Advances and Applications in Mathematical Sciences
Volume 21, Issue 11, September 2022, Pages 6401-6414

EXPONENTIATED QUASI AKASH DISTRIBUTION: PROPERTIES AND APPLICATIONS

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Abstract

In this paper, we introduced a new generalization of quasi Akash distribution called as exponentiated quasi Akash distribution. The statistical properties are derived and the model parameters are estimated by maximum likelihood estimation. Finally two real data sets are presented to examine the significance of newly introduced model.

1. Introduction

The quasi Akash distribution is a newly lifetime model formulated by Rama Shanker (10) for several engineering and medical science applications and calculated its various properties including moments, mean deavation, median, stochastic ordering, order statistics, Renyi entropy, mean residual life, and ML estimation.

The probability density function and cumulative density function of quasi Akash distribution are given as

$$h(z;\, \alpha,\, \theta) = \frac{\theta^2}{\theta\alpha + 2}(\alpha + \theta z^2)e^{-\theta z} \quad z > 0,\, \alpha,\, \theta > 0$$

and its corresponding cdf is given by

2020 Mathematics Subject Classification: Primary 62H10, secondary 62J12.
Keywords: Quasi Akash distribution, Exponentiation technique, Structural properties and maximum likelihood estimation.
Received July 1, 2021; Accepted March 21, 2022

$$H(z; \theta, \alpha) = 1 - \left[1 + \frac{\theta z(\theta z + 2)}{\alpha\theta + 2}\right] e^{-\theta z} \quad z > 0, \theta, \alpha > 0$$

2. Exponentiated Quasi Akash Distribution

Exponentiation technique for generalizing the probability models is one of the prominent techniques which have been used by researchers from past decades for bringing flexibility in applying probability models to data and for capturing extra variation from data. The exponentiated family of distribution is derived by powering a positive real number to the cumulative distribution function (cdf) of an arbitrary parent distribution. Its pdf is given by

$$f(z) = \beta h(z) [H(z)]^{\beta - 1} \tag{2.1}$$

The corresponding cdf is given by

$$F(z) = [H(z)]^{\beta}, \beta > 0 \tag{2.2}$$

$H(z)$ and $h(z)$ are cdf and pdf of the parent distribution.

The work has been carried out widely in that particular area since 1995 and a number of researchers have derived different classes of distributions using exponentated technique. Mudholkar et al. [6] derived the exponentiated Weibull distribution. Gupta et al. [4] first introduced a new life time model, called the exponentiated exponential (EE) distribution. Lemonte and Cordeiro [5] proposed the exponentiated generalized inverse Gaussian distribution. Cordeiro et al. [3] derived the beta exponentiated Weibull distribution. Adepoju, Chukwu and Shittu [2] derived the exponentiated nakagami distribution. Al-Kadim and Mahdi [1] proposed an exponentiated transmuted exponential (ETE) distribution that comes out to be more flexible than some other distributions. The results revealed that the MLE of the unknown parameters can be acquired numerically. Nasiru et al., [6] derived exponentiated generalized power series family of distributions which shows more flexibility than the classical distribution. Hassan et al. [9] introduced a new generalization of Ishita distribution and obtained vital properties of the distribution along with applications of the proposed model.

The cdf and pdf of exponentiated quasi Akash distribution are given as

$$F(z) = \left[1 - \left[1 + \frac{\theta z(\theta z + 2)}{\alpha\theta + 2}\right]e^{-\theta z}\right]^{\beta} \quad z > 0,\ \theta,\ \alpha > 0,\ \beta > 0 \tag{2.3}$$

$$f(z) = \beta\left[1 - \left[1 + \frac{\theta z(\theta z + 2)}{\alpha\theta + 2}\right]e^{-\theta z}\right]^{\beta-1} \frac{\theta^2}{\theta\alpha + 2}(\alpha + \theta z^2)e^{-\theta x},\ z > 0,\ \theta,\ \alpha > 0,\ \beta > 0 \tag{2.4}$$

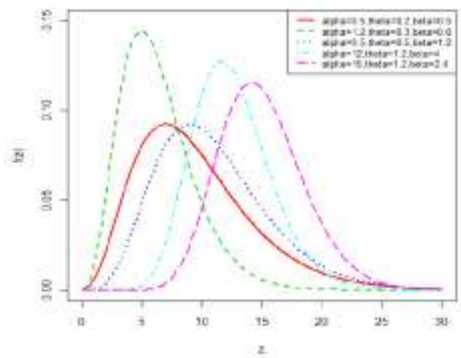

Figure 1. Graphical Overview of pdf plot of exponentiated quasi Akash distribution.

