Advances and Applications in Mathematical Sciences
Volume 21, Issue 5, March 2022, Pages 2901-2909

PD MEAN LABELING OF SPLITTING GRAPHS

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Abstract

A graph G with p vertices and q edges is said to be PD mean labeling if it is possible to label $h : V \to \{1, 2, \ldots, p\}$ is distinct and in such a way that each edge $h^* : E \to \left\{1, 2, \ldots, \frac{p(p-1)}{2}\right\}$ is defined as $h^*(uv) = \left\lfloor \frac{P + D}{2} \right\rfloor$, where $P = h(u)h(v)$ and $D = \left\lfloor \frac{h(u)}{h(v)} \right\rfloor$, $u > v$ is distinct. A graph that admits PD mean labeling is called PD mean graph.

1. Introduction

Graph labeling is one of the popular and comprehensively researched subject. Here we considered only finite, simple and undirected graphs. The term mean labeling was first introduced by R. Ponraj and S. Somasundaram. A graph G with p vertices and q edges is said to be mean labeling if it is

2020 Mathematics Subject Classification: 05C78.
Keywords: PD Mean Graphs, Splitting Graphs.
Received December 15, 2021; Accepted January 21, 2022

possible to label $h : V \to \{0, 1, 2, \ldots, p\}$ is distinct and in such a way that each edge $h(uv) = \begin{cases} \frac{h(u)+h(v)}{2} \text{ if } h(u)+h(v) \text{ is even} \\ \frac{h(u)+h(v)+1}{2} \text{ if } h(u)+h(v) \text{ is odd} \end{cases}$.

Then the resulting edges are distinct. A graph that admits mean labeling is called mean graph. Based on these labeling pattern we introduce a new labeling pattern called PD mean labeling. In this paper we investigate that the existence of PD mean labeling of splitting of some standard graphs.

2. Main Result

Definition 2.1. For each vertex v of a graph G take a new vertex v' and join v' to all the vertices of G adjacent to v. The resulting graph is splitting graph of G. It is denoted as $Spl(G)$.

Theorem 2.2. *$Spl(P_\alpha)$ is a PD mean graph for $\alpha > 1$.*

Proof. Let $G = Spl(P_\alpha)$ be the splitting graph of path P_α. Let $V(G) = \{[u_\theta, 1 \le \theta \le \alpha] \cup [u'_\theta, 1 \le \theta \le \alpha]\}$ be the vertices of G and $E(G) = \{[u_\theta u_{\theta+1}, 1 \le \theta \le \alpha - 1] \cup [u_\theta u'_{\theta+1}, 1 \le \theta \le \alpha - 1] \cup u'_\theta u_{\theta+1}, 1 \le \theta \le \alpha - 1]\}$ be the edges of G. Clearly $|V(G)| = 2\alpha$ and $|E(G)| = 3(\alpha - 1)$. Define a map $h : V \to \{1, 2, \ldots, 2\alpha\}$ by $h(u_{3\theta-2}) = 6\theta - 5, 1 \le \theta \le \left\lfloor \frac{\alpha+2}{3} \right\rfloor$, $h(u_{3\theta-1}) = 6\theta - 2, 1 \le \theta \le \left\lfloor \frac{\alpha+1}{3} \right\rfloor$, $h(u_{3\theta}) = 6\theta, 1 \le \theta \le \left\lfloor \frac{\alpha}{3} \right\rfloor$, $h(u'_{3\theta-2}) = 6\theta - 3, 1 \le \theta \le \left\lfloor \frac{\alpha+1}{3} \right\rfloor$, $h(u'_{3\theta-1}) = 6\theta - 4, 1 \le \theta \le \left\lfloor \frac{\alpha+1}{3} \right\rfloor$, $h(u'_{3\theta}) = 6\theta - 1, 1 \le \theta \le \left\lfloor \frac{\alpha}{3} \right\rfloor$, $h(v_\alpha) = 2\alpha$, if $\alpha = 3\alpha + 1$, $\theta \in N$.

