Advances and Applications in Mathematical Sciences
Volume 21, Issue 11, September 2022, Pages 6303-6318

SOLVING PROCEDURE FOR FUZZY TIME COST TRADE OFF PROBLEMS USING MULTIPLE ATTRIBUTE GROUP DECISION MAKING

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Abstract

This research article states the Mathematical structure and design for solving a time cost trade-off problems by using fuzzy linear programming problem and Multiple Attribute Group Decision Making (MAGDM) problems. A linear numerical illustration for project time cost trade-off problem is evolved through this work which gives the optimum solution. The activities presented in the network take decision matrices form which are solved by using aggregation operators available in the literature. Many aggregated operators are evolved in the decision process and the best alternative which comprises the normal cost, normal duration, crash cost and crash duration is selected for each activity and then the optimal solution of the network is obtained. The proposed method is explored through numerical illustration.

1. Introduction

The time cost trade-off project linking the project estimated cost and the project fulfilment duration and the uncertainty of the habitat issues that are considerable for all actual life project decision builders. In the previous literature there are many approaches put forward over the years to find the minimal cost with optimum duration [1, 6, 7, 8, 15]. Zadeh [20] introduced the concept of fuzzy sets and today almost all research areas have depended on

2020 Mathematics Subject Classification: 90B10, 90-08.

Keywords: Fuzzy sets, Fuzzy Numbers, Fuzzy Time Cost Trade off Problems, Fuzzy Linear programming problem Project network, Decomposition technique, Aggregation of m-LPPs, MAGDM problems, OWA operators.

Received January 12, 2022; Accepted March 5, 2022

the development of the same. Ghazanfari et al. [6] proposed the innovative optimal method for fuzzy time cost trade off problem using goal programming problem. Evangeline Jebaseeli et al. [7, 8] presented a new way out for time cost trade off problems with time and cost are fuzzy variables in the same period. Pandian and Jayalakshmi [9] give a brand new method meant as decomposition method which solves integer linear programming problems by using triangular fuzzy variables. Shakeela and Ganesan [15] give out the fully fuzzy Time Cost Trade off problem. Decision making problems are broadly grow in all real life circumstances. Multiple Attribute Group Decision Making (MAGDM) problems have gained much importance in the recent days. An extensive work has been done by researchers in MAGDM problems and the aggregations done for those decision problems [10-14, 16-19]. In this work, the activities involved in the Time-Cost Trade off problems are represented in the form of decision matrices which has to be aggregated against some conflicting criteria. After successful aggregation of the alternatives, the activities are employed in the Time-Cost Trade off problem and an optimal solution is obtained for the same. This work pioneers with coupling of the concept of MAGDM and Time-Cost Trade off problems. The proposed algorithm in this work is an effective method of reducing the decision matrices into normalised activities for the Time-Cost Trade off problems.

2. Preliminaries

Definition 1. The characteristic function μ_A in a crisp set $A \subseteq S$ assigns a value either 0 or 1 for each member in S. The function is generalised to a function $\mu_{\widetilde{A}}$ such that the value assigned with the element of S lies within a specified range i.e. $\mu_{\widetilde{A}} : S \to [0, 1]$. The assigned values $\mu_{\widetilde{A}}(s)$ for each $s \in S$ denote the membership grade of the element in the set A. The set $\widetilde{A} = \{A, \mu_A(x) : x \in X\}$ is called Fuzzy Set.

Definition 2. Triangular fuzzy number is a fuzzy number represented with three points as follows:

$\widetilde{A} = (g_1, g_2, g_3)$ This representation is interpreted as membership functions:

We use $F(R)$ to denote the set of all triangular fuzzy numbers.

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & \text{if } x < g_1 \text{ and } g_3 > x \\ \dfrac{x - g_1}{g_2 - g_1} & \text{if } g_1 \leq x < g_2 \\ \dfrac{g_3 - x}{g_3 - g_2} & g_2 \leq x \leq g_3 \end{cases}$$

Definition 3. Let (g_1, g_2, g_3) and (h_1, h_2, h_3) be two triangular fuzzy numbers. Then

$$(g_1, g_2, g_3) \oplus (h_1, h_2, h_3) = (g_1 + h_3, g_2 + h_2, g_3 + h_1)$$

$$(g_1, g_2, g_3) - (h_1, h_2, h_3) = (g_1 - h_3, g_2 - h_2, g_3 - h_1)$$

$$(g_1, g_2, g_3) = (cg_1, cg_2, cg_3), \text{ for } c \geq 0.$$

$$c(g_1, g_2, g_3) = (cg_3, cg_2, cg_1), \text{ for } c < 0.$$

$$\frac{(g_1, g_2, g_3)}{(h_1, h_2, h_3)} = \left(\frac{g_1}{h_3}, \frac{g_2}{h_2}, \frac{g_3}{h_1}\right)$$

