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ON SOLVING NEUTROSOPHIC UNCONSTRAINED OPTIMIZATION PROBLEMS BY NEWTON'S METHOD

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Abstract

In this paper, we proposed a method for solving unconstrained optimization problems by Newton's method with single valued neutrosophic triangular and trapezoidal fuzzy number coefficients. Also, some numerical examples demonstrate the effectiveness of the proposed algorithm. MATLAB programs are also developed for the proposed method.

Introduction

The concept of Neutrosophic Set (NS) was first introduced by Smarandache which is a generalization of fuzzy sets. Zadeh's classical concepts of fuzzy set is a strong mathematics tool to deal with the complexity generally arising from uncertainty in the form of ambiguity in real life scenario. In 1965, Zadeh invented the "Fuzzy sets", which play a significant role in dealing with ambiguity and impreciseness. In 1970, Bellman and Zadeh developed "a method for making decisions in a fuzzy environment". In 1983, Atanassov introduced his intuitionistic fuzzy set. "A Newton method for nonlinear unconstrained optimization problems with two variables" was

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proposed by Porchelvi and Sathya [15]. Irene Hepzibah R. et al. [21] proposed a "neutrosophic multi objective linear programming problems". This paper, deals with fuzzy Newton's method with single valued neutrosophic triangular coefficient, single valued neutrosophic trapezoidal coefficient to solve unconstrained optimization problems. This paper is organised as follows. The second section provides some background information on this research topic. Several strategies for solving unconstrained optimization problems in a neutrosophic fuzzy environment are proposed in section three. In section four, some illustrative cases are offered to demonstrate this method.

Preliminaries

This section provides an introduction to fuzzy unconstrained optimization models and stressed the importance to consider the topics like linear and nonlinear optimization problems in fuzzy environment using arithmetic operations and provides certain definitions which are related to this research work. In this section, the concept of single valued neutrosophic number, single valued trapezoidal neutrosophic numbers and single valued triangular neutrosophic number with operations are introduced.

Definition 1[7]. Let $\widetilde{A} = \{x, \mu_{\widetilde{A}}(x) : x \in \mu_{\widetilde{A}}(x) \in [0, 1]\}$ is a fuzzy set. The first element $\mathbb{X}$ in the pair $(x, \mu_{\widetilde{A}}(x))$ belongs to the classical set A, whereas the second element $\mu_{\widetilde{A}}(x)$ belongs to the interval $[0, 1]$ known as membership function, indicated by $\widetilde{A} = \{\mu_{\widetilde{A}}(x) \setminus x : x \in A, \mu_{\widetilde{A}}(x) \in [0, 1]\}$.

Definition 2[7]. Let $\Re$ be the set of real numbers and $\widetilde{A} : \Re \to [0, 1]$ be a fuzzy set. Then we say that $\widetilde{A}$ is a fuzzy number that contains the following properties:

(i) 0 is normal, i.e., there exist $x_0 \in \Re$ such that $\widetilde{A}(x_0) = 1$,

(ii) $\widetilde{A}$ is convex, i.e., $\widetilde{A}(tx + (1 - t))y \geq \min\{\widetilde{A}(x), \widetilde{A}(y)\}$, where $x, y \in \Re$ and $t \in [0, 1]$.

(iii) $\widetilde{A}(x)$ is upper semi-continuous on $\Re$, i.e., $\left\{\frac{x}{\widetilde{A}(x)} \geq \alpha\right\}$ is a closed subset of $\Re$ for each $\alpha \in [0, 1]$.

Definition 3[7]**.** Let us take a fuzzy number $\widetilde{\mathbb{A}}$ on $\Re$ is said to be a Triangular Fuzzy Number (TFN) or linear fuzzy number if its membership function $\widetilde{\mathbb{A}} : \Re \to [0, 1]$ meets the following features. It is a fuzzy number represents with three points as follows $\widetilde{\mathbb{A}} = (a_1, a_2, a_3)$. The following conditions apply to this representative, which is regarded as membership functions:

(i) a_1 to a_2 is increasing function.

(ii) a_2 to a_3 is decreasing function.

(iii) $a_1 \leq a_2 \leq a_3$

$$\mu_{\widetilde{\mathbb{A}}}(x) = \begin{cases} 0 \text{ for } x < a_1 \\ \dfrac{x - a_1}{a_2 - a_1} \text{ for } a_1 \leq x \leq a_2 \\ \dfrac{a_3 - x}{a_3 - a_2} \text{ for } a_2 \leq x \leq a_3 \\ 0 \text{ otherwise} \end{cases}$$

The TFN is diagrammatically shown below.