Then the labels of edges are given by $h(u_1u_2) = 4$,

$h(u_{\alpha-1}v_\alpha) = 2\alpha(\alpha - 1)$, if $\alpha = 3\theta + 1$, $\theta \in N$,

$h(u_1u'_2) = 2$, $h(u_{3\theta+1}u_{3\theta+2}) = 18\theta^2 + 15\theta + 2$, $1 \le \theta \le \left\lfloor \frac{\alpha-2}{3} \right\rfloor$,

$h(u_{3\theta-1}u_{3\theta}) = 6\theta(3\theta - 1), 1 \leq \theta \leq \left\lfloor \frac{\alpha}{3} \right\rfloor, h(u'_2u_3) = 7,$

$h(u_{3\theta}u_{3\theta+1}) = 3\theta(6\theta + 1), 1 \leq \theta \leq \left\lfloor \frac{\alpha - 1}{3} \right\rfloor,$

$h(u_{3\theta+1}u'_{3\theta+2}) = 18\theta^2 + 9\theta + 1, 1 \leq \theta \leq \left\lfloor \frac{\alpha - 2}{3} \right\rfloor,$

$h(u_{3\theta-1}u'_{3\theta}) = 18\theta^2 - 9\theta + 1, 1 \leq \theta \leq \left\lfloor \frac{\alpha}{3} \right\rfloor,$

$h(u_{3\theta}u'_{3\theta+1}) = 9\theta(2\theta + 1), 1 \leq \theta \leq \left\lfloor \frac{\alpha - 2}{3} \right\rfloor,$

$h(u'_{3\theta+2}u_{3\theta+3}) = 6(3\theta^2 + 4\theta + 1), 1 \leq \theta \leq \left\lfloor \frac{\alpha - 3}{3} \right\rfloor,$

$h(u'_{3\theta}u_{3\theta+1}) = 18\theta^2, 1 \leq \theta \leq \left\lfloor \frac{\alpha - 1}{3} \right\rfloor,$

which are distinct. Hence $Spl(P_\alpha)$ is a PD mean graph for $\alpha > 1$.

Theorem 2.3. *$Spl(K_{1,\alpha})$ is a PD mean graph for all α.*

Proof. Let $G = Spl(K_{1,\alpha})$ be the splitting graph of star $K_{1,\alpha}$. Let $V(G) = \{[u] \cup [u'] \cup [u_\theta, 1 \leq \theta \leq \alpha] \cup [u'_\theta, 1 \leq \theta \leq \alpha]\}$ be the vertices of G and $E(G) = \{[uu_\theta, 1 \leq \theta \leq \alpha] \cup [uu_\theta, 1 \leq \theta \leq \alpha] \cup [u'u_\theta, 1 \leq \theta \leq \alpha]\}$ be the edges of G. Clearly $|V(G)| = 2(\alpha + 1)$ and $|E(G)| = 3\alpha$. Define a map

$h : V \to \{1, 2, \ldots, 2(\alpha + 1)\}$ by $h(u) = 1, h(u') = 2(\alpha + 1),$

$h(u_\theta) = \alpha + \theta + 1, 1 \leq \theta \leq \alpha, h(u'_\theta) = \theta + 1, 1 \leq \theta \leq \alpha,$

Then the labels of edges are given by

$h(uu_\theta) = \alpha + \theta + 1, 1 \leq \theta \leq \alpha, h(uu'_\theta) = \theta + 1, 1 \leq \theta \leq \alpha,$

$h(u'u_\theta) = \alpha^2 + 2\alpha + 1 + \alpha\theta + \theta, 1 \leq \theta \leq \alpha,$ which are distinct. Hence $Spl(K_{1,\alpha})$ is a PD mean graph for all α.

Theorem 2.4. *$Spl(B(\alpha, \beta))$ is a PD mean graph for all α, β.*

Proof. Let $G = Spl(B(\alpha, \beta))$ be the splitting graph of bistar $B(\alpha, \beta)$. Let $V(G) = \{[u] \cup [v] \cup [u'] \cup [v'] \cup [u_\theta, 1 \le \theta \le \alpha] \cup [u'_\theta, 1 \le \theta \le \alpha] \cup [v_\theta, 1 \le \theta \le \beta] \cup [v'_\theta, 1 \le \theta \le \beta]$ be the vertices of G and $E(G) = \{[uv] \cup [uu_\theta, 1 \le \theta \le \alpha] \cup [uu'_\theta, 1 \le \theta \le \alpha] \cup [u'u_\theta, 1 \le \theta \le \alpha] \cup [u'v] \cup [uv'] \cup [vv_\theta, 1 \le \theta \le \beta] \cup [vv'_\theta, 1 \le \theta \le \beta] \cup [v'v_\theta, 1 \le \theta \le \beta]\}$ be the edges of G. Clearly $|V(G)| = 2(\alpha + \beta + 2)$ and $|E(G)| = 3(\alpha + \beta + 1)$. Define a map $h : V \to \{1, 2, \ldots, 2(\alpha + \beta + 2)\}$ by $h(u) = 1$, $h(u_\theta) = \theta + 1, 1 \le \theta \le \alpha$, $h(v) = 2(\alpha + \beta + 2)$,