Definition 4. Let $F(R)$ represents the set of triangular fuzzy numbers. Define a ranking function $\mathcal{R} : F(R) \to R$ maps triangular fuzzy numbers into R. Let $\tilde{A} = (f, g, h)$ be a triangular fuzzy number, and then Graded Mean Integration Representation (GMIR) method to defuzzify the number is noted as $R(\tilde{A}) = \left(\frac{f + 2g + h}{4}\right)$.

Definition 5. A fuzzy project network is an acyclic digraph, where the points represent events and the oriented lines represents activities. Let us represent the fuzzy project network by $\tilde{P} = \langle N, L, \tilde{O} \rangle$. Let $N = \{n_1, n_2, \ldots, n_m\}$ be the set of all points (events), n_m and n_1 are the head and tail events of the project. Let $L \subset N \times N$ be the set of all oriented lines $L = \{l_{ij} = (n_i, n_j)/n_i, n_j \in N\}$, which denote the activities to be represented in the project. A critical path is a longest path between initial event n_1 and terminal event n_m and an activity l_{ij} on a critical path is known as critical activity.

Definition 6. Linear programming problem is one among the most habitually applied operations research technique by assuming that all variables and parameters are real numbers. But in real life circumstance we do not have proper data. So, the fuzzy variables and fuzzy numbers are used in Linear programming problem. The standard form fully fuzzy linear programming problems with n fuzzy variables and m fuzzy constants are given below:

Maximimize or Minimize $(\widetilde{A}^T \otimes \widetilde{Y})$

Subject to $\widetilde{B}\widetilde{Y} = \widetilde{d}$

$\widetilde{Y}$ is a non-negative fuzzy number.

$\widetilde{A}^T = \widetilde{a}_{j_{1xn}} \widetilde{Y} = \widetilde{y}_{i_{nx1}}, \widetilde{B} = [\widetilde{b}_{ij}]_{mxn}, \widetilde{d} = [\widetilde{d}_i]_{mx1}$ and

where $\widetilde{c}_j, \widetilde{y}_j, \widetilde{b}_{ij}, d_i \in F(R)$

where $i = 1, 2, \ldots m$ and $j = 1, 2, \ldots, n$

Definition 7. A fuzzy project network can be defined by an activity-on-activity arc network $P = (N, L)$ where $N = \{1, 2, \ldots, m\}$ is the set of nodes (points) and A is the set of arcs (oriented lines) represents the activities. In the fuzzy project network, node 1 and n denotes the initial and terminal of the project respectively. The complete fuzzy Mathematical model for fully fuzzy time cost trade-off problems is given as follows:

$$Min\ \widetilde{Z} = \sum_k \sum_l A_{kl}$$

subject to

$$\widetilde{D}_1 = 0,\ \widetilde{D}_l - \widetilde{D}_k - \widetilde{y}_{kl} \geq 0,\ \widetilde{D}_m \leq \widetilde{D}; \widetilde{a}_{kl} = \widetilde{s}^*(N\widetilde{D}_{kl} - \widetilde{y}_{kl}),\ A\widetilde{D}_{kl} \leq \widetilde{y}_{kl} \leq N\widetilde{D}_{kl}$$

$$\forall (k, 1) \in P,\ \widetilde{A}_{kl} = \sum_k \sum_l \widetilde{a}_{kl} + \widetilde{I}^*(\widetilde{D}_m - \widetilde{D}_1) + \sum m\widetilde{K}_m;$$ where

$a = (1, 2, \ldots, m)$ and $b = (1, 2, \ldots, m)$.

Theorem 1. *A triangular fuzzy number* $\tilde{y} = (\tilde{y}_1, \tilde{y}_2, \tilde{y}_3)$ *is an optimal result of the problem* (Q) *if and only if* $\tilde{y}_1, \tilde{y}_2$ *and* $\tilde{y}_3$ *are optimal results of the prescribed crisp linear programming problems* $(Q2)$,

(Q1) *and* (Q3) *respectively where*:

(Q) *Maximize* $\tilde{Z} = Ay$ *Subject to By* $\tilde{B}y_1 \leq \tilde{d},\ y \geq 0$