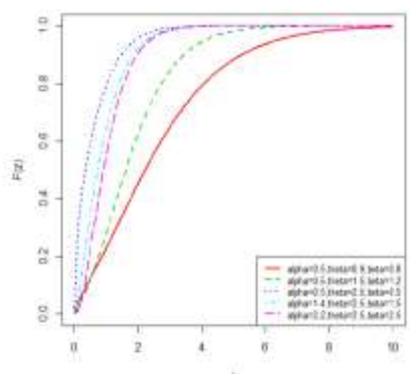

Figure 1.2. Graphical overview of CDF plot of exponentiated quasi Akash distribution.

3. Special Cases

Case 1. If we put $\beta = 1$, then exponentiated quasi Akash distribution reduces to quasi Akash distribution with pdf as

$$h(z;\theta,\alpha)=\frac{\theta^2}{\theta\alpha+2}(\alpha+\theta z^2)e^{-\theta x},\ z>0,\ \theta,\ a>0.$$

4. Reliability Analysis

In this section, we have obtained the reliability, hazard rate, reverse hazard rate and millis ratio of the proposed exponentiated quasi Akash distribution.

4.1 Reliability function

$$R(z)=1-\left[1-\left[1+\frac{\theta z(\theta z+2)}{\alpha\theta+2}\right]e^{-\theta z}\right]^{\beta}$$

4.2 Hazard Function

$$=\frac{\beta\left[1-\left[1+\frac{\theta z(\theta z+2)}{\alpha\theta+2}\right]e^{-\theta z}\right]^{\beta-1}\frac{\theta^2}{\alpha\theta+2}(\alpha+\theta z^2)e^{-\theta z}}{1-\left[1-\left[1+\frac{\theta z(\theta z+2)}{\alpha\theta+2}\right]e^{-\theta z}\right]^{\beta}}$$

4.3 Reverse Hazard Rate

$$h(z)=\frac{\beta\left[1-\left[1+\frac{\theta z(\theta z+2)}{\alpha\theta+2}\right]e^{-\theta z}\right]^{\beta-1}\frac{\theta^2}{\alpha\theta+2}(\alpha+\theta z^2)e^{-\theta z}}{\left[1-\left[1+\frac{\theta z(\theta z+2)}{\alpha\theta+2}\right]e^{-\theta z}\right]^{\beta}},$$

4.4 Mills Ratio

$$=\frac{\left[1-\left[1+\frac{\theta z(\theta z+2)}{\alpha\theta+2}\right]e^{-\theta z}\right]^{\beta}}{\beta\left[1-\left[1+\frac{\theta z(\theta z+2)}{\alpha\theta+2}\right]e^{-\theta z}\right]^{\beta-1}\frac{\theta^2}{\alpha\theta+2}(\alpha+\theta z^2)e^{-\theta z}}$$

5. Statistical Properties

In this section, the different structural properties of the proposed model have been evaluated. These include moments, harmonic mean, moment generating function and characteristic function.

5.1 Moments. Suppose Z is a random variable following exponentiated quasi Akash distribution with parameter $\theta, \alpha, \beta,$ and then the r^{th} moment for a given probability distribution is given by

$$\mu'_r = \int_0^\infty z^r f(z, \theta, \alpha, \beta) dz$$

$$\mu'_r = \int_0^\infty z^r \beta \left[1 - \left[1 + \frac{\theta z(\theta z + 2)}{\alpha\theta + 2}\right] e^{-\theta z}\right]^{\beta-1} \frac{\theta^2}{\theta^4 + 6} (\alpha + \theta z^2) e^{-\theta z} dx$$

