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RAYLEIGH WAVE'S PROPAGATION UNDER THE INFLUENCE OF ROTATION, MAGNETIC FIELD AND INITIAL STRESS IN THERMOELASTIC HALF SPACE WITH IMPEDANCE BOUNDARY CONDITIONS

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Abstract

The governing equation of generalized thermo-elasticity under the effect of rotation parameter, initial stress parameter and magnetic flux parameter is considered including impedance boundary conditions (IBC). The equation is obtained for surface wave solution satisfying the radiation condition. To obtain the secular equation for Rayleigh wave the solution so obtained also satisfied impedance boundary condition (IBC). To observe the effect of impedance parameter and various others parameter relevant material parameter are used. Effect of magnetic field, initial stress, frequency, rotation and impedance boundary condition (IBC) on the velocity of transmission for Rayleigh-*S*-wave (*R*-*S*-*W*) is depicted graphically.

1. Introduction

The classical dynamical theory of coupled thermo-elasticity was established by Biot [1], which was broaden to generalized thermo-elasticity by Lord and Shulman [2] and Green and Lindsay [3] which introduce hyperbolic

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equations which define heat in the form of wave. The phenomenon of wave propagation has vast applications in the field of engineering, science and technology, oil extraction, seismology, minerals extraction as well as geophysics. The maximum destruction on earth during the earthquake is caused due to Rayleigh wave as the rate at which the energy attenuates is very slow as compare to the body waves. The study of propagation of Rayleigh-S-wave through thermo-elastic material has various applications in science and technology. The transmission of Rayleigh-S-waves over the exterior of an elastic solid was first investigated by Lord Rayleigh [4]. Many researchers have studies the Rayleigh waves, as many problems can be solved using secular equation thus obtained and it also helps to estimate whether the wave velocity is dependent on the material parameter and vice versa. Using the theory of linear thermo-elasticity Lockett [5] studies Rayleigh wave propagation in semi-infinite solid which was later reconsidered. Chadwick and Windle [6] also studied Rayleigh wave for insulated and isothermal boundaries. Impedance boundary conditions are extensively used in physics. It is the linear combination of unknown function along with derivative is prescribed on the boundary, although Rayleigh wave with IBC is important in science but very few work is done in this direction. Godoy [7] examined the existence of Rayleigh-S-waves for elastic half-space in presence of IBC. Vinh and Hue [8, 9] also studied. Rayleigh waves in presence of IBC for anisotropic solids as well as in incompressible anisotropic half-spaces. B. Singh [10] also considered Rayleigh wave through thermoelastic half space in presence of IBC. In this work the secular equation of Rayleigh-S-wave under the effects of magnetic flux parameter, rotation parameter, and initial stress in thermo-elastic half space in the presence of impedance Boundary Conditions (IBC) is calculated.

2. Governing Equations

(i) Constitutive Equations

$$\sigma_{ij} = 2\mu e_{ij} + \lambda(e_{kk} - \beta_1 T)\delta_{ij} \tag{1}$$

$$e_{ij} = \frac{(u_{j,i} + u_{i,j})}{2} \tag{2}$$

(ii) Equation of Motion:

$$(\mu - p/2)u_{i,jj} + (\lambda + \mu + p/2)u_{j,ij} - \beta_1 T_i = [(\ddot{u}_{ii} + \bar{\Omega} \times \bar{\Omega} \times u_i)_i + (2\bar{\Omega} \times \dot{\bar{u}})_i]\rho \tag{3}$$

(iii) Heat Equation:

$$KT_{ii} = \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}\right)(\rho c_E T + \beta_1 T_0 e_{ii}). \tag{4}$$

Applying Helmholtz's equations

$$u_1 = \frac{\partial \psi}{\partial y} + \frac{\partial \phi}{\partial x},\ u_2 = \frac{\partial \phi}{\partial y} - \frac{\partial \psi}{\partial x}. \tag{5}$$

In $x - y$ plane, these equations were deduced.