(Q2) *Maximize* $\tilde{Z}_2 = Ay_2$ *Subject to* $By_2 \leq d_2,\ y_2 \geq 0$

(Q1) *Maximize* $\tilde{Z}_1 = Ay_1$ *Subject to* $By_1 \leq d_1,\ y_1 \geq 0,\ y_1 \leq y_2$

(Q3) *Maximize* $Z_3 = Ay_3$ *Subject to* $By_3 \leq d_3,\ y_3 \geq 0,\ y_3 \leq y_2$

Aggregation of m-LPPs [2]:

Notations

k : k^{th} problem $(k = 1, 2, m)$

l : l^{th} problem $(l = 1, 2, n_k)$

y_{kl} : l^{th} variable of the k^{th} problem

a_{kl} : Constant coefficient of the l^{th} variable of the k^{th} problem

n_k : Number of variables in the k^{th} problem

r_k : Number of constraints in the k^{th} problem

d_{kr_k} : RHS value of the r_k^{th} constraints of the k^{th} problem

General LPP structure of the k^{th}-problem $(k = 1, 2, \ldots, m)$ can be given as:

$$Max\ Z_k = a_{kl}\, y_{kl} + a_{k2}\, y_{k2} + \ldots + a_{kn_k} y_{kn_k}$$

Subject to the constraints:

$$b_{k11}\, y_{k1} + b_{k12}\, y_{k2} + \ldots + b_{k1n_k}\, y_{kn_k} \{\leq, =, \geq\} d_{k1}$$

$$b_{k21}\, y_{k1} + b_{k22}\, y_{k2} + \ldots + b_{k2n_k}\, y_{kn_k} \{\leq, =, \geq\} d_{k2}$$

....

....

$b_{ki_k1}\ y_{k1} + b_{ki_k2}\ y_{k2} + \ldots + b_{ki_kn_k}\ y_{kn_k}\{\leq, =, \geq\}d_{ki_k}$

$y_{kl} \geq 0, \{k = 1, m, 1 = 1, 2, \ldots n_k\}$

Aggregated structure of m-LPPs together

$$Max\ Z = \sum_{k=1}^{m}\sum_{l=1}^{n_k} a_{kl}\ y_{kl}$$

Subject to the constraints:

$b_{11}\ y_{11} + b_{12}\ y_{12} + \ldots + b_{11n_1}\ y_{1n1}\{\leq, =, \geq\}d_{11}$

....

$b_{1i_{11}}\ y_{11} + b_{1i_{12}}\ y_{12} + \ldots + b_{1i_{1n_1}}\ y_{1n1}\{\leq, =, \geq\}d_{1k_1}$

....

....

$b_{m_{11}}\ y_{11} + b_{m_{12}}\ y_{12} + \ldots + b_{m_{1n1}}\ y_{1n1}\{\leq, =, \geq\}d_{m_1}$

....

$b_{m_{km1}}\ y_{11} + b_{m_{km2}}\ y_{12} + \ldots + b_{m_{kmn1}}\ y_{mn1}\{\leq, =, \geq\}d_{m_{km}}$

$x_{kl} \geq 0, \{k = 1, \ldots m, l = 1, 2, \ldots, n_k\}.$

3. Intuitionistic Fuzzy Sets

Let A be the universe of discourse. An intuitionistic fuzzy set H in A is given buy: $H = \{\langle a, u_H(a), v_H(a) \mid \forall\ a \in A\}$, where $u_H(a), v_H(a) : A \to [0, 1]$ denote membership function and non-membership function, respectively, of H and satisfy $0 \leq u_H(a), v_H(a) \leq 1$ for every $a \in A$. $u_H(a)$ a represents the lowest bound of membership derived from entities of supporting $a; v_H(a)$ is the lowest bound of non-membership from entities of rejecting a. It is clear that the membership degree of Intuitionistic Fuzzy set H has been restricted in $[u_H(a), 1 - v_H(a)]$ which is a subinterval of $[0, 1]$. For each IFS H in A we call $\pi_H(a) = 1 - u_H(a) - v_H(a)$ as the hesitation index of a in H. It can be observed that $0 \leq u_H(a) \leq 1$ for each $a \in A$. For $H, G \in IFS(A)$, Atanassov [3, 4] defined the notion of containment as: $G \subseteq H \Leftrightarrow u_H(a) \leq u_G(a)$ and $v_H(a) \geq v_G(a), \forall a \in A$.

4. Introduction to Decision Making Methods

Decision making is the study of identifying and choosing alternatives based on the values and preferences of the decision maker. Making a good decision comprises the choice of the best alternative to be considered, and in such a case we want not only to identify as many of these alternatives as possible but to choose the one that best fits with our goals, objectives, desires, values, and so on.