$h(u'_\theta) = \alpha + \theta + 1, 1 \le \theta \le \alpha$, $h(v_\theta) = 2\alpha + \beta + \theta + 3, 1 \le \theta \le \beta$,

$h(v'_\theta) = 2\alpha + \theta + 3, 1 \le \theta \le \beta$, $h(u') = 2\alpha + 3$, $h(v') = 2(\alpha + 1)$.

Then the labels of edges are given by $h(uv) = 2(\alpha + \beta + 2)$,

$h(uu_\theta) = \theta + 1, 1 \le \theta \le \alpha$, $h(uv') = 2(\alpha + 1)$,

$h(uu'_\theta) = \alpha + \theta + 1, 1 \le \theta \le \alpha$,

$$h(u'u_\theta) = \left\lfloor \frac{(2\alpha + 3)(\theta^2 + 2\theta + 2)}{2(\theta + 1)} \right\rfloor, 1 \le \theta \le \alpha,$$

$$h(vv_\theta) = \left\lfloor \frac{(\alpha + \beta + 2)[(2\alpha + \beta + 3 + \theta)^2 + 1]}{2\alpha + \beta + \theta + 3} \right\rfloor, 1 \le \theta \le \beta,$$

$$h(vv'_\theta) = \left\lfloor \frac{(\alpha + \beta + 2)[(2\alpha + 3 + \theta)^2 + 1]}{2\alpha + \theta + 3} \right\rfloor, 1 \le \theta \le \beta,$$

$$h(v'v_\theta) = \left\lfloor \frac{(2\alpha + \beta + 3 + \theta)[(2\alpha + 2)^2 + 1]}{4(\alpha + 1)} \right\rfloor, 1 \le \theta \le \beta,$$

$h(u'v) = \left\lfloor \frac{(\alpha + \beta + 2)(4\alpha^2 + 12\alpha + 10)}{2\alpha + 3} \right\rfloor$, which are distinct. Hence $Spl(B(\alpha, \beta))$ is a PD mean graph for all α, β.

Theorem 2.5. *$Spl(P_\alpha \odot K_1)$ is a PD mean graph for α.*

Proof. Let $G = Spl(P_\alpha \odot K_1)$ be the splitting graph of comb $P_\alpha \odot K_1$. Let

$V(G) = \{[u_\theta, 1 \leq \theta \leq \alpha] \cup [u'_\theta, 1 \leq \theta \leq \alpha] \cup [v_\theta, 1 \leq \theta \leq \alpha] \cup [v'_\theta, 1 \leq \theta \leq \beta]\}$ be the vertices of G and $E(G) = \{[u_\theta u_{\theta+1}, 1 \leq \theta \leq \alpha - 1] \cup [u_\theta v_\theta, 1 \leq \theta \leq \alpha] \cup [u_\theta v'_\theta, 1 \leq \theta \leq \alpha] \cup [v_\theta u'_\theta, 1 \leq \theta \leq \alpha] \cup [u_\theta u'_{\theta+1}, 1 \leq \theta \leq \alpha - 1] \cup [u'_\theta u_{\theta+1}, 1 \leq \theta \leq \alpha - 1]\}$ be the edges of G. Clearly $|V(G)| = 4\alpha$ and $|E(G)| = 3(2\alpha - 1)$. Define a map $h : V \to \{1, 2, \ldots, 4\alpha\}$ by $h(u_\theta) = 4\theta - 2$, $1 \leq \theta \leq \alpha$, $h(v_\theta) = 4\theta - 1$, $1 \leq \theta \leq \alpha$, $h(u'_\theta) = 4\theta$, $1 \leq \theta \leq \alpha$, $h(v'_\theta) = 4\theta - 3$, $1 \leq \theta \leq \alpha$.