$$\mu'_r = \int_0^\infty \sum_{j=0}^\infty \sum_{k=0}^j \sum_{l=0}^k (-1)^j \binom{\beta - 1}{j} \binom{j}{k} \binom{k}{l} \frac{\theta^{2k+2-l}}{(\alpha\theta + 2)^{k+1}} \beta 2^l z^{2k-l+r} (\alpha + \theta z^2) e^{-\theta z(1+j)} dz$$

$$\mu'_r = \int_0^\infty \phi_1^l z^{2k-l+r} (\alpha + \theta z^2) e^{-\theta z(1+j)} dz$$

$$\mu'_r = \phi_1 \left[\frac{\alpha(2k - l + r)!}{\theta^{2k-l+r}(1 + j)^{2k-l+r+1}} + \frac{(2k - l + r + 2)!}{\theta^{2k-l+r+1}(1 + j)^{2k-l+r+3}}\right]$$

$$\mu'_1 = \phi_1 \left[\frac{\alpha(2k - l + 1)!}{\theta^{2k-l+1}(1 + j)^{2k-l21}} + \frac{(2k - l + 3)!}{\theta^{2k-l+2}(1 + j)^{2k-l+4}}\right]$$

$$\mu'_2 = \phi_1 \left[\frac{\alpha(2k - l + 2)!}{\theta^{2k-l+r}(1 + j)^{2k-l+3}} + \frac{(2k - l + 4)!}{\theta^{2k-l+r+1}(1 + j)^{2k-l+5}}\right]$$

$$\mu_2 = \phi_1 \left[\frac{\alpha(2k - l + 2)!}{\theta^{2k-l+r}(1 + j)^{2k-l+3}} + \frac{(2k - l + 4)!}{\theta^{2k-l+r+1}(1 + j)^{2k-l+5}}\right]$$

$$- (\phi_1)^2 \left[\frac{\alpha(2k - l + 1)!}{\theta^{2k-l+1}(1 + j)^{2k-l21}} + \frac{(2k - l + 3)!}{\theta^{2k-l+2}(1 + j)^{2k-l+4}}\right]^2$$

where $\phi_1 = \sum_{j=0}^\infty \sum_{k=0}^j \sum_{l=0}^k (-1)^j \binom{\beta - 1}{j} \binom{j}{k} \binom{k}{l} \frac{\theta^{2k+2-l}}{(\alpha\theta + 2)^{k+1}} \beta 2^l$

5.2. Harmonic Mean

The harmonic mean of the proposed model is computed as

$$H.M = E\left[\frac{1}{Z}\right] = \int_0^\infty \frac{1}{z} f(z;\ \alpha,\ \theta)dz$$

$$= \int_0^\infty \frac{1}{z}\beta\left[1-\left[1+\frac{\theta z(\theta z+2)}{\alpha\theta+2}\right]e^{-\theta z}\right]^{\beta-1}\frac{\theta^2}{\alpha\theta+2}(\alpha+\theta z^2)e^{-\theta z}dz$$

$$H.M = \phi_1\left(\frac{\alpha(2k-l-1)!}{(\theta(1+j))^{2k-l}}+\frac{\theta(2k-l)!}{(\theta(1+j))^{2k-l+1}}\right),\ \theta,\ \alpha,\ \beta > 0$$

where $\phi_1 = \sum_{j=0}^{\infty}\sum_{k=0}^{j}\sum_{l=0}^{k}(-1)^j\binom{\beta-1}{j}\binom{j}{k}\binom{k}{l}\frac{\theta^{2k+2-l}}{(\alpha\theta+2)^{k+1}}\beta 2^l$

5.3. Moment generating function and Characteristic function of EQAD

Theorem 1.1. *If Z has the EQAD, then the moment generating function* $M_Z(t)$ *and the characteristic function* $\psi_Z(t)$ *has the following form*

$$M_Z(t) = \phi_1\left(\frac{\alpha(2k-l)!}{(\theta(1+j+t))^{2k-l+1}}+\frac{\theta(2k-l+2)!}{(\theta(1+j+t))^{2k-l+3}}\right)$$

and

$$\psi_Z(t) = \phi_1\left(\frac{\alpha(2k-l)!}{(\theta(1+j+it))^{2k-l+1}}+\frac{\theta(2k-l+2)!}{(\theta(1+j+it))^{2k-l+3}}\right) \textit{ respectively.}$$