4.1 Decision Making with Score and Accuracy Functions

Definition 8 [16]**.** If $\tilde{b} = (u, v)$ is an intuitionistic fuzzy number, a score function S of an intuitionistic fuzzy value is given by: $S(\tilde{b}) = u - v$, $S(\tilde{b}) \in [-1, 1]$.

Definition 9 [16]**.** If $\tilde{b} = (u, v)$ is an intuitionistic fuzzy number, an accuracy function H of an intuitionistic fuzzy value can be represented as follows: $H(\tilde{b}) = u + v$, $H(\tilde{b}) \in [0, 1]$.

The larger the value of $H(\tilde{b})$, the more the degree of accuracy of the intuitionistic fuzzy value $\tilde{b}$. Based on the score function S and the accuracy function H, we can give an order relation between two intuitionistic fuzzy values, which is defined as follows:

Definition 10 [16]**.** Let $\tilde{b}_1 = (u_1, v_1)$ and $\tilde{b}_2 = (u_2, v_2)$ be two intuitionistic fuzzy values, $S(\tilde{b}_1) = u_1, v_1$ and $S(\tilde{b}_2) = u_2, v_2$ be the scores of $\tilde{b}_1$ and $\tilde{b}_2$ respectively, and let $H(\tilde{b}_1) = u_1 + v_1$ and $H(\tilde{b}_2) = u_2 + v_2$ be the accuracy degree of $\tilde{b}_1$ and $\tilde{b}_2$ respectively, then if $S(\tilde{b}_1) < S(\tilde{b}_2)$, then $\tilde{b}_1$ is smaller than $\tilde{b}_2$, denoted by $\tilde{b}_1 < \tilde{b}_2$; if $S(\tilde{b}_1) = S(\tilde{b}_2)$, then, if $H(\tilde{b}_1) = H(\tilde{b}_1)$, then $\tilde{b}_1$ and $\tilde{b}_2$, represent the same information, denoted by $\tilde{b}_1 \dots \tilde{b}_2$; if $H(\tilde{b}_1) < H(\tilde{b}_2)$, b_1 is smaller than $\tilde{b}_2$, denoted by $\tilde{b}_1 < \tilde{b}_2$.

4.2 The I-IFOWA Operator

Definition 11. Let $\tilde{b}_j = (u_j, v_j), (j = 1, 2, \dots, n)$ be a collection of intuitionistic fuzzy values, and let $IFWA : Q^n \to Q$. Then the Intuitionistic Fuzzy Weighted Averaging (IFWA) operator is defines as $IFWA_{\omega}(\tilde{b}_1, \tilde{b}_2, \dots, \tilde{b}_n) = \sum_{j=1}^{n} \omega_j \tilde{b}_j = \left(1 - \prod_{j=1}^{n} (1 - u_j)^{\omega_j}, \prod_{j=1}^{n} v_j^{\omega_j}\right)$, where $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ is the weight vector of $\tilde{b}_j(u_j, v_j)$, $(j = 1, 2, \dots, n)$ and $\omega_j > 0, \sum_{j=1}^{n} \omega_j = 1$.

Definition 12. Let $\tilde{b}_j = (u_j, v_j), (j = 1, 2, \dots, n)$ be a collection of intuitionistic fuzzy values. An Intuitionistic Fuzzy Ordered Weighted Averaging (IFOWA) operator of dimension n is a mapping $IFOWA : Q^n \to Q$, then has the weight vector $w = (w_1, w_2, \dots, w_n)^T$ such that $w_j > 0$ and $\sum_{j=1}^{n} w_j = 1$. Then, $IFOWA_w(\tilde{b}_1, \tilde{b}_2, \dots, \tilde{b}_n) = \sum_{j=1}^{n} w_j \tilde{b}_{\sigma(j)}$ $= \left(1 - \prod_{j=1}^{n} (1 - u_{\sigma(j)})^{w_j}, \prod_{j=1}^{n} v_{\sigma(j)}^{w_j}\right)$ where $(\sigma(1), \sigma(2), \dots, \sigma(n))$ is a permutation of $(1, 2, \dots, n)$, such that $\tilde{\alpha}_{\sigma(j-1)} \geq \tilde{\alpha}_{\sigma(j)}$ for all $j = 2, \dots, n$.