Then the labels of edges are given by $h(u_1 u_2) = 7$, $h(u_1 v'_1) = 2$,

$h(u_{\theta+1} u_{\theta+2}) = 2(4\theta^2 + 8\theta + 3)$, $1 \leq \theta \leq \alpha - 2$,

$h(u_\theta v_\theta) = 8\theta^2 - 6\theta + 1$, $1 \leq \theta \leq \alpha$,

$h(v_\theta u'_\theta) = 2\theta(4\theta - 1)$, $1 \leq \theta \leq \alpha$,

$h(u_1 u'_2) = 10$, $h(u_2 u'_3) = 37$,

$h(u_{\theta+1} v'_{\theta+1}) = 8\theta^2 + 6\theta + 1$, $1 \leq \theta \leq \alpha - 1$,

$h(u_{\theta+1} u'_{\theta+3}) = 4(2\theta^2 + 9\theta + 9)$, $1 \leq \theta \leq \alpha - 2$,

$h(u'_\theta u_{\theta+1}) = 4\theta(2\theta + 1)$, $1 \leq \theta \leq \alpha - 1$, which are distinct. Hence $Spl(P_\alpha \odot K_1)$ is a PD mean graph for α.

Theorem 2.6. *$Spl(L_\alpha)$ is a PD mean graph for all α.*

Proof. Let $G = Spl(L_\alpha)$ be the splitting graph of ladder L_α. Let $V(G) = \{[u_\theta, 1 \leq \theta \leq \alpha] \cup [u'_\theta, 1 \leq \theta \leq \alpha] \cup [v_\theta, 1 \leq \theta \leq \alpha] \cup [v'_\theta, 1 \leq \theta \leq \alpha]\}$ be the vertices of G and $E(G) = \{[u_\theta u_{\theta+1}, 1 \leq \theta \leq \alpha - 1] \cup [v_\theta v_{\theta+1}, 1 \leq \theta \leq \alpha - 1] \cup [u_\theta v_\theta, 1 \leq \theta \leq \alpha] \cup [u_\theta u'_{\theta+1}, 1 \leq \theta \leq \alpha - 1] \cup [u'_\theta u_{\theta+1}, 1 \leq \theta \leq \alpha - 1] \cup [v_\alpha v'_{\theta+1}, 1 \leq \theta \leq \alpha - 1] \cup [v'_\alpha v_{\theta+1}, 1 \leq \theta \leq \alpha - 1] \cup [v_\alpha u'_{\theta+1}, 1 \leq \theta \leq \alpha]\}$ be the edges of G. Clearly $|V(G)| = 4\alpha$ and $|E(G)| = 3(3\alpha - 2)$.

Define a map $h : V \to \{1, 2, \ldots, 4\alpha\}$ by

$h(u_{4\theta-3}) = 16\theta - 13$, $1 \leq \theta \leq \left\lfloor \frac{\alpha + 3}{4} \right\rfloor$,

$$h(u_{4\theta-2}) = 16\theta - 9,\ 1 \le \theta \le \left\lfloor \frac{\alpha + 2}{4} \right\rfloor,$$

$$h(u_{4\theta-1}) = 4(4\theta - 1),\ 1 \le \theta \le \left\lfloor \frac{\alpha + 1}{4} \right\rfloor,$$

$$h(u_{4\theta}) = 16\theta,\ 1 \le \theta \le \left\lfloor \frac{\alpha}{4} \right\rfloor,$$

$$h(u_{4\theta-3}) = 16\theta - 15,\ 1 \le \theta \le \left\lfloor \frac{\alpha + 3}{4} \right\rfloor,$$

$$h(u'_{4\theta-2}) = 16\theta - 11,\ 1 \le \theta \le \left\lfloor \frac{\alpha + 2}{4} \right\rfloor,$$

$$h(u'_{4\theta-1}) = 2(8\theta - 3),\ 1 \le \theta \le \left\lfloor \frac{\alpha + 1}{4} \right\rfloor,$$

$$h(u'_{4\theta}) = 2(8\theta - 1),\ 1 \le \theta \le \left\lfloor \frac{\alpha}{4} \right\rfloor,$$

$$h(v_{4\theta-3}) = 4(4\theta - 3),\ 1 \le \theta \le \left\lfloor \frac{\alpha + 3}{4} \right\rfloor,$$

