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SCALENE TRIANGULAR FUZZY NUMBERS AND ITS OPERATIONS

THANGARAJ BEAULA and R. SELVAKUMARI

Department of Mathematics
T.B.M.L College
Porayar-609307, India

Abstract

Generalized Fuzzy number is a new concept obtained by removing the property of normality. In this paper we introduce the notion of scalene triangular fuzzy number and discussed the algebra of this fuzzy number by developing all arithmetic operations.

1. Introduction

The concepts of fuzzy numbers and fuzzy arithmetic were introduced by Zadeh [7]. Since, then general authors have investigated properties and proposed applications of fuzzy numbers. Practical problems require effective fuzzy arithmetic which would enable solving uncertain linear ones. Fuzzy numbers are used in statistics, computer programming, engineering and experimental science. The concept of fuzzy number has been defined as a fuzzy subset of real line by D. Dubois and H. Prade [3]. Possibility theory (Zadeh 1978; Dubois and Prade 1988), formal concept analysis (FCA) (Ganter and Wille 1999), extensional fuzzy sets (Hohle 1988) and rough sets (Pawlak 1991) [8].

In general, the arithmetic operations on fuzzy numbers can be approached either by the direct use of the membership function or by the equivalent use of the cuts representations. The fuzzy calculations are not immediate to be performed and in many cases they require to solve mathematically or computationally hard sub problems for which a closed form is not available.

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In many fields of different sciences (physics, engineering, political sciences etc.,) and disciplines where fuzzy sets and fuzzy logic are applied (eg., fuzzy decision making operations research and optimization) fuzzy numbers and arithmetic play a central role are frequently and increasingly the main instruments [2].

Generalized fuzzy number is a new concept obtained by removing the property of normality. In this paper we introduce the notion of scalene triangular fuzzy number and discuss the algebra of this fuzzy number by developing all arithmetic operations and non-linear arithmetic operations.

2. Preliminaries

Definitions 2.1. The Characteristic function μ_A of a crisp set $A \in X$ assigns a value either 0 or 1 to each member in X. This function can be generalized to a function $\mu_{\widetilde{A}}$ such that the value assigned to the element of the universal set X fall within a specified range ie. $\mu_{\widetilde{A}} : X \to [0, 1]$. The assigned value indicate the membership grade of the element in the set A. The function $\mu_{\widetilde{A}}$ is called the member ship function and the set $\widetilde{A} = \{(x, \mu_{\widetilde{A}}(x)) : x \in X\}$ defined by $\mu_{\widetilde{A}}(x)$ for each $x \in X$ is a called fuzzy set.

Definitions 2.2. A fuzzy set $\widetilde{A}$ defined on the universal set of real number $\mathbb{R}$, is said to be a triangular fuzzy number if its membership function has the following characteristics:

(i) $\mu_{\widetilde{A}}(x) : \mathbb{R} \to (0, 1]$ is continuous.

(ii) $\mu_{\widetilde{A}}(x) = 0$ for all $x \in (-\infty, a]U[b, \infty)$

(iii) $\mu_{\widetilde{A}}(x)$ is strictly increasing on $[a, m]$ and strictly decreasing on $[m, b]$ where $a < m < b$

(iv) $\mu_{\widetilde{A}}(x) = 1$ for $x = m$. where $0 < \lambda \leq 1$

Definitions 2.3. A fuzzy set $\widetilde{A}$ defined on the universal set of real number $\mathbb{R}$, is said to be a generalized triangular fuzzy number if its membership function has the following characteristics:

(i) $\mu_{\tilde{A}}(x) : \mathbb{R} \to (0, 1]$ is continuous.

(ii) $\mu_{\tilde{A}}(x) = \alpha$ for all $x \in (-\infty, a]U[b, \infty)$

(iii) $\mu_{\tilde{A}}(x)$ is strictly increasing on $[a, m]$ and strictly decreasing on $[m, b]$ where $a < m < b$

(iv) $\mu_{\tilde{A}}(x) = \lambda$ for $x = m$, where $0 < \lambda \leq 1$.

