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COMPUTATION OF NUMEROUS TOPOLOGICAL INDICES OF DUTCH WINDMILL GRAPH D_n^m

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Abstract

In this paper, we evaluate sum and product versions of the Forgotten index, Symmetric Degree Division index, Inverse Sum index, Balaban index, first, second, and the third kind of Revan indices along with their hyper versions for D_n^m. Finally, the Sanskruti index is also computed for the Dutch windmill graphs in terms of n and the number of copies m.

1. Introduction

The graphs we considered in this paper are simple finite and connected together. Topological indices are numerical parameters of a graph portraying its topology, which is regularly invariants of graphs. A few other topological indices have been utilized in different research, and comprehensive research has been performed on a wide extend of graph types on these indices. Inspired by this research, we talk about a few topological indices for certainly related graphs of a specific graph class, specifically the Dutch windmill graph which is an undirected and planar graph.

The Dutch windmill graph is denoted as D_n^m and the graph obtained takes m copies of the C_n with a mutual common vertex. Often the Dutch

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windmill graph is called as a Friendship graph if $n = 3$ (i.e.) D_3^m. The D_n^m Dutch windmill graph comprises $(n-1)m+1$ nodes and edges of mn.

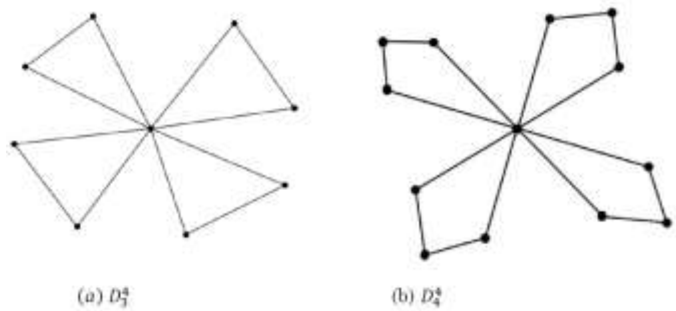

Figure 1. Dutch Windmill Graphs.

Note 1. The $E(D_n^m)$ is mn from that, the edges are partitioned with respect to the degree of end vertices in each edge as given below.

Edges of the type $E_{(d_u, d_v)}$	Number of edges
$E_{(2,2)}$	$(n-2)m$
$E_{(2m,2)}$	$2m$

Note 2. The $E(D_n^m)$ is mn from that the edges are partitioned with respect to the degree sum of neighbors of end vertices of an each edge as given below.

Edges of the type $E_{(d_u, d_v)}$	Number of edges
$E^*_{(2m+2, 2m+2)}$	m
$E^*_{(2m+2, 4m)}$	$2m$

Figure 2. Edge partitions based on Degree Sum Neighbourhood whenever $n = 3$.

Edges of the type $E_{(d_u, d_v)}$	Number of edges
$E^*_{(4,4)}$	$(n-4)m$
$E^*_{(4,2m+2)}$	$2m$
$E^*_{(2m+2,4m)}$	$2m$

Figure 3. Edge partitions based on Degree Sum Neighbourhood whenever $n > 3$.

In this paper, we have examined sum and product versions of certain topological indices of Dutch windmill graphs based on the edge partitions with respect to the degrees of end vertices and the degree sum of neighbors of end vertices of an each edge. We also followed the topological indices notations, edge partitions based on the degree of end vertices and the indices formulae from the articles [[4]-[5]].

2. Computed Topological Index Results for the Dutch Windmill Graphs

Theorem 2.1. *The forgotten index of* D_n^m *is* $F(D_n^m) = 8m(m^2 + n - 1)$.

Proof. From the note 1, we compute the forgotten index of D_n^m as

$$F(D_n^m) = |E_{(2,2)}|(2^2 + 2^2) + |E_{(2m,2)}|((2m)^2 + 2^2)$$

$$= (n-2)(8m) + 8m(1 + m^2)$$

$$= 8m(m^2 + n - 1) \quad \blacksquare$$

Note 3. The multiplicative version of $F(D_n^m)$ is $64(nm^4 + nm^2 - 2m^4 - 2m^2)$.