$$\left(\frac{\lambda + 2\mu + \mu_e {H_0}^2}{\rho}\right)(\phi_{11} + \phi_{22}) - \frac{\beta_1 T}{\rho} = \ddot{\phi} - \Omega^2 \phi - 2\Omega \frac{\partial \psi}{\partial t} \tag{6}$$

$$\left(\frac{\mu - p/2}{\rho}\right)(\psi_{11} + \psi_{22}) = \frac{\partial^2 \psi}{\partial t^2} - \Omega^2 \psi + 2\Omega \frac{\partial \phi}{\partial t} \tag{7}$$

$$K(T_{11} + T_{22}) = \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}\right)(\rho c_E T + \beta_1 T_0 (\phi_{11} + \phi_{22})) \tag{8}$$

3. Nomenclature

$\bar{\lambda}, \bar{\mu}$: Lame's Constant

$T = \theta - T_0$: minute increment in temperature

T_0 : Uniform temperature of the medium such that $\left|\frac{T}{T_0}\right| << 1$.

ρ : Density

q_i : conduction vector

K : thermal conductivity

α : Expansion coefficient (thermal)

α_{ij} : stress tensor components

u_i : displacements vector components

e_{ij} : strain tensor components

w_{ij} : small rotation tensor components

δ_{ij} : Kronecker delta

S : entropy per unit mass

p : initial pressure

H_0 : Magnetic field vector

h : perturbed magnetic flux over H_0

μ_e : magnetic permeability

4. Problem Formulation and Solution

Using thermoelastic half-space which occupies the space i.e. $y \geq 0$ in the considered configuration having boundary $y = 0$ and Rayleigh-S-wave transmission along x-axis having wave speed and wave number. Using the solution in xy-plane of equations (6)-(8).

$$[\phi, T, \psi] = [\bar{\phi}(y), \bar{T}(y), \bar{\psi}(y)]\, e^{i\eta(x-\chi t)} \tag{9}$$

where χ denotes phase speed, η denotes wave number.

With the help of equation (9), the equations (6)-(8) becomes

$$D^6 + A_1 D^4 + B_1 D^2 + C_1 = 0 \tag{10}$$

$$A_1 = 3\eta^2 - B(1 + v^2) - \frac{\eta^2\chi^2(\epsilon + 1)}{K_1}$$

$$B_1 = \frac{B\chi^2\eta^2(v^2(\epsilon + 1) + (1 + v^2))}{K_1} + 3\eta^2 + (B^2 - C^2)v^2 - 2\eta^2 B(1 + v^2)n$$

$$C_1 = \frac{-\eta^6\chi^6(\epsilon+1)+\eta^4\chi^2 B(\epsilon+1)v^2+\eta^2\chi^2 v^2(B^2+C^2)}{K_1}$$

$$-\eta^4 B(v^2+1)-\eta^2 v^2(C^2-B^2)+\eta^6.$$

Where

$$c_1^2 = \frac{\lambda+2\mu+\mu_e H_0^2}{\rho_0}, \qquad c_2^2 = \frac{\mu-\frac{p}{2}}{\rho_0}$$

$$\epsilon = \frac{\beta_1^2 T_0}{\rho^2 c_1^2 c_E}, \quad K^* = \frac{K}{\rho c_E}.$$

Using following radiation conditions on $\bar{\phi}, \bar{T}$ and $\bar{\psi}$

$$\bar{\phi}(y)\to 0, \bar{T}(y)\to 0 \quad , \bar{\psi}(y)\to 0 \text{ as } y\to\infty. \tag{11}$$

The solution of equation (10) are taken in the following form

$$\phi(y) = P_i e^{-m_i z} expi(\eta x-\chi t), \tag{12}$$

$$\psi(y) = F_i P_i e^{-m_i z} expi(\eta x-\chi t) \tag{13}$$

$$T(y) = F_i^* P_i e^{-m_i z} expi(\eta x-\chi t), \tag{14}$$

where

$$F_i = \frac{2i\frac{\Omega}{\chi}}{\left(1-\frac{m_i^2}{\eta^2}\right)\frac{1}{c^2 v^2}-\left(1+\frac{\Omega^2}{\chi^2}\right)}. \tag{15}$$

$$F_i^* = \frac{-\epsilon\chi^2[\tau_0+\frac{i}{\chi}][-\eta^2+m_i^2]}{K^*(-\eta^2+m_i^2)(i\chi+\tau_0\chi^2)}. \tag{16}$$