Proof. We begin with the well known definition of the moment generating function given by

$$M_Z(t) = E(e^{tz}) = \int_0^\infty e^{tz} f(z;\ \theta,\ \beta,\ \alpha)\,dz$$

$$= \int_0^\infty e^{tz}\beta\left[1-\left[1+\frac{\theta z(\theta z+2)}{\alpha\theta+2}\right]e^{-\theta z}\right]^{\beta-1}\frac{\theta^2}{\alpha\theta+2}(\alpha+\theta z^2)e^{-\theta z}dx$$

$$= \int_0^\infty \sum_{j=0}^{\infty} \sum_{k=0}^{j} \sum_{l=0}^{k} (-1)^j \binom{\beta-1}{j} \binom{j}{k} \binom{k}{l} \frac{\theta^{2k+2-l}}{(\alpha\theta+2)^{k+1}} \beta 2^l z^{2k-l} (\alpha + \theta z^2) e^{-z(\theta(1+j)+t)} dz$$

$$\Rightarrow M_Z(t) = \phi_1 \left(\frac{\alpha(2k-l)!}{(\theta(1+j+t))^{2k-l+1}} + \frac{\theta(2k-l+2)!}{(\theta(1+j+t))^{2k-l+3}} \right)$$

Where $\phi_1 = \sum_{j=0}^{\infty} \sum_{k=0}^{j} \sum_{l=0}^{k} (-1)^j \binom{\beta-1}{j} \binom{j}{k} \binom{k}{l} \frac{\theta^{2k+2-l}}{(\alpha\theta+2)^{k+1}} \beta 2^l$

Also we know that $\psi_Z(t) = M_Z(it)$

Therefore,

$$\psi_Z(t) = \phi_1 \left(\frac{\alpha(2k-l)!}{(\theta(1+j+it))^{2k-l+1}} + \frac{\theta(2k-l+2)!}{(\theta(1+j+it))^{2k-l+3}} \right)$$

6. Renyi Entropy

The renyi entropy for a proposed model is given as

$$e(\sigma) = \frac{1}{1-\sigma} \log \left(\int_0^\infty f^{\sigma}(z) dz \right)$$

$$e(\sigma) = \frac{1}{1-\sigma} \log \int_0^\infty \left(\beta \left[1 - \left[1 + \frac{\theta z(\theta z+2)}{\alpha\theta+2} \right] e^{-\theta z} \right]^{\beta-1} \frac{\theta^2}{\theta\alpha+2} (\alpha + \theta z^2) e^{-\theta x} \right)^{\sigma}$$

$$e(\sigma) = \frac{1}{1-\sigma} \log \left[\sum_{j=0}^{\infty} \sum_{i=0}^{\sigma} \sum_{k=0}^{j} (-1)^j \binom{\sigma(\beta-1)}{j} \binom{\sigma}{i} \binom{j}{k} \alpha^{\sigma-i} 2^k \frac{\beta^{\sigma} \theta^{2(\sigma+j)+i-k} (2(j+i)-k)}{(\alpha\theta+2)^{\sigma+j} (\theta(\sigma+j))^{2(j+i)-k+1}} \right]$$

7. Order Statistics

Let $Z_{(1)}, Z_{(2)}, Z_{(3)}, ..., Z_{(n)}$ be the ordered statistics of the random sample $Z_1, Z_2, Z_3, ..., Z_n$ drawn from the continuous distribution with cumulative distribution function $F_Z(z)$ and probability density function $f_Z(z)$, then the probability density function of r^{th} order statistics $Z_{(r)}$ is given by:

$$f_{z(r)}(z, c, \theta) = \frac{n!}{(r-1)!\,(n-r)!} f(z)[F(z)]^{r-1}[1 - F(z)]^{n-r} \cdot r = 1, 2, 3, \ldots, n$$