Theorem 2.2. *The symmetric Degree Division index of* D_n^m *is*

$$SDD(D_n^m) = 2m^2 + 2mn - 4m + 2.$$

Proof. From the note 1, we compute the Symmetric Degree Division index of D_n^m as

$$SDD(D_n^m) = |E_{(2,2)}|\left[\frac{2^2+2^2}{(2)(2)}\right] + |E_{(2m,2)}|\left[\frac{(2m)^2+2^2}{(2m)(2)}\right]$$

$$= (n-2)(2) + 2m\left[\frac{4(1+m^2)}{4m}\right] = 2m^2 + 2mn - 4m + 2 \quad \blacksquare$$

Note 4. The multiplicative version of $SDD(D_n^m)$ is $4m^3n + 4mn - 8m^3 - 8m.$

Theorem 2.3. *The Inverse sum Degree index of* D_n^m *is*

$$ISI(D_n^m) = \frac{2m^2 + m^2n + mn - 2m}{1+m}.$$

Proof. From the note 1, we compute the inverse sum degree index of D_n^m as

$$ISI(D_n^m) = |E_{(2,2)}|\left[\frac{(2)(2)}{2+2}\right] + |E_{(2m,2)}|\left[\frac{(2m)(2)}{(2m)+(2)}\right]$$

$$= (n-2)(1) + 2m\left[\frac{4m}{2m+2}\right] = \frac{2m^2 + m^2n + mn - 2m}{1+m} \quad \blacksquare$$

Note 5. The multiplicative version of $ISI(D_n^m)$ is $\dfrac{4m^3n - 8m^3}{1+m}$.

Now, we compute the three Revan indices of Dutch Windmill graphs. For that we have the $|V(D_n^m)| = (n-1)m + 1$ and $|E(D_n^m)| = mn.$ From the graph structure, the mutual vertex which is shared by all copies of C_n will have be the maximum degree $\Delta(D_n^m) = 2m,$ where m is the number of copies in D_n^m and $\delta(D_n^m) = 2.$ For calculating Revan indices we need to find $r(v).$

For $D_n^m,$ we have the edge partitions based on the edges end point degrees, the partitions are $|E_{(2,2)}|$ and $|E_{(2m,2)}|.$ If $|E_{(2,2)}|$ is considered then $r(v) = 2m$ and suppose $| E_{(2m, 2)} |$ is considered then $r(v) = 2$.

Theorem 2.4. *The first Revan index of* D_n^m *is* $R_1(D_n^m) = 4(m^2 n - m^2 + 4)$.

Proof. From the note 1, we compute the first Revan index of D_n^m as

$$R_1(D_n^m) = | E_{(2,2)} |(2m + 2m) + | E_{(2m,2)} |(2m + 2)$$

$$= (n - 2)(4m^2) + 4m(1 + m) = 4(m^2 n - m^2 + 4) \quad \blacksquare$$

Note 6. The multiplicative version of $R_1(D_n^m)$ is $16(m^4 n - 2m^4 + 2m^3)$.

Theorem 2.5. *The second Revan index of* D_n^m *is* $R_2(D_n^m) = 4(m^3 n - 2m^3 + 2m^2)$.

Proof. From the note 1, we compute the second Revan index of D_n^m as

$$R_2(D_n^m) = | E_{(2,2)} |(2m)(2m) + | E_{(2m,2)} |(2m)(2)$$

$$= (n - 2)(4m^3) + 8m^2 = 4(m^3 n - 2m^3 + 2m^2) \quad \blacksquare$$

Note 7. The multiplicative version of $R_2(D_n^m)$ is $32m^5 n - 64m^5$.

Theorem 2.6. *The third Revan index of* D_n^m *is* $R_3(D_n^m) = 4m - 4m^2$.

Proof. From the note 1, we compute the third Revan index of D_n^m as

$$R_3(D_n^m) = | E_{(2,2)} |(| 2m - 2m |) + | E_{(2m,2)} |(| 2 - 2m |)$$

$$= (2m)(| 2 - 2m |) = 4m - 4m^2 \quad \blacksquare$$

Note 8. The multiplicative version of $R_3(D_n^m)$ is 0.