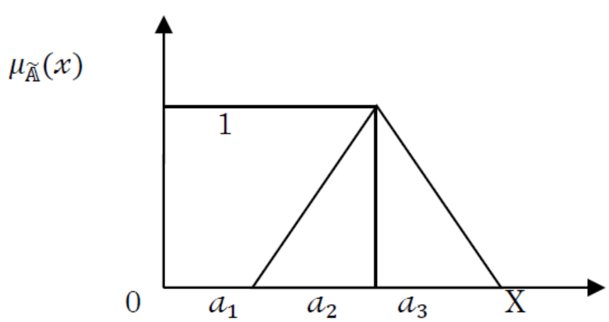

Figure 1.1. Triangular fuzzy number.

Let $F(\Re)$ to denote the set of all TFNs. The α level set of $\widetilde{\mathbb{A}}$ is defined as

$$\widetilde{\mathbb{A}}_\alpha = [(a_2 - a_1)\alpha + a_1,\ a_3 - (a_3 - a_2)\alpha].$$

Definition 4[7]**.** Let take a fuzzy number $\widetilde{\mathbb{A}}$ on $\Re$ is said to be a trapezoidal fuzzy number (TrFN) or linear fuzzy number if the membership function $\widetilde{\mathbb{A}} : \Re \to [0, 1]$ of $\widetilde{\mathbb{A}}$ on $\Re$ meets the has the following features, it is said to be a trapezoidal fuzzy number (TrFN) or linearly fuzzy number. It's a fuzzy number that would be expressed by four points $\widetilde{\mathbb{A}} = (a_1, a_2, a_3, a_4)$ such that $a_1 \leq a_2 \leq a_3 \leq a_4$ with the membership function defined as

$$\mu_{\widetilde{\mathbb{A}}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1} \text{ for } a_1 \leq x \leq a_2 \\ 1, \text{ for } a_2 \leq x \leq a_3 \\ \dfrac{a_4 - x}{a_4 - a_3} \text{ for } a_2 \leq x \leq a_3 \\ 0 \text{ otherwise} \end{cases}$$

The TrFN is diagrammatically shown below.

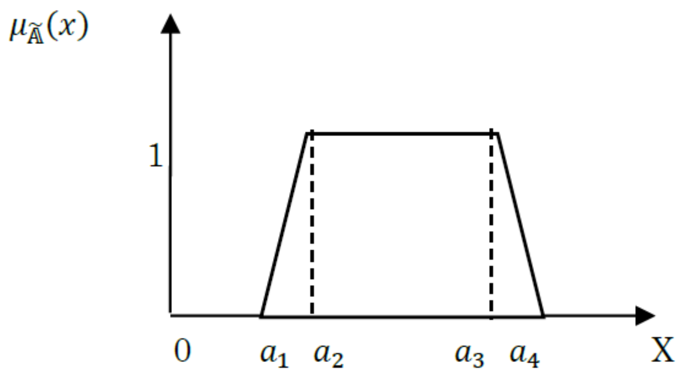

Figure 1.1. Trapezoidal fuzzy number.

Definition 5[21]**.** Let G be a universe. A Neutrosophic Set (NS) A in G is characterised by a truth-membership function T_A, a indeterminacy-membership function I_A, and a falsity membership function F_A. $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard elements of $[0, 1]$. It can be written as $A = \{x, (T_A(x), I_A(x) F_A(x)) >: x \in G, T_A(x), I_A(x), F_A(x) \in]^{-}0, 1[^{+}\}$. There is no restriction on the sum of $T_A(x), I_A(x)$ and $F_A(x)$, so on $0^{-} \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^{+}$.

Definition 6[21]**.** Let G be a universe. A Single Valued Neutrosophic Set (SVNS) A, which can be used I a real scientific and engineering applications, in G is characterised by a truth membership function T_A, an indeterminacy-membership function I_A and a falsity membership function F_A. $T_A(x)$, $I_A(x)$ and $F_A(x)$, are real standard elements of $[0, 1]$. It can be written as $A = \{\langle x, (T_A(x), I_A(x) F_A(x))\rangle : x \in E, T_A(x), I_A(x)$ and $F_A \in [0, 1]\}$. There is no restriction on the sum of $T_A(x), F_A(x)$ and $I_A(x)$, so $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