$$h(v_{4\theta-2}) = 8(2\theta - 1),\ 1 \le \theta \le \left\lfloor \frac{\alpha + 2}{4} \right\rfloor,$$

$$h(v_{4\theta-1}) = 16\theta - 5,\ 1 \le \theta \le \left\lfloor \frac{\alpha + 1}{4} \right\rfloor,$$

$$h(v_{4\theta}) = 16\theta - 1,\ 1 \le \theta \le \left\lfloor \frac{\alpha}{4} \right\rfloor,$$

$$h(v'_{4\theta-3}) = 2(8\theta - 7),\ 1 \le \theta \le \left\lfloor \frac{\alpha + 3}{4} \right\rfloor,$$

$$h(v'_{4\theta-2}) = 2(8\theta - 5),\ 1 \le \theta \le \left\lfloor \frac{\alpha + 2}{4} \right\rfloor,$$

$$h(v'_{4\theta-1}) = 16\theta - 7,\ 1 \le \theta \le \left\lfloor \frac{\alpha + 1}{4} \right\rfloor,$$

$$h(v'_{4\theta}) = 16\theta - 3,\ 1 \le \theta \le \left\lfloor \frac{\alpha}{4} \right\rfloor.$$

Then the labels of edges are given by

$$h(u_1u_2) = 11,\ h(u_{4\theta+1}u_{4\theta+2}) = 128\theta^2 + 80\theta + 11,\ 1 \le \theta \le \left\lfloor \frac{\alpha-2}{4} \right\rfloor,$$

$$h(u_{4\theta-2}u_{4\theta-1}) = 2(64\theta^2 - 52\theta + 9),\ 1 \le \theta \le \left\lfloor \frac{\alpha-1}{4} \right\rfloor,$$

$$h(u_{4\theta-1}u_{4\theta}) = 32\theta(4\theta - 1),\ 1 \le \theta \le \left\lfloor \frac{\alpha}{4} \right\rfloor,$$

$$h(u_{4\theta}u_{4\theta+1}) = 8\theta(16\theta + 3),\ 1 \le \theta \le \left\lfloor \frac{\alpha-1}{4} \right\rfloor,$$

$$h(v_1v_2) = 17,\ h(v_{4\theta+1}v_{4\theta+2}) = 16(8\theta^2 + 6\theta + 1),\ 1 \le \theta \le \left\lfloor \frac{\alpha-2}{4} \right\rfloor,$$

$$h(v_{4\theta-2}v_{4\theta-1}) = 4(32\theta^2 - 26\theta + 5),\ 1 \le \theta \le \left\lfloor \frac{\alpha-1}{4} \right\rfloor,$$

$$h(v_{4\theta-1}v_{4\theta}) = 16\theta(8\theta - 3),\ 1 \le \theta \le \left\lfloor \frac{\alpha}{4} \right\rfloor,$$

$$h(v_{4\theta}v_{4\theta+1}) = 2(64\theta^2 + 12\theta - 1),\ 1 \le \theta \le \left\lfloor \frac{\alpha-1}{4} \right\rfloor,$$

$$h(u_{4\theta-3}u'_{4\theta-2}) = 8(16\theta^2 - 24\theta + 9),\ 1 \le \theta \le \left\lfloor \frac{\alpha+2}{4} \right\rfloor,$$

$$h(u_{4\theta-2}u'_{4\theta-1}) = 128\theta^2 + 120\theta + 27,\ 1 \le \theta \le \left\lfloor \frac{\alpha+1}{4} \right\rfloor,$$

$$h(u_{4\theta-1}u'_{4\theta}) = 4(32\theta^2 - 12\theta + 1),\ 1 \le \theta \le \left\lfloor \frac{\alpha}{4} \right\rfloor,$$

$$h(u_{4\theta}u'_{4\theta+1}) = 8\theta(16\theta + 1),\ 1 \le \theta \le \left\lfloor \frac{\alpha-1}{4} \right\rfloor,$$

$$h(u'_1u_2) = 7,\ h(u'_{4\theta+1}u_{4\theta+2}) = 4(32\theta^2 + 16\theta + 1),\ 1 \le \theta \le \left\lfloor \frac{\alpha-2}{4} \right\rfloor,$$

$$h(u'_2u_3) = 31,\ h(u'_{4\theta+2}u_{4\theta+3}) = 2(64\theta^2 - 68\theta + 15),\ 1 \le \theta \le \left\lfloor \frac{\alpha-3}{4} \right\rfloor,$$