Theorem 2.7. *The first hyper Revan index of* D_n^m *is*

$$HR_1(D_n^m) = 16m^3 n + 16m^2 + 8m - 24m^3.$$

Proof. From the note 1, we compute the first hyper Revan index of D_n^m as

$$HR_1(D_n^m) = |E_{(2,2)}|(2m+2m)^2 + |E_{(2m,2)}|(2m+2)^2$$

$$= (n-2)16m^3 + 8m(1+m)^2$$

$$= 16m^3n + 16m^2 + 8m - 24m^3$$ ■

Note 9. The multiplicative version of $HR_1(D_n^m)$ is

$$HR_1(D_n^m) = 128m^3(m^2n + 2mn + n - 4m - 2m^2 - 2).$$

Theorem 2.8. *The second hyper Revan index of* D_n^m *as*

$$HR_2(D_n^m) = 16m^5n + 32m^3 - 32m^5.$$

Proof. From the note 1, we compute the second hyper Revan index of D_n^m as

$$HR_2(D_n^m) = |E_{(2,2)}|((2m)(2m))^2 + |E_{(2m,2)}|((2m)2)^2$$

$$= (n-2)16m^5 + 32m^3$$

$$= 16m^5n + 32m^3 - 32m^5$$ ■

Note 10. The multiplicative version of $HR_2(D_n^m)$ is $2^9m^8(n-2)$.

Theorem 2.9. *The third hyper Revan index of* D_n^m *is*

$$HR_3(D_n^m) = 8m^3 - 16m^2 + 8m.$$

Proof. From the note 1, we compute the second hyper Revan index of D_n^m as

$$HR_3(D_n^m) = |E_{(2,2)}|(|2m-2m|)^2 + |E_{(2m,2)}|(|2-2m|)^2$$

$$= 2m(4(1-m)^2) = 8m^3 - 16m^2 + 8m$$ ■

Note 11. The multiplicative version of $HR_3(D_n^m)$ is 0.

Theorem 2.10. *The Balaban index of* D_n^m *is*

$$J(D_n^m) = \frac{m^2 n}{m+1}\left(\frac{n-2}{2} + \frac{1}{\sqrt{m}}\right).$$

Proof. From the note 1, we compute the Balaban index of D_n^m. Here we have for D_n^m, $|V(D_n^m)| = (n-1)m+1$ and $|E(D_n^m)| = mn$.

Therefore, we have

$$J(D_n^m) = \frac{mn}{mn - ((n-1)m+1)+2} \sum_{uv \in E} \frac{1}{\sqrt{d_u d_v}}$$

$$J(D_n^m) = \frac{mn}{m+1} | E_{(2,2)} | \left(\frac{1}{\sqrt{(2)(2)}}\right) + | E_{(2m,2)} | \left(\frac{1}{\sqrt{(2)(2m)}}\right)$$

$$= \frac{m^2 n}{m+1}\left(\frac{n-2}{2} + \frac{1}{\sqrt{m}}\right) \qquad \blacksquare$$

Note 12. The multiplicative version of $J(D_n^m)$ is $\dfrac{\sqrt{m}(mn^2 - 2mn)}{2m+2}$.

Theorem 2.11. *The Sanskruti index of* D_n^m *is*

$$S(D_n^m) = \begin{cases} \dfrac{8m(1+m)^3}{27(2m+1)^3}(155m^3 + 177m^2 + 177m + 43) & \textit{for } n = 3, \\ 128m\left(\dfrac{m^4 + 32m^3 + 90m^2 + 4mn + 72m + 8n - 3}{27(m+2)}\right) & \textit{for } n > 3. \end{cases}$$

Proof. From the note2, we compute the Sanskruti index D_n^m. We have two cases depending on the vertices C_n.