Using the equations (2.1) and (2.2), the probability density function of r^{th} order statistics of exponentiated quasi Akash distribution is given by:

$$f_{(r)}(z, \theta, a, \beta) = \frac{n!}{(r-1)!\,(n-r)!} \beta \left[1 - \left[1 + \frac{\theta z(\theta z + 2)}{\alpha\theta + 2}\right] e^{-\theta z}\right]^{\beta - 1} \frac{\theta^2}{\alpha\theta + 2} (\alpha + \theta z^2) e^{-\theta z}$$

$$\left[\left[1 - \left[1 + \frac{\theta z(\theta z + 2)}{\alpha\theta + 2}\right] e^{-\theta z}\right]^{\beta}\right]^{r-1} \left[1 - \left[1 - \left[1 + \frac{\theta z(\theta z + 2)}{\alpha\theta + 2}\right] e^{-\theta z}\right]^{\beta}\right]^{n-r}.$$

Then, the pdf of first order $Z_{(1)}$ exponentiated quasi Akash distribution is given by:

$$f_{(1)}(z, \theta, a, \beta) = n\beta \left[1 - \left[1 + \frac{\theta z(\theta z + 2)}{\alpha\theta + 2}\right] e^{-\theta z}\right]^{\beta - 1} \frac{\theta^2}{\alpha\theta + 2} (\alpha + \theta z)^2 e^{-\theta z}$$

$$\left[1 - \left[1 - \left[1 + \frac{\theta z(\theta z + 2)}{\alpha\theta + 2}\right] e^{-\theta z}\right]^{\beta}\right]^{n-1}.$$

and the pdf of nth order $Z_{(n)}$ exponentiated quasi Akash model is given as:

$$f_{(n)}(z, \theta, a, \beta) = n\beta \left[1 - \left[1 + \frac{\theta z(\theta z + 2)}{\alpha\theta + 2}\right] e^{-\theta z}\right]^{\beta - 1} \frac{\theta^2}{\alpha\theta + 2} (\alpha + \theta z^2) e^{-\theta z}$$

$$\left[\left[1 - \left[1 + \frac{\theta z(\theta z + 2)}{\alpha\theta + 2}\right] e^{-\theta z}\right]^{\beta}\right]^{n-1}.$$

8. Method of Maximum Likelihood Estimation of Exponentiated Quasi Akash Distribution

This is one of the most useful method for estimating the different parameters of the distribution. Let $Z_1, Z_2, Z_3, \ldots.., Z_n$ be the random sample of size n drawn from exponentiated quasi Akash distribution, then the likelihood function of exponentiated quasi Akash distribution is given as:

$$L(z \mid \alpha, \theta, \beta) = \prod_{i=1}^{n} f(z; \alpha, \theta, \beta) = \prod_{i=1}^{n} \beta \left[1 - \left[1 + \frac{\theta z(\theta z + 2)}{\alpha\theta + 2}\right] e^{-\theta z}\right]^{\beta - 1}$$

$$\frac{\theta^2}{\alpha\theta + 2} (\alpha + \theta z^2) e^{-\theta z} \tag{8.1}$$

The log likelihood function becomes:

$$\log L = 2n \log \theta^2 - n \log(\alpha\theta + 2) + n \log \beta + \sum_{i=1}^{n} \log(\alpha + \theta z^2) - \theta \sum_{i=1}^{n} z_i + (\beta - 1)$$

$$\sum_{i=1}^{n} \log\left(1 - \left[1 + \frac{\theta z(\theta z + 2)}{\alpha\theta + 2}\right] e^{-\theta z}\right) \tag{8.2}$$

Differentiating the log-likelihood function with respect to θ, α and β. This is done by partially differentiate (8.2) with respect to θ, α and β and equating the result to zero, we obtain the following normal equations,

$$\frac{d \log L}{d\theta} = \frac{2n}{\theta} - \frac{n\alpha}{(\alpha\theta + 2)} - \sum_{i=1}^{n} z_i - \sum_{i=1}^{n} \frac{z^2}{(\alpha + \theta z^2)} + (\beta - 1)$$

$$\sum_{i=1}^{n} \frac{e^{-\theta z}\left(z\left(1 + \frac{\theta z(\theta z + 2)}{\alpha\theta + 2}\right) - \frac{(\alpha\theta + 2)(2\theta z^2 + z) - \theta z \alpha(\theta z + 2)}{(\alpha\theta + 2)^2}\right)}{\left(1 - \left[1 + \frac{\theta z(\theta z + 2)}{\alpha\theta + 2}\right] e^{-\theta z}\right)} = 0. \tag{8.3}$$

