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PAIRED DOMINATION IN OXIDE NETWORKS

J. ANITHA

Department of Mathematics
Arulmigu Kapleeshwarar
College of Arts and Science
Chennai-99, India
E-mail: anithaharish78@gmail.com

Abstract

A set S of vertices in a graph G is called a dominating set of G if every vertex in $V(G)\setminus S$ is adjacent to some vertex in S. A dominating set S with the additional property that the subgraph induced by S contains a perfect matching is called a paired-dominating set. The domination number and paired domination number of G are the minimum cardinality of a dominating set and the minimum cardinality of a paired dominating set respectively of G. In this paper, we obtain paired domination number for oxide networks.

1. Introduction

The open v neighborhood, denoted as $NG(v)$, for $v \in V(G)$, is the set of vertices adjacent to v, and the closed v neighborhood, denoted by $NG[v]$, is $NG(v) \cup \{v\}$. The open neighborhood of S is defined as $NG(S)$ $= \bigcup v \in SNG(v)$ for a set of $S \subseteq V(G)$ and the closed neighborhood of S is defined as $NG[S] = NG(S) \cup S$. We denote brevity for $N_G(S)$ by $N(S)$ and $N_G[S]$ by $N[S]$.

A v vertex is said to dominate a u vertex in a graph G if $u = v$ or if u and v are neighbors in G. A dominant set of G is a subset of S of G vertices, such that at least one vertex in S dominates any vertex outside of S. The G domination number, denoted by $\gamma(G)$, is the minimum cardinality of the G.

If no two edges in it are adjacent to G, that is, an independent edge set is

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a set of edges without common vertices, the set of edges in the graph G is independent. A matching G in a graph is a group of independent G edges. The matching graph number G, denoted $\alpha(G)$, is the maximum matching cardinality of G. A perfect matching of M is a matching of any vertex of M. A graph G paired-dominant set is a S dominant set of G, with the additional property that the $G[S]$ subgraph induced by S contains a perfect M match (not necessarily induced). The paired-domination number of G, denoted by $\gamma pr(G)$, is the minimum paired-dominant cardinality set in G.

Paired-domination was implemented as a model for assigning backups to guards for security purposes by Haynes and Slater [2] and is shown to be *NP*-complete. For any graph G of the order n, $2 \leq \gamma pr(G) \leq n$, the lower bound and upper bound on the paired dominance number were obtained from [2]. In the same article, $\gamma pr(G)$ well investigated that $\gamma(G) \leq \gamma_t(G) \leq \gamma_{pr}(G)$ [2]. Bresar et al. have shown in 2007 that the problem of finding the paired dominance number in graph [3] for Cartesian products is the problem. Chen et al. discussed cubic graphs [4] in 2007. For interval and circular-arc graphs [5] and so on.

2. Main Results

In this section, we determine the paired domination number for oxide networks.

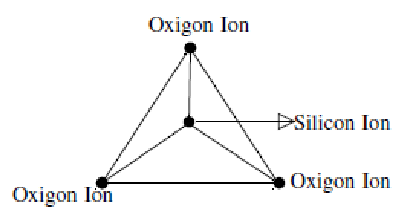

Figure 1. SiO_4 Tetrahedra.

2.1. Chain Oxide. Silicates are by far the largest, most interesting and the most complex mineral type. The SiO_4 tetrahedron is the fundamental chemical unit of silicates. A silicate sheet is a ring of tetrahedrons that, in a two-dimensional plane that creates a sheet-like structure, are connected by mutual oxygen nodes to other rings. Silicates are obtained through the fusion of metal oxides or carbonates Uh, with sand. Essentially, all the silicates contain tetrahedral of SiO_4. In chemistry, the SiO_4 tetrahedron's corner vertices reflect oxygen ions and the silicon ion is represented by the middle vertex. We refer to the corner vertices as oxygen nodes in graph theory and the middle vertex as the silicon node [6]. When we detach all the silicon nodes from a SiO_4 tetrahedron silicate, we get a Oxide networks. See Figure 1.

A chain oxide of dimension n, denoted by COX_n, is obtained by arranging n, K_3 linearly. The number of vertices of COX_n, $n \geq 1$, is $2n+1$ and the number of edges is $3n$ [6].