Definition 7[21]**.** Let $j_{\widetilde{a}}, h_{\widetilde{a}}, q_{\widetilde{a}} \to [0, 1]$ be any real numbers. A single valued neutrosophic number $\widetilde{a} = (((s_1, d_1, f_1, g_1), j_{\widetilde{a}}), ((s_2, d_2, f_2, g_2), h_{\widetilde{a}}), ((s_3, d_3, f_3, g_3), q_{\widetilde{a}}))$, is defined as a special single valued neutrosophic set on the set of real numbers R, whose truth-membership function $\mu_{\widetilde{a}} : R \to [0, j_{\widetilde{a}}]$, a determinacy-membership function $\nu_{\widetilde{a}} : R \to [0, h_{\widetilde{a}}]$ and a falsity-membership function $\lambda_{\widetilde{a}} : R \to [0, q_{\widetilde{a}}]$ as given by

$$\mu_{\widetilde{a}}(x) = \begin{cases} f_{\mu l}(x)\ (s_1 \leq x \leq d_1) \\ j_{\widetilde{a}}\ (d_1 \leq x \leq f_1) \\ f_{\mu r}(x)\ (f_1 \leq x \leq g_1) \\ 0 \text{ otherwise} \end{cases}$$

$$\nu_{\widetilde{a}}(x) = \begin{cases} f_{\lambda l}(x)\ (s_3 \leq x \leq d_3) \\ h_{\widetilde{a}}\ (d_3 \leq x \leq f_3) \\ f_{\lambda r}(x)\ (f_3 \leq x \leq g_3) \\ 1 \text{ otherwise} \end{cases}$$

$$\lambda_{\widetilde{a}}(x) = \begin{cases} f_{\nu l}(x)\ (s_2 \leq x \leq d_2) \\ q_{\widetilde{a}}\ (d_2 \leq x \leq f_2) \\ f_{\nu r}(x)\ (f_2 \leq x \leq g_2) \\ 1 \text{ otherwise.} \end{cases}$$

2.2 Arithmetic operations of single valued triangular neutrosophic numbers [21]. Let $\widetilde{a} = ((s_1, d_1, f_1), j_{\widetilde{a}}, h_{\widetilde{a}}, q_{\widetilde{a}})$ and

$\tilde{b} = ((s_2, d_2, f_2), j_{\tilde{b}}, h_{\tilde{b}}, q_{\tilde{b}})$ be two single valued triangular neutrosophic numbers. Then

Addition.

$$\tilde{a} + \tilde{b} = ((s_1 + s_2, d_1 + d_2, f_1 + f_2), (j_{\tilde{a}} \wedge j_{\tilde{b}}, h_{\tilde{a}} \vee h_{\tilde{b}}, q_{\tilde{a}} \vee q_{\tilde{b}}))$$

Subtraction.

$$\tilde{a} - \tilde{b} = ((s_1 - f_3, d_2 - d_1, f_3 - s_1), (j_{\tilde{a}} \wedge j_{\tilde{b}}, h_{\tilde{a}} \vee h_{\tilde{b}}, q_{\tilde{a}} \vee q_{\tilde{b}}))$$

Multiplication.

$$\tilde{a} \cdot \tilde{b} = ((s_1 \cdot R(\tilde{b}), d_1 \cdot R(\tilde{b}), f_1 \cdot R(\tilde{b})), (j_{\tilde{a}} \wedge j_{\tilde{b}}, h_{\tilde{a}} \vee h_{\tilde{b}}, q_{\tilde{a}} \vee q_{\tilde{b}})), \text{ where}$$

$$R(\tilde{b}) = ((s_2 + d_2 + f_2) \times (2 + j_b + h_{\tilde{b}} - q_{\tilde{b}}))/8$$

Division.

$$\tilde{a}/\tilde{b} = ((s_1/R(\tilde{b}), d_1/R(\tilde{b}), f_1/R(\tilde{b})), (j_{\tilde{a}} \wedge j_{\tilde{b}}, h_{\tilde{a}} \vee h_{\tilde{b}}, q_{\tilde{a}} \vee q_{\tilde{b}})), \text{ where}$$

$$R(\tilde{b}) = ((s_2 + d_2 + f_2) \times (2 + j_b + h_{\tilde{b}} - q_{\tilde{b}}))/8$$

Scalar Multiplication.

$$\gamma\tilde{a} = ((\gamma s_1, \gamma d_1, \gamma f_1), j_{\tilde{a}}, h_{\tilde{a}}, q_{\tilde{a}}) \text{ where } \gamma > 0$$

$$\gamma\tilde{a} = ((\gamma g_1, \gamma f_1, \gamma d_1), j_{\tilde{a}}, h_{\tilde{a}}, q_{\tilde{a}}) \text{ where } \gamma < 0.$$