Definition 13. An Induced Intuitionistic Fuzzy Ordered Weighted Averaging (I-IFOWA) operator is defined as follows:

$$I - IFOWA_w(\langle y_1, \tilde{b}_1 \rangle, \langle y_2, \tilde{b}_2 \rangle, \dots, \langle y_n, \tilde{b}_n \rangle)$$

$$= \sum_{j=1}^{n} w_j \tilde{g}_j = \left(1 - \prod_{j=1}^{n} (1 - \bar{u}_j)^{w_j}, \prod_{j=1}^{n} \bar{v}_j^{w_j}\right).$$

Where $w = (w_1, w_2, \dots, w_n)^T$ is a weighting vector, such that $w_j \in [0, 1]$,

$\sum_{j=1}^{n} w_j = 1$, $j = 1, 2, n$, $\tilde{g}_j(\bar{u}_j, \bar{v}_j)$ is the $\tilde{b}_j$ value of the IFOWA pair $\langle y_i, \tilde{b}_i \rangle$ having the j^{th} largest $y_i (y_i \in [0, 1])$, and y_i in $y_i \langle y_i, \tilde{b}_i \rangle$ is referred to as the order inducing variable and $\tilde{b}_i (\tilde{b}_i = (u_i, v_i))$ as the intuitionistic fuzzy values.

5. Solving procedure for Group Decision Making with Intuitionistic Fuzzy Information and its Application in the Fuzzy Time Cost Trade-off Problem

Let $A = \{A_1, A_2, \ldots, A_m\}$ be a set of alternatives, and $G = \{G_1, G_2, \ldots, G_n\}$ be the set of attributes, $\omega = (\omega_1, \omega_2, \ldots, \omega_n)$ is the weighting vector of the attribute $G_j (j = 1, 2, \ldots, n)$, where $\omega_j \in [0, 1]$, $\sum_{j=1}^{n} \omega_j = 1$. Let $D = \{D_1, D_2, \ldots, D_t\}$ be the set of decision makers and $\tilde{R}_k (\tilde{r}_{ij}^{(k)})_{m \times n} = (u_{ij}^{(k)}, v_{ij}^{(k)})_{m \times n}$ is the intuitionistic fuzzy decision matrix, where $u_{ij}^{(k)}$ indicates the degree that the alternative A_i satisfies the attribute G_j given by the decision maker D_k, $v_{ij}^{(k)}$ indicates the degree that the alternative A_i doesn't satisfy the attribute G_j given by the decision maker D_k, $u_{ij}^{(k)} \subset [0, 1]$, $v_{ij}^{(k)} \subset [0, 1]$, $u_{ij}^{(k)} + v_{ij}^{(k)} \leq 1$, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$, $k = 1, 2, \ldots, t$. The decision algorithm for solving the Fuzzy Time-Cost Trade off problem is given as:

Step 1. Utilize the decision information given in matrix $\tilde{R}_k$, and the I-IFOWA operator to aggregate all the decision matrices $\tilde{R}_k (k = 1, 2, \ldots, t)$ into a collective decision matrix $\tilde{R}_k = (\tilde{r}_{ij})_{m \times n}$.

Step 2. Utilize the decision information given in matrix $\tilde{R}_k$, and the IFWA operator to derive the collective overall preference values $\tilde{r}_i (i = 1, 2, \ldots, m)$ of the alternative A_i.

Step 3. Calculate the scores $S(\tilde{r}_i) (i = 1, 2, \ldots, m)$ of the collective overall intuitionistic fuzzy preference value $\widetilde{r}_i (i = 1, 2, \ldots, m)$ to rank all the alternatives $A_i (i = 1, 2, \ldots, m)$ and then to select the best one (s). If there is no difference between two scores $S(\widetilde{r}_i)$ and $S(\widetilde{r}_j)$, then we need to calculate the accuracy degrees $H(\widetilde{r}_i)$ and $H(\widetilde{r}_j)$ of the collective overall intuitionistic fuzzy preference values $\widetilde{r}_i$ and $\widetilde{r}_j$, respectively, and then rank the alternatives.

(or)

Calculate The Hamming distance $d'(A, B)$ for intuitionistic fuzzy sets preference value $\widetilde{r}_i (i = 1, 2, \ldots, m)$ and $\widetilde{r}^+ = (1, 0) \cdot d'(A, B)$ $= \frac{1}{2} \sum_{i=1}^{n} [| u_A(a_i) - u_B(a_i) | + v_A(a_i) - v_B(a_i) |]$.

(or)

Calculate The Hamming distance $d''(A, B)$ for intuitionistic fuzzy sets preference value $\widetilde{r}_i (i = 1, 2, \ldots, m)$ and $\widetilde{r}^+ = (1, 0) \cdot d''(A, B)$ $= \frac{1}{2} \sum_{i=1}^{n} [| u_A(a_i) - u_B(a_i) | + | v_A(a_i) - v_B(a_i) | + | \pi_A(a_i) - \pi_B(a_i) |]$.