A cyclic oxide of dimension n, denoted by $CCOX_n$, is obtained by connecting n, K_3 into a cyclic structure. The numbers of vertices and edges of $CCOX_n$, $n \geq 3$, are $2n$ and $3n$ respectively [6].

2.2. Paired Domination in Oxide Network

Definition 1 [6]**.** Consider a $HC(n)$ honeycomb network with a n dimension. Place the ions of silicon on all the $HC(n)$ vertices. Each edge of $HC(n)$ is subdivided once. Oxygen ions are positioned on the new vertices. At the 2-degree silicon ions of $HC(n)$, add $6n$ new pendant edges, one each and put oxygen ions at the pending vertices. The three adjacent oxygen ions bind with each silicon ion and form a tetrahedron. A silicate network of dimension n, denoted $SL(n)$, is the resulting network. We get a new network when we remove all the silicon nodes from a silicate network, which we will call an oxide network. $OX(n)$ denotes a r-dimensional oxide network. For $OX(n)$, the number of vertices is $9n^2 + 3n$. See the 2(b).

A critical subgraph of H of G is defined by the following lemma in the sense that H contains at least one of two vertices of any paired dominating set.

Lemma 1. *Let G be a graph and H as shown in Figure* 4(a) *be a subgraph G. Then H is a critical subgraph of G.*

Proof. Let S be a paired dominating set of G. We are claiming that $|S| = 2$. Assume that H does not include any of the S members. Let

$V(H) = \{u_1, u_2, u_3, u_4, u_5, x_1, x_2\}$ as shown in Figure 4(a). We note that H is connected to the rest of the graph only through the $u_i s$, $1 \le i \le 5$. Now we have the following two cases:

1. $N(S) \cap V(H) \neq \phi$.

2. $N(S) \cap V(H) = \phi$.

In both these cases, the $x_j s$, $1 \le j \le 2$, are not dominated, a contradiction. □

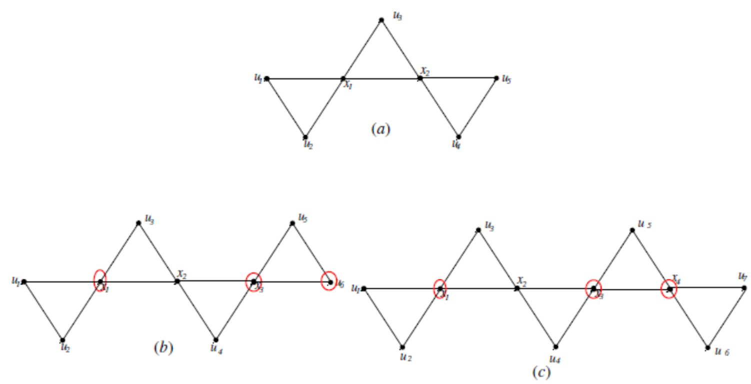

Figure 2. Circled vertices indicates paired dominating set of (a) Critical subgraph H induced by G (b) Chain Silicate Network COX_3 (c) Chain Silicate Network COX_4 (d) Chain Silicate Network COX_5.

Lemma 2. *Let G be a chain oxide network COX_n; $n \ge 3$ or cyclic oxide network $CCOX_{nn}$, $n \ge 3$. Then*

$$\gamma_{pr}(G) = \begin{cases} \dfrac{2n}{3} & \text{if } n \equiv 0 (\text{mod } 3) \\ \dfrac{2n}{3} + 2 & \text{if } n \equiv 1 (\text{mod } 3) \\ \dfrac{2n}{3} + 2 & \text{if } n \equiv 2 (\text{mod } 3). \end{cases}$$

Proof. Let S be a paired dominating set of G.

Case (i). $n \equiv 0 (\text{mod } 3)$

$(\ge:)$ When $n = 3$ the result is trivial by Lemma 2.

In COX_n, there are $\frac{n}{3}$ edge disjoint copies of H. By Lemma 1, H contains two vertices from a paired dominating set. Therefore, $|S| \geq \frac{2n}{3}$.