Case 1. For $n = 3$,

$S(D_n^m) =$

$$| E_{(2m+2, 2m+2)} | \left(\frac{(2m+2)(2m+2)}{2(2m+2)-2} \right)^3 + | E_{(2m+2, 4m)} | \left(\frac{(2m+2)(4m)}{2m+2+4m-2} \right)^3$$

$$= m\left(\frac{(2m+2)^2}{4m+2} \right)^3 + 2m\left(\frac{4m(2m+2)}{6m} \right)^3$$

$$= m\left(\frac{64(1+m)^6}{2^3(2m+1)^3} \right) + \frac{8^3 m^3 (m+1)^3}{108m^2}$$

$$= \left(\frac{8m(1+m)^6}{(2m+1)^3} \right) + \frac{128m(m+1)^3}{27}$$

$$= 8m(1+m)^3 \left(\frac{(1+m)^3}{(2m+1)^3} + \frac{16}{27} \right)$$

$$= 8m(1+m)^3 \left(\frac{27(m^3+3m^2+3m+1)+16(8m^3+6m^2+6m+1)}{27(2m+1)^3} \right)$$

$$= \frac{8m(1+m)^3}{27(2m+1)^3}(155m^3+177m^2+177m+43).$$

Case 2. For $n > 3$.

$$S(D_n^m) = | E_{(4, 4)} | \left(\frac{(4)(4)}{4+4-2} \right)^3 + | E_{(4, 2m+2)} | \left(\frac{(4)(2m+2)}{4+(2m+2)-2} \right)^3$$

$$+ | E_{(2m+2, 4m)} | \left(\frac{(4)(2m+2)}{(2m+2)+4m-2} \right)^3$$

$$= (n-4)m\left(\frac{8}{3} \right)^3 + (2m)\left(\frac{4(1+m)}{2+m} \right)^3 + (2m)\left(\frac{4(1+m)}{3} \right)^3$$

$$= (4^3 m)\left((n-4)\left(\frac{8}{27} \right) + 2\left(\frac{1+m}{2+m} \right)^3 + 2\,\frac{(1+m)^3}{27} \right)$$

$$= (64m)\left(\frac{8n-32+2(1+m)^3}{27} + \frac{2(1+m)^3}{m+2} \right)$$

$$= 128m\left(\frac{m^4 + 32m^3 + 90m^2 + 4mn + 72m + 8n - 3}{27(m + 2)}\right) \blacksquare$$

Result 2.1. The multiplicative version of $S(D_n^m)$ when $n = 3$ is

$$S(D_n^m) =$$

$$\left(|E_{(2m+2, 2m+2)}|\left(\frac{(2m+2)(2m+2)}{2(2m+2)-2}\right)^3\right) \cdot \left(|E_{(2m+2, 4m)}|\left(\frac{(2m+2)(4m)}{2m+2+4m-2}\right)^3\right)$$

$$= \left(\frac{8m(1+m)^6}{(2m+1)^3}\right)\left(\frac{128m(m+1)^3}{27}\right) = \frac{1024m^2(1+m)^9}{27(2m+1)^3} = \left(\frac{2^{10}}{3^3}\right)\left(\frac{m^2(1+m)^9}{(2m+1)^3}\right).$$

Result 2.2. The multiplicative version of $S(D_n^m)$ when $n > 3$ is

$$S(D_n^m) = |E_{(4, 4)}|\left(\frac{(4)(4)}{4+4-2}\right)^3 \cdot |E_{(4, 2m+2)}|\left(\frac{(4)(2m+2)}{4+(2m+2)-2}\right)^3 \cdot$$

$$|E_{(2m+2, 4m)}|\left(\frac{(4)(2m+2)}{(2m+2)+4m-2}\right)^3$$

$$= (n-4)m\left(\frac{8}{3}\right)^3 \cdot (2m)\left(\frac{4(1+m)}{2+m}\right)^3 \cdot (2m)\left(\frac{4(1+m)}{3}\right)^3 = (n-4)\left(\frac{2^{21}(1+m)^6}{3^6(2+m)^3}\right).$$

3. Conclusion

A few topological indices of the Dutch windmill graphs are discussed in this paper. Sum and product versions of the forgotten index, Symmetric Degree Division index, Inverse Sum index, first, second and third kind, and its hyper versions of Revan indices, Balaban index are evaluated. Finally, the Sanskruti index is also computed for the Dutch windmill graphs.

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