Step 4. Rank all the alternatives $A_i (i = 1, 2, \ldots, m)$ and select the best one (s) in accordance with $S(\widetilde{r}_i)$ and $H(\widetilde{r}_i)(i = 1, 2, \ldots, m)$, $d'(A, B)$ and $d''(A, B)$.

Step 5. Find the direct cost and the cost slope of the fuzzy time cost trade-off problem using triangular fuzzy variable.

Step 6. Fully fuzzy mathematical model is used to transform the fuzzy time cost trade-off problem into fuzzy linear programming problem.

Step 7. Using decomposition technique fuzzy linear programming problem is split up into crisp linear programming problems.

Step 8. Crisp linear programming problems are aggregated into unique linear programming problems.

Step 9. Optimum solution of fully fuzzy mathematical model is obtained by using LINGO solver package.

Step 10. The optimum result of the crash cost and crash duration for all the activities can be found in the respective variables.

6. Numerical Illustration

List of activities for construction of house is shown below with the required data. Table 1 gives the description of the project. In the construction project time and cost parameters of the project are taken as triangular fuzzy number. (100, 100, 100) is taken as the indirect cost per day. The project manager wishes to complete the project within 90 days. Activities required data are shown table 2.

Table 1. Project description.

Activity	Description
$1 \to 2(E)$	Preparing the site location
$2 \to 3(F)$	Raise the Building
$2 \to 4(G)$	Plumbing and Electricity works
$3 \to 4(H)$	Plastering works

The four possible alternatives $A(i = 1, 2, 3, 4)$ are to be tested using the intuitionistic fuzzy numbers given by the three decision makers and constructed as matrices are given in the following:

$$\tilde{R}_1 = \begin{pmatrix} (0.4, 0.3) & (0.5, 0.2) & (0.2, 0.5) & (0.1, 0.6) \\ (0.6, 0.2) & (0.6, 0.1) & (0.6, 0.1) & (0.3, 0.4) \\ (0.5, 0.3) & (0.4, 0.3) & (0.4, 0.2) & (0.5, 0.2) \\ (0.7, 0.1) & (0.5, 0.2) & (0.2, 0.3) & (0.1, 0.5) \end{pmatrix}$$

$$\tilde{R}_2 = \begin{pmatrix} (0.5, 0.4) & (0.6, 0.3) & (0.3, 0.6) & (0.2, 0.7) \\ (0.7, 0.3) & (0.7, 0.2) & (0.7, 0.2) & (0.4, 0.5) \\ (0.6, 0.4) & (0.5, 0.4) & (0.5, 0.3) & (0.6, 0.3) \\ (0.8, 0.1) & (0.6, 0.3) & (0.3, 0.4) & (0.2, 0.6) \end{pmatrix}$$

$$\widetilde{R}_3 = \begin{pmatrix} (0.4,\ 0.5) & (0.5,\ 0.4) & (0.2,\ 0.7) & (0.1,\ 0.8) \\ (0.6,\ 0.4) & (0.6,\ 0.3) & (0.6,\ 0.3) & (0.3,\ 0.6) \\ (0.5,\ 0.5) & (0.4,\ 0.5) & (0.4,\ 0.4) & (0.5,\ 0.4) \\ (0.7,\ 0.2) & (0.5,\ 0.4) & (0.2,\ 0.5) & (0.1,\ 0.7) \end{pmatrix}$$

Then, use the approach proposed to get the most desirable alternative(s).

Ranking with score and accuracy functions.

Step 1. Using the computations mentioned in the algorithm we get:

$$\widetilde{R}_2 = \begin{pmatrix} (0.421,\ 0.380) & (0.521,\ 0.276) & (0.221,\ 0.583) & (0.121,\ 0.684) \\ (0.622,\ 0.276) & (0.622,\ 0.169) & (0.622,\ 0.169) & (0.321,\ 0.482) \\ (0.522,\ 0.380) & (0.421,\ 0.380) & (0.421,\ 0.276) & (0.522,\ 0.276) \\ (0.723,\ 0.127) & (0.522,\ 0.276) & (0.221,\ 0.380) & (0.121,\ 0.583) \end{pmatrix}$$

Step 2. Using the computations mentioned in the algorithm we get:

$\widetilde{r}_1 = (0.266,\ 0529); \widetilde{r}_2 = (0.522,\ 0.284); \widetilde{r}_3 = (0.484,\ 0.304); \widetilde{r}_4 (0.367,\ 0.351);$

Step 3. Calculate the scores of collective overall intuitionistic fuzzy preference values $\widetilde{r}_1 (i = 1,\ 2,\ 3,\ 4)$. $S(\widetilde{r}_1) = 0.266 - 0.529 = -0.263$. Similarly all other values are calculated. $S(\widetilde{r}_2) = 0.239; S(\widetilde{r}_3) = 0.180; S(\widetilde{r}_4) = 0.0160$.