($\leq$:) Select $\frac{n}{3}$ alternate edges in P^*, which are at distance 2 apart on P^* as shown in Figure 2(b). Selected edges from S dominates $\frac{n}{3}$ edge disjoint copies of $V(H)$ in COX_n. The subgraph induced by the set S is perfect matching. Moreover, it is possible to take pairwise $\frac{2n}{3}$ adjacent vertices which can dominate the vertices of COX_n. Hence S is a minimum paired dominating set of CS_n. Therefore, $|S| = \frac{2n}{3}$. See Figure 2(a).

Case (ii). $n \equiv 1 \pmod 3$.

($\geq$:) When $n = 4$.

COX_4 contains $H \cup K_3$, where H is described in Lemma 1. We claim that $|S| = 4$. Suppose not, let $S = \{x_1, x_2, x_6\}$. See Figure 2(c). Even if, all the vertices of COX_4 are dominated by S then the induced subgraph of S does not induce a perfect matching, a contradiction. Therefore, $|S| = 4$.

In general, COX_n, contains $\frac{n}{3}$ edge disjoint copies of H and a copy of K_3. We claim that $|S| \geq \frac{2n}{3} + 2$. Suppose not, by Lemma 2, $\frac{2n}{3}$ vertices in S dominate vertices of $\frac{n}{3}$ edge disjoint copies of H and one more vertex from S dominates a copy of K_3 then the induced subgraph of S does not induce a perfect matching, a contradiction. Therefore, $\gamma_{pr}(G) \geq \frac{2n}{3} + 2$.

($\leq$:) Select $\frac{n}{3}$ alternate edges in P^*, which are at distance 2 apart on P^* in S as shown in Figure 2(b). Apart from these edges, select an edge from n^{th}

copy of $COX_{(n)}$ in S then $\frac{n}{3}$ independent edges in S dominates $\frac{n}{3}$ edge disjoint copies of $V(H)$ and one an edge in S dominates the n^{th} copy of $COX_{(n)}$. Therefore, $\gamma_{pr}(G) = \frac{2n}{3} + 2$. See Figure 2(b).

Case (iii). $n \equiv 2 (\text{mod } 3)$

($\geq$:) When $n = 5$. CS_5 contains a copy of H and a copy of twin K_3. We claim that $|S| = 4$. Suppose not, let $S = \{x_1, x_3, x_4\}$. Even if, all the vertices of COX_5 are dominated by S then the induced subgraph of S does not induce a perfect matching, a contradiction. Therefore $|S| \geq 4$. In general, we claim that $|S| \geq \frac{2n}{3} + 2$. Suppose not, COX_n contains $\frac{n}{3}$ edge disjoint copies of H and a copy of twin K_3. Even if, all the vertices of COX_n are dominated by S then the induced subgraph of S does not induce a perfect matching, a contradiction. Therefore, $\gamma_{pr}(G) \geq \frac{2n}{3} + 2$.

($\leq$:) Select $\frac{n}{3} + 1$ alternate edges in P^*, which are at a distance 2 apart on P^* in S as shown in Figure 2(b). By Lemma 1, $\frac{n}{3} + 1$ independent edges in S dominates $\frac{n}{3}$ edge disjoint copies of $V(H)$ and one more edge in S dominates the vertices of twin K_3. The subgraph induced by the set S has induce a perfect matching, a contradiction. Moreover, it is possible to take pairwise $\frac{2n}{3} + 2$ adjacent vertices which can dominates the vertices of COX_n. Hence S is a minimum paired dominating set of COX_n. Therefore, $\gamma_{pr}(G) = \frac{2n}{3} + 2$. See Figure 2(c). □

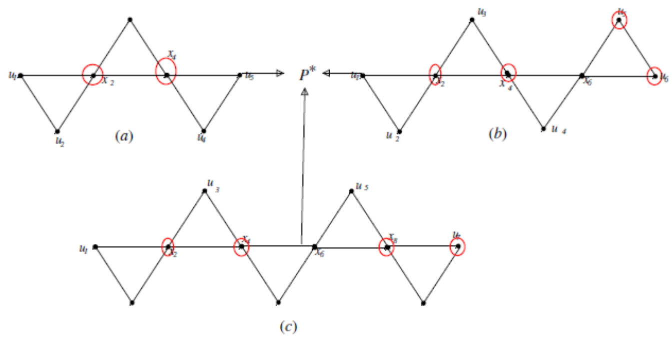

Figure 3. Circled vertices indicates paired dominating set of (a) Chain Oxide Network COX_3 (b) Chain Oxide Network COX_4 (c) Chain Oxide Network COX_5.