Definition 8[21]**.** Let $\tilde{a} = ((s_1, d_1, f_1), j_{\tilde{a}}, h_{\tilde{a}}, q_{\tilde{a}})$ be a single valued triangular neutrosophic number. Then $S(\tilde{a}) = ((s_1 + d_1 + f_1) \times (2 + j_{\tilde{a}} + h_{\tilde{a}} - q_{\tilde{a}}))/8$ and $A(\tilde{a}) = ((s_1 + d_1 + f_1) \times (2 + j_{\tilde{a}} - h_{\tilde{a}} - q_{\tilde{a}}))/8$ are called score and accuracy degrees of $\tilde{a}$ respectively.

Definition 9[21]**.** A Single Valued Trapezoidal Neutrosophic Number (SVTN) $\tilde{a} = ((s_1, d_1, f_1, g_1), j_{\tilde{a}}, h_{\tilde{a}}, q_{\tilde{a}})$ is defined as a special neutrosophic set on the real number set R, whose truth-membership, indeterminacy, and a falsity membership are given as follows:

$$\mu_{\tilde{a}}(x) = \begin{cases} (x - a) j_{\tilde{a}} / (d_1 - s_1) \ (s_1 \le x \le d_1) \\ j_{\tilde{a}} \ (d_1 \le x \le f_1) \\ (g_1 - x) j_{\tilde{a}} / (g_1 - f_1) \ (f_1 \le x \le g_1) \\ 0 \text{ otherwise} \end{cases}$$

$$\nu_{\tilde{a}}(x) = \begin{cases} (s_1 - x + h_{\tilde{a}}(x - d_1)) \ (s_1 \le x \le d_1) \\ h_{\tilde{a}} \ (d_1 \le x < f_1) \\ ((x - f_1 + h_{\tilde{a}}(g_1 - x))/(g_1 - f_1) \ (f_1 \le x < g_1) \\ 1 \text{ otherwise} \end{cases}$$

$$\lambda_{\tilde{a}}(x) = \begin{cases} (d_1 - x) h_{\tilde{a}}(x - s_1)/(d_1 - d_1) \ (s_1 \le x \le d_1) \\ h_{\tilde{a}} \ (d_1 \le x < f_1) \\ ((x - f_1 + h_{\tilde{a}}(g_1 - x))/(g_1 - f_1) \ (f_1 \le x < g_1) \\ 1 \text{ otherwise} \end{cases}$$

respectively. If $s_1 \ge 0$ and at least $d_1 > 0$, then $\tilde{a} = ((s_1, d_1, f_1, g_1), j_{\tilde{a}}, h_{\tilde{a}}, q_{\tilde{a}})$ is called a positive single valued neutrosophic number denoted by $\tilde{a} < 0$. Likewise, if $d_1 > 0$ and $\tilde{a} < 0$ at least, then $\tilde{a} = ((s_1, d_1, f_1, g_1), j_{\tilde{a}}, h_{\tilde{a}}, q_{\tilde{a}})$ is called a negative single valued trapezoidal neutrosophic number, denoted by $\tilde{a} < 0$. A single value trapezoidal neutrosophic number $\tilde{a} = ((s_1, d_1, f_1, g_1), j_{\tilde{a}}, h_{\tilde{a}}, q_{\tilde{a}})$ may be represented as an ill-known quantity of the range, which is approximately equal to the interval $[d_1, f_1]$.

2.3 Arithmetic operations of single valued trapezoidal neutrosophic numbers [21]. Let $\tilde{a} = ((s_1, d_1, f_1, g_1), j_{\tilde{a}}, h_{\tilde{a}}, q_{\tilde{a}})$ and $\tilde{b} = ((s_2, d_2, f_2, g_2), j_{\tilde{a}}, h_{\tilde{a}}, q_{\tilde{a}})$ be two single valued trapezoidal neutrosophic numbers. Then

Addition.

$$\tilde{a} + \tilde{b} = ((s_1 + s_2, d_1 + d_2, f_1 + f_2, g_1 + g_2), (j_{\tilde{a}} \wedge j_{\tilde{b}}, h_{\tilde{a}} \vee h_{\tilde{b}}, q_{\tilde{a}} \vee q_{\tilde{b}}))$$

Subtraction.