$$h(u'_{4\theta-1}u_{4\theta}) = 16\theta(8\theta - 3),\ 1 \leq \theta \leq \left\lfloor \frac{\alpha}{4} \right\rfloor,$$

$$h(u'_{4\theta}u_{4\theta+1}) = 128\theta^2 + 8\theta - 3,\ 1 \leq \theta \leq \left\lfloor \frac{\alpha - 1}{4} \right\rfloor,$$

$$h(v_{4\theta-3}v'_{4\theta-2}) = 4(32\theta^2 - 44\theta + 15),\ 1 \leq \theta \leq \left\lfloor \frac{\alpha + 2}{4} \right\rfloor,$$

$$h(v_{4\theta-2}v'_{4\theta-1}) = 4(32\theta^2 - 30\theta + 7),\ 1 \leq \theta \leq \left\lfloor \frac{\alpha + 1}{4} \right\rfloor,$$

$$h(v_{4\theta-1}v'_{4\theta}) = 8(16\theta^2 - 8\theta + 1),\ 1 \leq \theta \leq \left\lfloor \frac{\alpha}{4} \right\rfloor,$$

$$h(v_{4\theta}v'_{4\theta+1}) = 128\theta^2 + 8\theta - 1,\ 1 \leq \theta \leq \left\lfloor \frac{\alpha - 1}{4} \right\rfloor,$$

$$h(v'_2v_2) = 10,\ h(v'_{4\theta+1}v_{4\theta+2}) = 8(16\theta^2 + 10\theta + 1),\ 1 \leq \theta \leq \left\lfloor \frac{\alpha - 2}{4} \right\rfloor,$$

$$h(v'_{4\theta-2}v_{4\theta-1}) = 128\theta^2 + 120\theta + 25,\ 1 \leq \theta \leq \left\lfloor \frac{\alpha - 3}{4} \right\rfloor,$$

$$h(v'_{4\theta-1}v_{4\theta}) = 4(32\theta^2 - 16\theta + 1),\ 1 \leq \theta \leq \left\lfloor \frac{\alpha}{4} \right\rfloor,$$

$$h(v'_{4\theta}v_{4\theta+1}) = 2(64\theta^2 + 4\theta - 3),\ 1 \leq \theta \leq \left\lfloor \frac{\alpha - 1}{4} \right\rfloor,$$

$$h(u_{4\theta-3}v'_{4\theta-3}) = 128\theta^2 - 216\theta + 91,\ 1 \leq \theta \leq \left\lfloor \frac{\alpha + 3}{4} \right\rfloor,$$

$$h(u_{4\theta-2}v'_{4\theta-2}) = 128\theta^2 - 152\theta + 45,\ 1 \leq \theta \leq \left\lfloor \frac{\alpha + 2}{4} \right\rfloor,$$

$$h(u_{4\theta-1}v'_{4\theta-1}) = 2(64\theta^2 - 44\theta + 7),\ 1 \leq \theta \leq \left\lfloor \frac{\alpha + 1}{4} \right\rfloor,$$

$$h(u_{4\theta}v'_{4\theta}) = 8\theta(16\theta - 3),\ 1 \leq \theta \leq \left\lfloor \frac{\alpha}{4} \right\rfloor,$$

$$h(u'_1v_1) = 4,\ h(u'_{4\theta+1}v_{4\theta+1}) = 2(64\theta^2 + 20\theta + 1),\ 1 \leq \theta \leq \left\lfloor \frac{\alpha - 1}{4} \right\rfloor,$$

$$h(u'_{4\theta-2}v_{4\theta-2}) = 4(32\theta^2 - 38\theta + 11),\ 1 \leq \theta \leq \left\lfloor \frac{\alpha+2}{4} \right\rfloor,$$

$$h(u'_{4\theta-1}v_{4\theta-1}) = 128\theta^2 - 88\theta + 15,\ 1 \leq \theta \leq \left\lfloor \frac{\alpha+1}{4} \right\rfloor,$$

$$h(u'_{4\theta}v_{4\theta}) = 128\theta^2 - 24\theta + 1,\ 1 \leq \theta \leq \left\lfloor \frac{\alpha}{4} \right\rfloor,$$

which are distinct. Hence $Spl(L_\alpha)$ is a PD mean graph for all α.

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