The IBC at the surface when $y=0$

$$\sigma_{12}+\bar{\sigma}_{12}+wZ_1u_1 = 0, \qquad \sigma_{22}+\bar{\sigma}_{22}+wZ_2u_2 = -p, \qquad \frac{\partial T}{\partial y}+hT = 0, \tag{17}$$

where $w = kc$ denotes frequency of wave, Z_1, Z_2 denotes impedance parameters. $h \to 0$ belongs to thermally insulated surface and $h \to \infty$ corresponds with isothermal surface. The solution given by equation (12)-(14) satisfy the radiation condition (17). Hence the Rayleigh-s-wave frequency equation is obtained.

$$m_1 F_1^* (S_2 R_3 - S_3 R_2) - m_2 F_2^* (S_1 R_3 - R_1 S_3) + m_3 F_3^* (S_1 R_2 - R_1 S_2)$$
$$= h[F_1^* (S_2 R_3 - S_3 R_2) + F_2^* (S_3 R_1 - R_3 S_1) + F_3^* (S_1 R_2 - R_1 S_2)] \quad (18)$$

$$S_i = (2 + p_1) i m_i \eta + (\eta^2 (1 + p_1) + m_i^2) F_i + \frac{w Z_1 (i\eta - m_i)}{\rho_0}$$

$$r_i = -\eta^2 \left[1 - \frac{2}{v^2} - p_2\right] + m_i^2 - F_i^* + \left[\frac{2}{v^2} + p_2\right] i\eta m_i F_i - \frac{w Z_2 (m_i + i\eta F_i)}{\rho_0 c_1^2}$$

where $(i = 1, 2, 3)$.

5. Particular Cases

(i) For Thermally insulated Surface i.e. when $h \to 0$

$$m_1 F_1^* (S_2 R_3 - S_3 R_2) - m_2 F_2^* (S_1 R_3 - R_1 S_3) + m_3 F_3^* (S_1 R_2 - R_1 S_2) = 0 \quad (19)$$

(ii) For Isothermal Surface i.e. when $h \to \inf$

$$[F_1^* (S_2 R_3 - S_3 R_2) + F_2^* (S_3 R_1 - R_3 S_1) + F_3^* (S_1 R_2 - R_1 S_2)] \quad (20)$$

6. Special Cases

(a) In absence of rotation i.e. $\Omega = 0,$ initial stress i.e. $p = 0$ & magnetic field $H_o = 0$ the frequency equation (18) reduces to frequency equation for Rayleigh-S-wave as formulated in B. Singh [10]

(b) In the absence of IBC $(Z_1 = Z_2 = 0),$ rotation i.e. $\Omega = 0,$ initial stress i.e. $p = 0$ and magnetic field i.e. $H_o = 0$ the frequency equation (18) reduces to Rayleigh wave.

7. Numerical Result and Discussion

Frequency equation obtained for Rayleigh-S-wave is calculated using a computer program numerically.

The velocity of propagation of R-S-w so obtained is shown vis-a-vis frequency, magnetic field and rotation and is shown in Figure 1-5.

Following are the values of a suitable parameter at $T_0 = 300K$

$$E = 6.9 \times 10^{10} N.m^{-2}, \rho_0 = 2700 Kg.m^{-3}, \sigma = 0.33$$

$$K = 205.85 J.m^{-1}.s^{-1}.K^{-1}, c_v 987.9 J.Kg^{-1}.K^{-1}$$

$$\tau_o = 2.5, \omega = 0.5, \epsilon = 0.05, \mu_e = 1,$$

where σ is Poission ratio, E is Young's modulus and ζ is the initial stress parameter.