$$\tilde{a} - \tilde{b} = ((s_1 - g_4,\ d_2 - f_3,\ f_3 - d_2,\ g_4 - s_1),\ (j_{\tilde{a}} \wedge j_{\tilde{b}},\ h_{\tilde{a}} \vee h_{\tilde{b}},\ q_{\tilde{a}} \vee q_{\tilde{b}}))$$

Multiplication.

$$\tilde{a} \cdot \tilde{b} = ((s_1 \cdot R(\tilde{b}),\ d_1 \cdot R(\tilde{b}),\ f_1 \cdot R(\tilde{b}),\ g_1 \cdot R(\tilde{b})),\ (j_{\tilde{a}} \wedge j_{\tilde{b}},\ h_{\tilde{a}} \vee h_{\tilde{b}},\ q_{\tilde{a}} \vee q_{\tilde{b}})),$$

$$\text{where } R(\tilde{b}) = ((s_2 + d_2 + f_2 + g_2) \times (2 + j_{\tilde{b}} + h_{\tilde{b}} - q_{\tilde{b}}))/16$$

Division.

$$\tilde{a}/\tilde{b} = ((s_1/R(\tilde{b}),\ d_1/R(\tilde{b}),\ f_1/R(\tilde{b}),\ g_1/R(\tilde{b})),\ (j_{\tilde{a}} \wedge j_{\tilde{b}},\ h_{\tilde{a}} \vee h_{\tilde{b}},\ q_{\tilde{a}} \vee q_{\tilde{b}})),$$

$$\text{where } R(\tilde{b}) = ((s_2 + d_2 + f_2 + g_2) \times (2 + j_{\tilde{b}} + h_{\tilde{b}} - q_{\tilde{b}}))/16$$

Scalar Multiplication.

$$\gamma\tilde{a} = ((\gamma s_1,\ \gamma d_1,\ \gamma f_1,\ \gamma g_1),\ j_{\tilde{a}},\ h_{\tilde{a}},\ q_{\tilde{a}}) \text{ where } \gamma > 0$$

$$\gamma\tilde{a} = ((\gamma g_1,\ \gamma f_1,\ \gamma d_1,\ \gamma s_1),\ j_{\tilde{a}},\ h_{\tilde{a}},\ q_{\tilde{a}}) \text{ where } \gamma < 0.$$

Definition 10[21]**.** Let $\tilde{a} = ((s_1, d_1, f_1, g_1), j_{\tilde{a}}, h_{\tilde{a}}, q_{\tilde{a}})$ be a single valued trapezoidal neutrosophic number. Then $S(\tilde{a}) = ((s_1 + d_1 + f_1 + g_1) \times (2 + j_{\tilde{b}} - h_{\tilde{b}} - q_{\tilde{b}}))/16$ and $A(\tilde{a}) = ((s_1 + d_1 + f_1 + g_1) \times (2 + j_{\tilde{b}} + h_{\tilde{b}} - q_{\tilde{b}}))/16$ are called score and accuracy degrees of $\tilde{a}$ respectively.

3. Proposed Algorithms to Solve Unconstrained Optimization Problems

3.1 Newton's Method [13]**.** Gradient search can be viewed as pursuing the move direction suggested by the first order Taylors series approximation.

Aligning Δx with gradient $\nabla f(x^{(t)})$ produces the most rapid improvement in this first order approximation to $f(x)$.

To improve on the slow, zigzagging progress characteristic of gradient search requires more information.

Newton Step. Until the first order Taylor approximation, which is linear in directional components Δx_j, the quadratic second order version may have a local optimum of the second order approximation, we may fix $\lambda = 1$ and differentiable f_2 with respect to components of Δx with $\lambda = 1$, the scalar notation form of f_2 is

$$f_2(x^{(t)}) + \Delta x \triangleq f(x^{(t)}) + \sum_{i=1}^{n}\left(\frac{\partial f}{\partial x_i}\right)\Delta x_i + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=i}^{n}\left(\frac{\partial^2 f}{\partial x_i \partial x_j}\right)\Delta x_i \Delta x_j$$

$$\frac{\partial f_2}{\partial \Delta x_i} = \left(\frac{\partial f}{\partial x_i}\right) + \sum_{j=i}^{n}\left(\frac{\partial^2 f}{\partial x_i \partial x_j}\right)\Delta x_i,\ i = 1, 2, 3, \ldots.$$

or in matrix format $\Delta f_2(\Delta x) = \Delta f(x^{(t)}) + H(x^{(t)})\Delta x.$ Either way, setting $\Delta f_2(x) = 0$ to find a stationary point produces the famous Newton step.