3. Scalene Fuzzy Number and Arithmetic Operations

Definitions 3.1. The fuzzy number $\tilde{A}_{\lambda 1}^{\lambda 2(m)}[a, b]$ with membership function

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{\lambda 2 - \lambda 1}{m - a}(x - a) + \lambda 1; \; x \in [a, m] \\ \dfrac{\lambda 2}{m - b}(x - b); \; x \in [m, b]. \end{cases} \tag{1}$$

Where $\lambda \in (0, 1]$ is called a scalene triangular fuzzy number.

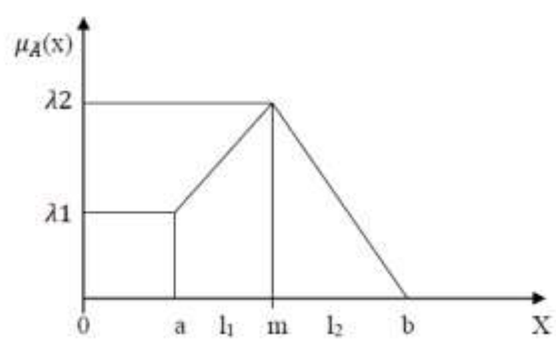

Figure 1. Scalene triangle fuzzy number $S_{\lambda 1}^{\lambda 2(m)}[a, b]$ or $S_{\lambda 1}^{\lambda 2}[a, b]$.

If we take $a = m - l_1$ and $b = m + l_2$ then this may be denoted by $A_{\lambda 1}^{\lambda 2}[a, b] = [m, l_1, l_2; \lambda_2, \lambda_1]$. The set of all these fuzzy number is denoted by ITFN $(\mathbb{R})$.

Using the definition (2.3), our proposed ITFN $(\mathbb{R})$ has the following characteristics:

(i) $\mu_{\widetilde{A}}(x) : \mathbb{R} \to (0, \lambda_2)$ is continuous

(ii) $\mu_{\widetilde{A}}(x) = \alpha$ for all $x \in (-\infty, a]$

(iii) $\mu_{\widetilde{A}}(x)$ is strictly increasing on $[a, m]$ and strictly decreasing on $[m, b]$

(iv) $\mu_{\widetilde{A}}(x) = \lambda_2$ for $x = m$, where $0 < \lambda_2 < 1$.

3.2. Arithmetic operations

Four arithmetic binary operations addition, subtraction, multiplication and division of fuzzy numbers is as follows:

Definitions 3.2. Let the fuzzy number $A_{\lambda 1}^{\lambda 2}[a, b], B_{\lambda 1}^{\lambda 2}[c, d] \in ITFN(\mathbb{R})$, then $\widetilde{A}[a, b] * \widetilde{B}[c, d] = \widetilde{N}[e, f] * \in \{+, -\times, \div\}$.

3.3. Addition

Let $\widetilde{A} = S_{\lambda 1}^{\lambda 2(m)}[a, b] = [m, l_1, l_2, \lambda_2, \lambda_1], \widetilde{B} = S_{\lambda 1}^{\lambda 2(n)}[c, d] = [n, l_3, l_4; \lambda_2, \lambda_1]$ $\in ITFN(\mathbb{R})$.

Then addition of two triangular fuzzy numbers $\widetilde{A}$ and $\widetilde{B}$ is given by

$$\widetilde{A} + \widetilde{B} = S_{\lambda 1}^{\lambda 2(m+n)}[e, f].$$

Whose membership function is defined by

$$\mu_{\widetilde{A}+\widetilde{B}}(x) = \begin{cases} \dfrac{\lambda 2 - \lambda 1}{m + n - e}(x - e) + \lambda 1; \; x \in [e, m + n] \\ \dfrac{\lambda_2}{m + n - f}(x - f); \; x \in [m + n, f] \end{cases}$$

Where $e = a + c$, $f = b + d$.