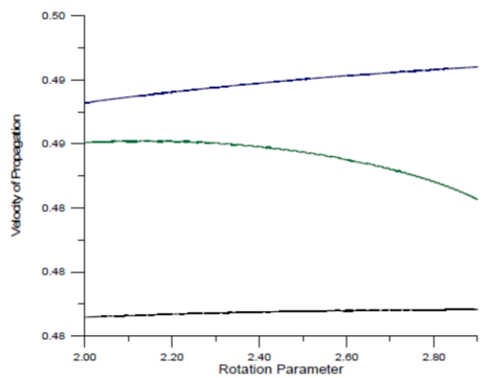

Figure 1. Variation of Velocity of propagation of R-S-W against rotation.

Figure 1. The velocity of propagation of R-S-W is depicted qualitatively vis-a-vis the rotation parameter. $2 < \Omega < 2.80$ where $Z_1 = Z_2 = 1$ are the impedance parameters at $\chi = 0.1$ and $H_0 = 10$ and when initial stress is not considered i.e. $p = 0$ the blue curve shows increase in velocity of propagation of R-S-W as the rotation parameter increases but with the small increase in the value of initial stress $p = 2,$ we can see only small increase in the velocity of propagation with respect to rotation parameter shown by black line but as

the value of initial stress increases i.e. when $p = 5$ the maximum velocity of propagation of R-S-W is 0.49056330 at $\Omega = 2.0$ after that it start decreasing with the increase of Ω parameter.

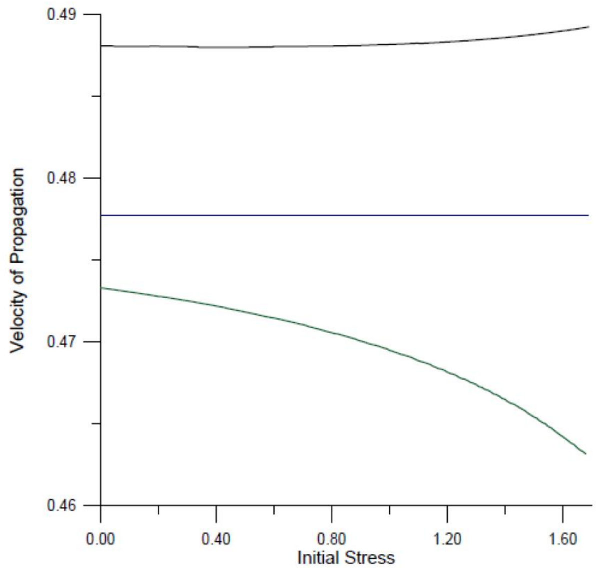

Figure 2. Variation of Velocity of propagation of R-S-W against initial stress.

Figure 2. The velocity of propagation of R-S-W is depicted qualitatively vis-a-vis initial stress $0/ < P < 1.60$, when $Z_1 = Z_2 = 1$ then the impedance parameter $\chi = 0.1$ and $H_0 = 10$ and when rotation parameter is not considered i.e. when $\Omega = 0$ the velocity of propagation becomes with the increment of initial stress as shown by the blue line, but with the slight increase in the value of rotation parameter $\Omega = 2.0$ there is a slight decrease in the value of velocity propagation initially as shown by black line but after a while it start rising due to rise in initial stress. As we increase the value of rotation parameter to $\Omega = 10$ we can see that green line shows the value of $\chi = 0.47329670$ at $p = 0$ but after that it deceases abruptly as the rotation parameter increases.

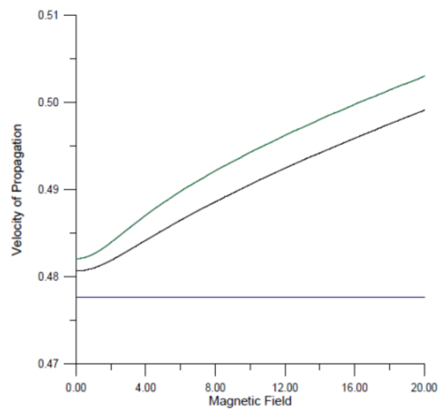

Figure 3. Variation of Velocity of propagation of R-S-W against magnetic field.