"Consider $H(x^{(t)})\Delta x = -\nabla f(x^{(t)}),$ solving the linear equation system, Newton step Δx can be obtained, which moves to a stationary point (if there is one) of the 2nd order Taylor series approximation to $f(x)$ at current point $X^{(t)}$ immediately" in [11].

3.2 Algorithm for Newton method [13]**.**

Step 1. Initialization. Choose any starting solution $(x^{(0)})$, pick stopping tolerance $\in < 0$, and the solution index $t \leftarrow 0$.

Step 2. Derivatives. Generate the gradient of both the objective function $\nabla f(x^{(t)})$ as well as the Hessian matrix $H(x^{(t)})$ well at current point $(x^{(t)})$.

Step 3. Stationary point. If $\| \nabla f(x^{(t)}) \| < \in,$ stop. Point $(x^{(0)})$ is sufficiently close to a stationary point.

Step 4. Newton move. The linear system should always be computed. For the Newton move $\Delta(x^{(t+1)}),\ H(x^{(t)})\Delta x = -\nabla f(x^{(t)}).$

Step 5. New point $(x^{(t+1)}) \leftarrow (x^{(t)}) + \Delta(x^{(t+1)}).$

Step 6. Increment $t \leftarrow t + 1,$ and return to step 1.

3.3 Single valued neutrosophic fuzzy Newton algorithm

Step 1. The objective function can be obtained by replacing crisp parameters with single valued neutrosophic numbers.

Step 2. Initialization. Choose any starting solution $(\widetilde{x}^{(0)})$, pick stopping tolerance $\in < 0$, and the solution index $t \leftarrow 0$.

Step 3. Derivatives. Generate the gradient of both the objective function with single valued neutrosophic numbers $\nabla\widetilde{f}(\widetilde{x}^{(t)})$ as well as the Hessian matrix $\widetilde{H}(\widetilde{x}^{(t)})$ well at current point $(\widetilde{x}^{(t)})$.

Step 4. Stationary point. If $\| \nabla\widetilde{f}(\widetilde{x}^{(t)}) \| < \in$, stop. Point $(\widetilde{x}^{(t)})$ is sufficiently close to a stationary point.

Step 5. Newton move. The linear system should always be computed. For the Newton move $\Delta(\widetilde{x}^{(t+1)})$, $\widetilde{H}(\widetilde{x}^{(t)})\Delta\widetilde{x} = -\nabla\widetilde{f}(\widetilde{x}^{(t)})$.

Step 6. New point $(\widetilde{x}^{(t+1)}) \leftarrow (\widetilde{x}^{(t)}) + \Delta(\widetilde{x}^{(t+1)})$.

Step 7. Increment $t \leftarrow t + 1$, and return to step 1.

4. Numerical Illustrations

Here are some numerical examples to test the efficiency of the proposed techniques.

Example 1.

Case (i). Let us consider the unconstrained optimization problem with single valued neutrosophic triangular fuzzy number coefficients,

$$\widetilde{\mathfrak{T}}(x, y) = ((0.5, 1, 1.5), (0.6, 0.5, 0.5))x^3 - ((2, 3, 4)(0.6, 0.6, 0.5))xy + ((0.5, 1, 1.5), (0.6, 0.5, 0.5))y^3.$$

Solving this problem by the algorithm proposed in section 3.3, the MATLAB outputs are tabulated here.

Iteration	(x_i, y_i)	(x_{i+1}, y_{i+1})
11	((0.5000000, 1.0000000, 1.5000000); (0.6600, 0.500, 0.500))	((0.21500000, 1.1428000, 2.0718000); (0.6600, 0.500, 0.500)
	((1.5000000, 2.0000000, 2.5000000); (0.6600, 0.500, 0.500))	((-0.1901000, 1.2860000, 2.7648000); (0.6600, 0.500, 0.500))
22	((0.2150000, 1.1428000, 2.0718000); (0.6600, 0.500, 0.500))	((0.0601000, 1.0279000, 2.0010000); (0.6600, 0.500, 0.500))
	((-0.1901000, 1.2860000, 2.7648000); (0.6600, 0.500, 0.500))	((-0.5543000, 1.0428000, 0.6427000); (0.6600, 0.500, 0.500))

Case (ii). Let us consider the following unconstrained optimization problems with single valued neutrosophic trapezoidal fuzzy number coefficients

$$\widetilde{\mathcal{T}}(x, y) = ((0.5, 0.75, 1, 1.75), (0.6, 0.5, 0.5))\widetilde{x}^3$$

$$-((1.5, 2.25, 3, 5.25), (0.7, 0.5, 0.5))\widetilde{x}\widetilde{y}$$

$$+((0.5, 0.75, 1, 1.75), (0.6, 0.5, 0.5))\widetilde{y}^3.$$

Solving this problem by the algorithm proposed in section 3.3, the MATLAB outputs are tabulated here.