Example 3.3. Let $\widetilde{A} = S_{0.2}^{0.6(2)}[1, 4] = [2, 1, 2; 0.2, 0.6], \widetilde{B} = S_{0.2}^{0.6(4)}[2, 5]$ $= [4, 2, 9; 0.2, 0.6]$ be two scalene triangular fuzzy numbers. Then the addition of these two fuzzy numbers is defined by membership function

$$\mu_{\widetilde{A}+\widetilde{B}}(x) = \begin{cases} 0.1x - 0.1;\ x \in [3,\ 6] \\ \\ 3 - 0.2x;\ x \in [6,\ 9] \end{cases}$$

3.4. Subtraction

Let $\widetilde{A} = S_{\lambda 1}^{\lambda 2(m)}[a, b] = [m, l_1, l_2; \lambda_2, \lambda_1]$, $\widetilde{B} = S_{\lambda 1}^{\lambda 2(n)}[c, d] = [n, l_3, l_4; \lambda_2, \lambda_1]$ $\in ITFN(\mathbb{R})$.

Then subtraction of two triangular fuzzy numbers $\widetilde{A}$ and $\widetilde{B}$ is given by

$$\widetilde{A} - \widetilde{B} = S_{\lambda 1}^{\lambda 2(m-n)} = [e,\ f].$$

Whose membership function is defined by

$$\mu_{\widetilde{A}-\widetilde{B}}(x) = \begin{cases} \dfrac{\lambda 2 - \lambda 1}{m - n - e}(x - e) + \lambda 1;\ x \in [e,\ m - n] \\ \dfrac{\lambda 2}{m - n - f}(x - f);\ x \in [m - n,\ f], \end{cases}$$

Where $e = a - c,\ f = b - d$.

Example 3.4. Let $\widetilde{A} = S_{0.2}^{0.6(2)}[1,\ 4] = [2,\ 1,\ 2;\ 0.2,\ 0.6]$, $\widetilde{B} = S_{0.2}^{0.6(4)}[2,\ 5]$ $= [4,\ 2,\ 9;\ 0.2,\ 0.6]$ be two scalene triangular fuzzy numbers. Then the subtraction of these two fuzzy numbers is defined by membership function

$$\mu_{\widetilde{A}-\widetilde{B}}(x) = \begin{cases} -0.4x - 0.2;\ x \in [-1,\ -2] \\ \\ -0.6x - 0.6;\ x \in [-2,\ 1]. \end{cases}$$

3.5. Multiplication

Let $\widetilde{A} = S_{\lambda 1}^{\lambda 2(m)}[a,\ b]$ and $\widetilde{B} = S_{\lambda 1}^{\lambda 2(n)}[c,\ d]$ be two scalene triangular fuzzy numbers. It can be shown that the shape of the membership function of $\widetilde{A} \cdot \widetilde{B}$ is not necessarily a triangular, but if the spreads of $\widetilde{A}$ and $\widetilde{B}$ are small compared to their mean values m and n then the shape of the membership function is closed to a triangle. A good approximation is as follows

Let $\widetilde{A} = S_{\lambda 1}^{\lambda 2(m)}[a, b] = [m, l_1, l_2; \lambda_2, \lambda_1]$, $\widetilde{B} = S_{\lambda 1}^{\lambda 2(n)}[c, d] = [n, l_3, l_4; \lambda_2, \lambda_1]$