Figure 3. The velocity of propagation of R-S-W is depicted qualitatively vis-a-vis the magnetic field $0 < H_0 < 20, \chi = 0.1\ Z_1 = Z_2 = 1$ and $p = 2$ in the absence of rotation parameter i.e. $\Omega = 0$ as shown by blue line the velocity of transmission of Rayleigh wave become stable with the increase of magnetic field as shown in black line but as we increase rotation parameter there is sudden increase in value of velocity of propagation as the magnetic field increases as shown in green line.

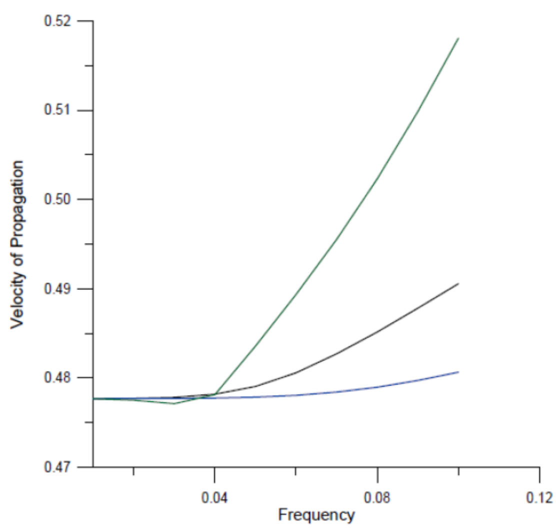

Figure 4. Variation of Velocity of propagation of R-S-W against frequency.

Figure 4. The velocity of propagation of R-S-W is depicted qualitatively vis-a-vis frequency $0 < \omega < 0.12, \Omega = 2, p = 2$ and $Z_1 = Z_2 = 1$ in the absence of magnetic field velocity of propagation slightly increases with the increase in frequency as shown by line blue. As we increase the value of $H_0 = 10,$ the velocity of propagation increases slightly as shown in Figure 4 as black line but during the application of strong field having strong magnetic character, velocity of transmission of R-S-W increases sharply as the increase of frequency parameter.

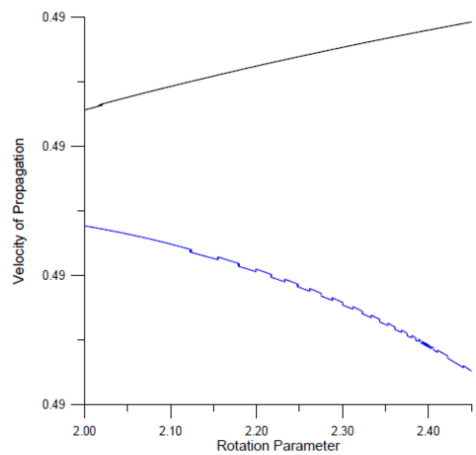

Figure 5. Variation of Velocity of propagation of R-S-W against rotation.

Figure 5. The velocity of propagation of R-S-W is depicted qualitatively vis-a-vis rotation $2 < \Omega < 2.5, p = 2$ and $Z_1 = 5, Z_2 = 0, H_0 = 10, \chi = 0.1.$ The velocity of transmission of Rayleigh wave sharply decreases with the increase of rotation parameter in the presence of Impedance parameter as shown by blue line whereas in absence of impedance parameter velocity of R-S-W will slowly increase with increase of rotation parameter as shown by black line.

8. Conclusion

The governing equation is solved using the technique of surface wave solution for generalized thermo-elasticity under the influence of rotation parameter, initial stress parameter and magnetic flux parameter and IBC. The secular equation so obtained for Rayleigh-S-Wave satisfies IBC, it is also solved numerically using the computer programming. In absence of rotation, IBC, initial stress and magnetic field the frequency equation so obtained reduces to R-S-W. The velocity of transmission of Rayleigh-S-Wave is considerably influenced by the presence of rotation, IBC, initial stress and magnetic field.

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