Iteration	(x_i, y_i)	(x_{i+1}, y_{i+1})
1	((0.5000000, 0.7500000, 1.0000000, 1.7500000); (0.600, 0.500, 0.500))	(0.7500000, 0.8928570, 1.1071430, 0.8214290); (0.600, 0.500, 0.500))
	(1.500000, 1.7500000, 2.0000000, 2.7500000); (0.600, 0.500, 0.500))	(0.2500000, 1.0357140, 1.4642860, 2.3928570); (0.600, 0.500, 0.500))

2	((0.7500000, 0.8928570, 1.1071430, 0.8214290); (0.600, 0.500, 0.500))	((0.5476, 0.7772, 1.0204350, 1.7636); (0.600, 0.500, 0.500))
	((0.2500000, 1.0357140, 1.4642860, 2.3928570); (0.600, 0.500, 0.500))	(-0.1723, 0.7943, 1.2832, 2.2721); (0.600, 0.500, 0.500))

Illustrative Example 2

Case (i). Consider the unconstrained optimization with single valued neutrosophic triangular fuzzy number coefficients as follows,

$$\tilde{\mathfrak{T}}(x,\ y) = ((0.5,\ 1,\ 1.5),\ (0.7,\ 0.5,\ 0.6))x - ((0.5,\ 1,\ 1.5),\ (0.6,\ 0.5,\ 0.5))y$$

$$+ ((1.5,\ 2,\ 2.5),\ (0.7,\ 0.5,\ 0.6))xy + ((1.5,\ 2,\ 2.5),\ (0.6,\ 0.5,\ 0.5))x^2$$

$$+ ((0.5,\ 1,\ 1.5),\ (0.6,\ 0.5,\ 0.7))y^2.$$

Solving this problem by the algorithm proposed in section 3.3, the MATLAB outputs are tabulated here.

Iteration	$(x_i,\ y_i)$	$(x_{i+1},\ y_{i+1})$
1	((0,0,0), (0.76000, 0.5000, 0.6000))	(-0.510, -1.0012, -1.501), (0.7600, 0.50, 0.60)
	((0,0,0), (0.76000, 0.5000, 0.6000))	((0.7510, 1.5010, 2.2510), (0.7600, 0.500, 0.600))

Case (ii). Consider the unconstrained optimization with single valued neutrosophic trapezoidal fuzzy number coefficients shown below.

$$\tilde{\mathfrak{T}}(x,\ y) = ((0.5,\ 0.75,\ 1,\ 1.75),\ (0.7,\ 0.5,\ 0.6))x$$

$$-((0.5,\ 0.75,\ 1,\ 1.75),\ (0.6,\ 0.5,\ 0.5))y$$

$$+((1.5,\ 1.75,\ 2,\ 2.75),\ (0.7,\ 0.5,\ 0.6))xy$$

$$+ ((1.5,\ 1.75,\ 2,\ 2.25),\ (0.6,\ 0.5,\ 0.5))x^2$$

$$+ ((0.5,\ 0.75,\ 1,\ 1.75),\ (0.6,\ 0.5,\ 0.7))y^2.$$

Solving this problem by the algorithm proposed in section 3.3, the MATLAB outputs are tabulated here.

Iteration	(x_i, y_i)	(x_{i+1}, y_{i+1})
1	(0, 0, 0, 0), (0.66, 0.5, 0.5)	(-1.5625, -1, -0.8125, -0.625); (0.66, 0.500, 0.500)
	(0, 0, 0, 0), (0.66, 0.5, 0.5)	(1.125, 1.3125, 1.5, 2.0625); (0.66, 0.500, 0.500)

Conclusion

A new technique for handling fuzzy unconstrained optimization algorithms is suggested in this work. In addition, single valued neutrosophic triangle and single valued neutrosophic trapezoidal coefficients are used. For tackling fuzzy unconstrained optimization problems, Newton's approach is employed, and the validity of the proposed method is tested using numerical examples and MATLAB programme outputs.