$\in ITFN(\mathbb{R})$. Then multiplication of two triangular fuzzy numbers $\widetilde{A}$ and $\widetilde{B}$ is given by

$$\alpha_{(\widetilde{A}\cdot\widetilde{B})} = \alpha_{\widetilde{A}} \cdot \alpha_{\widetilde{B}}$$

$$\alpha_{(\widetilde{A}\cdot\widetilde{B})} = \left[\frac{(\alpha'-\lambda 1)(m-a)}{(\lambda 2-\lambda 1)}+a, \frac{a'(m-b)}{\lambda 2}+b\right]\left[\frac{(\alpha'-\lambda 1)(n-c)}{(\lambda 2-\lambda 1)}+c, \frac{a'(n-d)}{\lambda 2}+d\right]$$

$$= \left[\min\left[\begin{array}{l} \frac{(\alpha'-\lambda 1)^2(m-a)(n-c)}{(\lambda 2-\lambda 1)^2}+\frac{c(\alpha'-\lambda 1)(m-a)}{(\lambda 2-\lambda 1)}+\frac{a(\alpha'-\lambda 1)(n-c)}{(\lambda 2-\lambda 1)} \\ +ac, \frac{\alpha'(\alpha'-\lambda_1)(m-a)(n-d)}{\lambda_1(\lambda_2-\lambda_1)}+\frac{d(\alpha'-\lambda_1)(m-a)}{(\lambda_2-\lambda_1)}+\frac{a\alpha'(n-d)}{\lambda_2}+ad \\ \frac{\alpha'(\alpha'-\lambda_1)(m-b)(n-c)}{\lambda_1(\lambda_2-\lambda_1)}+\frac{b(\alpha'-\lambda_1)(n-c)}{(\lambda_2-\lambda_1)}+\frac{c\alpha'(m-b)}{\lambda_2}+bc, \\ \frac{\alpha'^2(m-b)(n-d)}{\lambda_2^2}+\frac{d\alpha'(m-b)}{\lambda_2}+\frac{b\alpha'(n-d)}{\lambda_2}+bd \end{array}\right]\right]$$

$$\max\left[\begin{array}{l} \frac{(\alpha'-\lambda 1)^2(m-a)(n-c)}{(\lambda 2-\lambda 1)^2}+\frac{c(\alpha'-\lambda 1)(m-a)}{(\lambda 2-\lambda 1)}+\frac{a(\alpha'-\lambda 1)(n-c)}{(\lambda 2-\lambda 1)} \\ +ac, \frac{\alpha'(\alpha'-\lambda_1)(m-a)(n-d)}{\lambda_1(\lambda_2-\lambda_1)}+\frac{d(\alpha'-\lambda_1)(m-a)}{(\lambda_2-\lambda_1)}+\frac{a\alpha'(n-d)}{\lambda_2}+ad \\ \frac{\alpha'(\alpha'-\lambda_1)(m-b)(n-c)}{\lambda_1(\lambda_2-\lambda_1)}+\frac{b(\alpha'-\lambda_1)(n-c)}{(\lambda_2-\lambda_1)}+\frac{c\alpha'(m-b)}{\lambda_2}+bc, \\ \frac{\alpha'^2(m-b)(n-d)}{\lambda_2^2}+\frac{d\alpha'(m-b)}{\lambda_2}+\frac{b\alpha'(n-d)}{\lambda_2}+bd \end{array}\right]$$

Example 3.5. Let $\widetilde{A} = S_{0.2}^{0.6(2)}[1, 4] = [2, 1, 2; 0.2, 0.6]$, $\widetilde{B} = S_{0.2}^{0.6(4)}[2, 5] = [4, 2, 9; 0.2, 0.6]$ be two scalene triangular fuzzy numbers. Then the multiplication of these two fuzzy numbers is defined by membership function

$$\mu_{\widetilde{A}\cdot\widetilde{B}}(x) = \begin{cases} 0;\ x < 0.5,\ x > 20 \\ 0.25 \pm (0.002 + 0.128x)^{1/2}0.5;\ 1.938 \le x < 14.957 \\ 0.651 \pm (-0.6667)(1.631 - 0.1056x)^{1/2};\ 14.957 \le x < 16.982 \\ 0.071 \pm (-0.094)(0.669 - 0.026x)^{1/2},\ 16.982 \le x < 20 \end{cases}$$

3.6. Division

Let $\widetilde{A} = S_{\lambda 1}^{\lambda 2(m)}[a, b] = [m, l_1, l_2; \lambda_2, \lambda_1], \widetilde{B} = S_{\lambda 1}^{\lambda 2(n)}[c, d] = [n, l_3, l_4; \lambda_2, \lambda_1]$ $\in ITFN(\mathbb{R})$.

