contradiction.

(iii) $\Rightarrow$ (i) Assume that $U, \bigcap \{Mic\text{-}\alpha\text{g-cl}(L) : x \in L\} = \{x\}$, for each x in U. Consider for each $x \neq y$ in U, $y \notin Mic\text{-}\alpha\text{g-cl(L)}$ for every Mic- αg -open set L containing x. This implies that $L \cap M = \phi$ for some Mic- αg -open set M containing y. Hence U is Mic- αg-T_2.

Proposition 4.19. *Every Mic-* αg-T_1 *space is a Mic-* αg-T_0 *space*.

Proof. Obvious.

Proposition 4.20. Every Mic- αg-T_2 space is a Mic- αg-T_1 space.

Proof. Let $(U, \tau_R(X), \mu_R(X))$ be Mic- αg-T_2 space. Then by definition, for $x \neq y$, there exist two Mic- αg -open sets L and M such that $x \in L$ and $y \in M$ and $L \cap M = \phi$ which implies $x \notin M$ and $y \notin L$. Hence the space $(U, \tau_R(X), \mu_R(X))$ is Mic- αg-T_1.

Proposition 4.21. *Let* $(U, \tau_R(X), \mu_R(X))$ *be a Mic-* αg-T_i *space, then it is a Mic-* αg-D_i *space, for* $i = 0, 1, 2$.

Proof. Follows from the fact that, every Mic- αg -open set $A \neq U$ is Mic-$D\alpha g$ set.

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