References

[1] K. T. Atanassov, Intuitionistic Fuzzy Sets, VII ITKR's Session, Sofia, Bulgarian, 1983.

[2] Athar Kharal, A neutrosophic multi-criteria decision making method, New Mathematics and Natural Computation 10(2) (2014), 143-162.

[3] R. E. Bellman and L. A. Zadeh, Decision making in a fuzzy environment, Management Science 17(4) (1970), 141-164.

[4] F. Ben Aicha, F. Bouani and M. Ksouri, A multivariable multiobjective predictive controller, International Journal of Applied Mathematics and Computer Science 23(1) (2013), 35-45. DOI:10.2478/amcs-2013-004

[5] Y. Chalo-Cano, G. N. Silva and A. Rufian-Lizana, On the Newton method for solving fuzzy unconstrained optimization problems, Fuzzy Sets and Systems 272 (2015), 60-69.

[6] Dalbiner K. Our and Kajla Basu, Application of extended fuzzy programming technique to a real life Transformation problem in neutrosophic environment, Neutrosophic Sets and Systems 10 (2015), 74-87.

[7] R. Debski, An adaptive multi-spline refinement algorithm in simulation based sailboat trajectory optimization using onboard multi-core computer systems, International Journal of Applied Mathematics and Computer Science 26(2) (2016), 351-360. DOI:10.1515/amcs-2016-0025

[8] J. E. Dennis and R. B. Shanable, Numerical methods for unconstrained optimization and non linear equations, Prentice Hall, New York, (1983).

[9] R. Irene Hepzibah, An Algorithmic Approach to Fuzzy Linear and Complementarity Problems, Lambert Academic Publishing, Germany, (2021).

[10] Irfan Deli and Yusuf Subas, Single valued neutrosophic numbers and their applications to multicriteria decision making problems, viXra.org, General Mathematics (2014), viXra:1412.0012.

[11] W. B. Kandasamy and F. Smarandache, Neutrosophic Algebraic Structures, Hexis, Phoenix, 2006.

[12] G. S. Mahapatra and T. K. Roy, Intuitionistic fuzzy number and its arithmetic operation with application on system failure, Journal of Academic Systems 3(2) (2013), 92-107.

[13] Ronald L. Rardin, Optimization in Operations Research, Pearson Education Inc., 1998.

[14] F. Smarandache, Neutrosophic set, a generalisation of the intuitionistic fuzzy sets, International Journal of Pure and Applied Mathematics 24 (2002), 287-297.

[15] R. Sophia Porchelvi and S. Sathya, A Newton method for nonlinear unconstrained optimization problems with two variables, International Journal of Science and Research ISSN:2319-7064 2(1) (2013).

[16] T. C. E. Cheng and Mikhail Y. Kovalyov, An unconstrained optimization problem is NP-hard given an oracle representation of its objective function: a technical note, Computer and Operations Research 29(14) (2002), 2087-2091.

[17] J. R. Timothy, Fuzzy Logic with Engineering Applications, 3rd Edition, Wiley, New York, NY, (2010).

[18] P. Umamaheshwari and K. Ganesan, A solution approach to fuzzy nonlinear programming problems, International Journal of Pure and Applied Mathematics 113(13) (2017), 291-300.

[19] P. Umamaheshwari and K. Ganesan, A comparative solution of fuzzy unconstrained optimization problems with triangular fuzzy number, Journal Xidian University, ISSN No: 1001-2400 14(6) (2020), 1443-1455.

[20] R. Vidhya and R. Irene Hepzibah, A comparative study on interval arithmetic operations with intuitionistic fuzzy numbers for solving an intuitionistic fuzzy multi-objective linear programming problems, International Journal of Applied Mathematics and Computer Science 27(3) (2017), 563-573. DOI:10.1515/amcs-2017-0040.

[21] R. Vidhya, I. Hepzibah and N. Gani, Neutrosophic multi-objective linear programming problems, Global Journal of Pure and Applied Mathematics 13(2) (2017), 265-280.

[22] H. Wang, F. Smarandache, Q. Zhang and R. Sunderraman, Single valued neutrosophic sets, Multispace and Multistructure 4 (2010), 410-413.

[23] L. A. Zadeh, Fuzzy sets, Information and Control 8 (1965), 338-353.

[24] L. A. Zadeh, The concept of a linguistic variable and its application to approximate reasoning-1, Information Sciences 8 (1975), 199-249.

[25] Zhen-Jue Shi, Convergence of line search methods for unconstrained optimization, Applied Mathematics and Computation 157(2) (2004), 393-405.