Then division of two triangular fuzzy numbers $\widetilde{A}$ and $\widetilde{B}$ is given by

$$\alpha_{(\widetilde{A}/\widetilde{B})} = \alpha_{\widetilde{A}} / \alpha_{\widetilde{B}}$$

$$\alpha_{(\widetilde{A}/\widetilde{B})} = \left[\frac{(\alpha' - \lambda 1)(m - a)}{(\lambda 2 - \lambda 1)} + a, \frac{\alpha'(m - b)}{\lambda 2} + b\right] \Big/ \left[\frac{(\alpha' - \lambda 1)(n - c)}{(\lambda 2 - \lambda 1)} + c, \frac{\alpha'(n - d)}{\lambda 2} + d\right]$$

$$\left[\min\begin{bmatrix} \dfrac{(\alpha' - \lambda_1)(m - a) + a(\lambda_2 - \lambda_1)}{(\alpha' - \lambda_1)(n - c) + c(\lambda_2 - \lambda_1)}, \dfrac{\lambda_2((\alpha' - \lambda_1)(m - a) + a(\lambda_2 - \lambda_1))}{(\lambda_2 - \lambda_1)(\alpha'(n - d) + \lambda_2 d)}, \\ \dfrac{(\lambda_2 - \lambda_1)(\alpha'(m - b) + \lambda_2 b)}{\lambda_2((\alpha' - \lambda_1)(n - c) + c(\lambda_2 - \lambda_1))}, \dfrac{\alpha'(m - b) + \lambda_2 b}{\alpha'(n - d) + \lambda_2 d} \end{bmatrix}\right],$$

$$\left[\begin{bmatrix} \dfrac{(\alpha' - \lambda_1)(m - a) + a(\lambda_2 - \lambda_1)}{(\alpha' - \lambda_1)(n - c) + c(\lambda_2 - \lambda_1)}, \dfrac{\lambda_2((\alpha' - \lambda_1)(m - a)a(\lambda_2 - \lambda_1))}{(\lambda_2 - \lambda_1)(\alpha'(n - d) + \lambda_2 d)}, \\ \dfrac{(\lambda_2 - \lambda_1)(\alpha'(m - b) + \lambda_2 b)}{\lambda_2((\alpha' - \lambda_1)(n - c) + c(\lambda_2 - \lambda_1))}, \dfrac{\alpha'(m - b) + \lambda_2 b}{\alpha'(n - d) + \lambda_2 d} \end{bmatrix}\right].$$

Example 3.6. Let $\widetilde{A} = S_{0.2}^{0.6(2)}[1, 4] = [2, 1, 2; 0.2, 0.6], \widetilde{B} = S_{0.2}^{0.6(4)}[2, 5] =$ $[4, 2, 9; 0.2, 0.6]$ be two scalene triangular fuzzy numbers. Then the division of these two fuzzy numbers is defined by membership function

$$\mu_{\widetilde{A}/\widetilde{B}}(x) = \begin{cases} 0; \; x < -2, \; x > 1.333 \\ \dfrac{0.25 - 1.5x}{1 - 2x}; \; -2 \le x < 0.499 \\ \dfrac{3.75x - 3}{x - 2}; \; 0.499 \le x < 0.8 \\ \dfrac{0.375x + 0.75}{0.5 + 1.5x}; \; 0.8 \le x < 1.333 \end{cases}$$

4. Conclusion

This is attempt to define a new kind of Fuzzy number Scalene triangular fuzzy number. Using this fuzzy number many decision making problem can be solved. Wide variety of application in the field of Applied Mathematics.

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