Step 4. Ranking all the alternatives $A_i (i = 1,\ 2,\ 3,\ 4)$ according to the scores $S(\widetilde{r}_i)(i = 1,\ 2,\ 4,\ 5)$ we can observe that $A_2 > A_3 > A_4 > A_1$, and thus the most desirable alternative is A_2.

Ranking with hamming distance function excluding intuitionistic degree

Step 1 and Step 2 are same as in method-1.

Step 3. Calculate the Hamming distance between each entry of step-2 and the positive ideal solution $\widetilde{r}^+ = (1,\ 0)$. Hence $d'(\widetilde{r}^+,\ \widetilde{r}_1) = 0.6315; d'(\widetilde{r}^+,\ \widetilde{r}_2) = 0.3810; d'(\widetilde{r}^+,\ \widetilde{r}_3) = 0.410; d'(\widetilde{r}^+,\ \widetilde{r}_4) = 0.492$.

Step 4. Ranking all the alternatives $A_i (i = 1,\ 2,\ 3,\ 4)$ according with the Hamming distance $d'(A,\ B)$ of the collective overall intuitionistic fuzzy preference values $\widetilde{r}_i (i = 1,\ 2,\ 3,\ 4) : A_1 > A_4 > A_3 > A_2$, and thus the most desirable alternative is A_1.

Ranking with improved hamming distance function including intuitionistic degree

Step 1 and Step 2 are same as in method-1.

Step 3. Calculate the Improved Hamming distance between each entry of step-2 and the positive ideal solution $\tilde{r}^+ = (1, 0)$. Hence $d''(\tilde{r}^+, \tilde{r}_1) = 0.734; d''(\tilde{r}^+, \tilde{r}_2) = 0.478; d''(\tilde{r}^+, \tilde{r}_3) = 0516; d''(\tilde{r}^+, \tilde{r}_4) = 0633.$

Step 4. Ranking all the alternatives $A_i (i = 1, 2, 3, 4)$ according with the Improved Hamming distance $d''(A, B)$ intuitionistic fuzzy preference values $\tilde{r}_i (i = 1, 2, 3, 4) A_1 > A_4 > A_3 > A_2$, and thus the most desirable alternative is A_1.

Based on the above computations the final ranked values are normalised and utilised for further computations in the Fuzzy Time Cost Trade-off problem given in Table 2.

Table 2. Fuzzy Data of the proposed Project.

Activity	Crash Duration (CD)	Normal Duration (ND)	Crash Cost (CC)	Normal Cost (NC)
$1 \to 2(E)$	(20,21,22)	(24,24,24)	(500,500,500)	(800,800,800)
$2 \to 3(F)$	(15,16,17)	(18,18,18)	(263,263,263)	(239,239,239)
$2 \to 4(G)$	(38,38,38)	(40,41,42)	(631,631,631)	(492,492,492)
$3 \to 4(E)$	(46,48,50)	(52,52,52)	(734,734,734)	(633,633,633)

Step 5.

Table 3. Crash Slope of the proposed project.

Activity	ΔT	ΔC	Crash Slope $\Delta C / \Delta T$
$1 \to 2(A)$	(2,3,4)	(300,300,300)	(75,100,150)
$2 \to 3(B)$	(1,2,3)	(24,24,24)	(8,12,24)

$2 \to 4(C)$	(2,3,4)	(139,139,139)	(34.75,46.33,69.5)
$3 \to 4(D)$	(2,4,6)	(101,101,101)	(16.8,25.25,50.5)

Critical Path is $A \to B \to D$; Total duration is (94, 94, 94); Direct Cost of the project is (1672, 1672, 1672); Total Cost of the project is (11072, 11072, 11072).