Theorem 3. *Let G be the oxide networks* OX_n, $n \geq 1$. *Then* $\gamma_{pr}(G) \geq 4n^2$.

Proof. In $OX(n)$, $n \geq 2$, there are $2n^2$ edge disjoint copies of $H's$ as described in Lemma 7 each of which consists of two vertices from paired dominating sets. Therefore, $\gamma_{pr}(G) \geq 4n^2$. □

Paired Domination Algorithm in Oxide Networks $OX(n)$

Input: Oxide Networks $OX(n)$, $n \geq 1$.

Algorithm: Label the vertices of oxide networks $OX(n)$, $n \geq 1$ as 1 to $9n^2 + 3n$, sequentially from left to right, row wise beginning with the first row. Let C_k^*, $1 \leq k \leq n$ denote the bounding cycle of $OX(n)$. Select $4k - 2$ alternate edges in each of bounding cycle in C_k^*, which are at distance 2 apart on C_k^* in S. See Figure 4(b).

Output: $\gamma_{pr}(OX(n)) = 4n^2$.

Proof of Correctness. Let S be a paired dominating set of $OX(n)$. In bounding cycle C_k^* of $OX(n)$, select the alternate edges which are apart from two in S as shown in Figure 4(b). The end vertices of the edges form a paired dominating set S. Let (u_1, v_1) and (u_2, v_2) be the selected edges such that $d(u_1, v_1) = d(u_2, v_2) = 2$. Let us consider the vertices a, b, c, d, e, f, g, h between the edges of (u_1, v_1) and (u_2, v_2). Now the vertices a, b, c, d, e dominated by (u_1, v_1) the vertices e, d, f, g, h dominated by (u_2, v_2). See Figure 4(b). Proceeding inductively, in each bounding cycle C_k^*, then $N[S] = V(G)$ and the subgraph induced by the set S has no isolated vertices. Moreover, it is possible to take pairwise $4n^2$ adjacent vertices which can dominates the vertices of $OX(n)$. Hence S is a minimum paired dominating of $OX(n)$.

Theorem 4. *Let G be the Oxide Networks $OX(n)$, $n \geq 1$. Then* $\gamma_{pr}(G) = 4n^2$.

3. Conclusion

In this paper, we have obtained paired domination number for oxide networks.

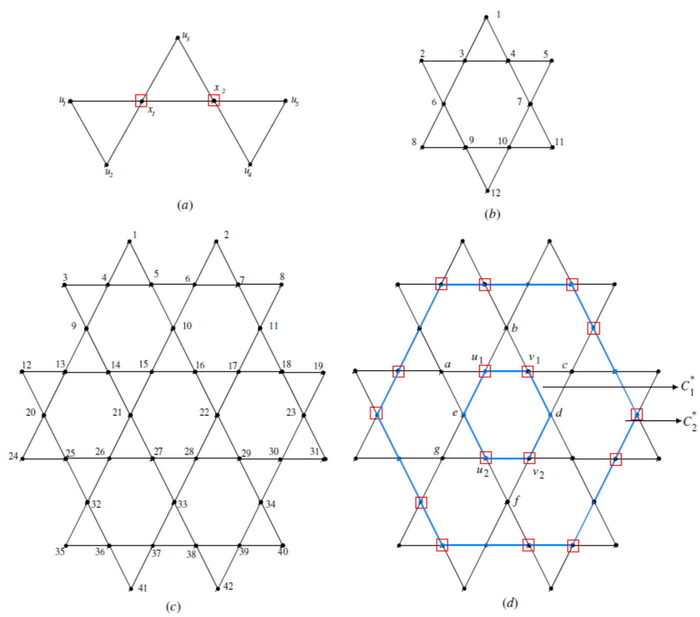

Figure 4. Cyclic Slicate CC_6.

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