Step. 6, 7 and 8

Hence we have: $Min\ \widetilde{Z} = [\sum_k \sum_l a_{kl} + \widetilde{I}^*(\widetilde{D}_m - \widetilde{D}_1) + \sum_m \widetilde{K}_m]$

Subject to the constraints:

$\widetilde{D} = 0$

$\widetilde{D}_{21} - \widetilde{D}_{11} - \widetilde{y}_{E1} \geq 0; \widetilde{D}_{22} - \widetilde{D}_{12} - y_{E2} \geq 0; \widetilde{D}_{23} - \widetilde{D}_{13} - \widetilde{y}_{E3} \geq 0; \widetilde{D}_{31}$

$- \widetilde{D}_{21} - \widetilde{y}_{F1} \geq 0;$

$\widetilde{D}_{32} - \widetilde{D}_{22} - \widetilde{y}_{F2} \geq 0; \widetilde{D}_{33} - \widetilde{D}_{23} - \widetilde{y}_{F3} \geq 0; \widetilde{D}_{41} - \widetilde{D}_{21} - \widetilde{y}_{G1} \geq 0; \widetilde{D}_{42}$

$- \widetilde{D}_{22} - \widetilde{y}_{G2} \geq 0;$

$\widetilde{D}_{43} - \widetilde{D}_{23} - \widetilde{y}_{G3} \geq 0; \widetilde{D}_{41} - \widetilde{D}_{31} - \widetilde{y}_{H1} \geq 0; \widetilde{D}_{42} - \widetilde{D}_{32} - \widetilde{y}_{H2} \geq 0; \widetilde{D}_{43}$

$- \widetilde{D}_{33} - \widetilde{y}_{H3} \geq 0$

$\widetilde{D}_4 \leq (90, 90, 90)$

$\widetilde{a}_{12} = \widetilde{s}_{12} * (N\widetilde{D}_{12} - \widetilde{y}_{E1}); \widetilde{a}_{12} = \widetilde{s}_{12} * (N\widetilde{D}_{12} - \widetilde{y}_{E2}); \widetilde{a}_{13} = \widetilde{s}_{13} * (N\widetilde{D}_{13} - \widetilde{y}_{E3});$

$\widetilde{a}_{23} = \widetilde{s}_{23} * (N\widetilde{D}_{23} - \widetilde{y}_{F1}); \widetilde{a}_{23} = \widetilde{s}_{23} * (N\widetilde{D}_{23} - \widetilde{y}_{F2}); \widetilde{a}_{23} = \widetilde{s}_{23} * (N\widetilde{D}_{23} - \widetilde{y}_{F3});$

$\widetilde{a}_{24} = \widetilde{s}_{24} * (N\widetilde{D}_{24} - \widetilde{y}_{G1}); \widetilde{a}_{24} = \widetilde{s}_{24} * (N\widetilde{D}_{24} - \widetilde{y}_{G2}); \widetilde{a}_{24} = \widetilde{s}_{24} * (N\widetilde{D}_{24} - \widetilde{y}_{G3});$

$\widetilde{a}_{34} = \widetilde{s}_{34} * (N\widetilde{D}_{34} - \widetilde{y}_{H1}); \widetilde{a}_{34} = \widetilde{s}_{34} * (N\widetilde{D}_{34} - \widetilde{y}_{H2}); \widetilde{a}_{34} = \widetilde{s}_{34} * (N\widetilde{D}_{34} - \widetilde{y}_{H3});$

$a\widetilde{D}_{12} \leq \widetilde{y}_E \leq N\widetilde{D}_{12}; a\widetilde{D}_{23} \leq \widetilde{y}_F \leq N\widetilde{D}_{23}; a\widetilde{D}_{24} \leq \widetilde{y}_G \leq N\widetilde{D}_{24}; a\widetilde{D}_{34}$

$\leq \widetilde{y}_H \leq N\widetilde{D}_{34}.$

All the triangular fuzzy variables are decomposed in to 3 crisp variables and then aggregated.

Step 9.

Variable	Value				
YE	24.00000; YF	16.00000 ; YG	41.00000; YH	48.00000; D4	88.00000;
D1	0.000000; D2	24.00000; D3	40.00000; AE	0.000000; AF	26.00000;
AG	0.000000;	AH 112.0000.			

Step 10. The Optimum Project Cost is Rs.20,010. Hence the Project Manager can able to finish the project within 88 days with the above costs and duration.

7. Conclusion

This article put in the I-IFOWA operator to group decision making with intuitionistic fuzzy information. Initially the fuzzy time cost trade-off problem with activity data in the form of intuitionistic fuzzy matrices are aggregated and then solved under numerous attributes and then ranked using distinct methods for normalised representation of the activities. The time and cost parameters are considered as triangular fuzzy variables and are utilised in the MAGDM problem. All the triangular fuzzy variables are divided into crisp variables later it is aggregated so as to obtain a fuzzy solution for a fuzzy variables and then optimum solution for the project is obtained.

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