$$\frac{d \log L}{d\alpha} = -\frac{n\theta}{(\alpha\theta + 2)} + \sum_{i=1}^{n} \frac{1}{(\alpha + \theta z^2)} + (\beta - 1)$$

$$\sum_{i=1}^{n} \frac{e^{-\theta z} \theta z(\theta z + 2)}{\left(1 - \left[1 + \frac{\theta z(\theta z + 2)}{\alpha\theta + 2}\right] e^{-\theta z}\right)(\theta\alpha + 2)^2} = 0 \tag{8.4}$$

$$\frac{d \log L}{d\beta} = \frac{n}{\beta} + \sum_{i=1}^{n} \log\left(1 - \left[1 + \frac{\theta z(\theta z + 2)}{\alpha\theta + 2}\right] e^{-\theta z}\right) = 0 \tag{8.5}$$

The above equations can be solved numerically by using R software (3.5.3-version).

9. Applications of Proposed Model

To illustrate the significance of the suggested model, real life examples are presented. The goodness of fit result of the suggested model (exponentiated quasi Akash distribution) is compared with the base model quasi Akash distribution and Lindley distribution.

Data set 9.1. This data set represents the strength of glass of the aircraft window reported by Fuller et al. (1994).

Table 9.1. Strength of glass of the aircraft window.

18.83	20.80	21.657	23.03	23.23	24.05	24.321	25.50
25.52	25.80	26.69	26.77	26.78	27.05	27.67	29.90
31.11	33.20	33.73	33.76	33.89	34.76	35.75	35.91
36.98	37.08	37.09	39.58	44.045	45.29	45.381	

Data set 9.2. The second data set is about the breaking stress of carbon fibres of 50 mm length (GPa). The data has been previously used by Nichols and Padgett (2006), Cordeiro and Lemonte (2011) and Al-Aqtash et al. (2014). The data is as follows.

Table 9.2. Breaking stress of carbon fibres of 50 mm length.

0.39	0.85	1.08	1.25	1.47	1.57	1.61	1.61	1.69	1.80	1.84
1.87	1.89	2.03	2.03	2.05	2.12	2.35	2.41	2.43	2.48	2.50
2.53	2.55	2.55	2.56	2.59	2.67	2.73	2.74	2.79	2.81	2.82
2.85	2.87	2.88	2.93	2.95	2.96	2.97	3.09	3.11	3.11	3.15
3.15	3.19	3.22	3.22	3.27	3.28	3.31	3.31	3.33	3.39	3.39
3.56	3.60	3.65	3.68	3.70	3.75	4.20	4.38	4.42	4.70	4.90

These data sets are used here only for illustrative purposes. The required numerical evaluations are carried out using R software version R i386 3.3.2. We have fitted exponentiated quasi Akash distribution, quasi Akash distribution and Lindley distribution to these two real life data sets. The summary statistic of these two data sets is given in table 9.3. The MLEs of the parameters with standard errors in parentheses, model functions are displayed in table 4 for these two data sets. The corresponding log-likelihood values, AIC, AICC, HQIC, BIC and Shannon's entropy are given in table 9.5 and 9.6 for data sets 9.1 and 9.2 respectively.

Table 9.3. Summary statistic of data sets 9.1 and 9.2.

Data Set	No. of observations	Min.	First quartile	median	mean	Third quartile	Max.
Data Set 9.1	31	18.83	25.51	29.90	30.81	35.83	45.38
Data Set 9.2	66	0.390	2.178	2.835	2.760	3.278	4.900

Table 9.4. ML Estimates, Standard Error of Estimates in parenthesis, model function of related models and proposed model for data sets 9.1 and 9.2.

Data set	Distribution	ML Estimates with Standard errors	Model Function
Data set 9.1	Exponentiated Quasi Akash Distribution	$\hat{\alpha} = 0.5925$ $\hat{\theta} = 0.2214$ (0.0257) $\hat{\beta} = 17.855$	$f(z) = \beta\left[1 - \left[1 + \frac{\theta z(\theta z + 2)}{\alpha\theta + 2}\right]e^{-\theta z}\right]^{\beta - 1}$ $\frac{\theta^2}{\theta\alpha + 2}(\alpha + \theta z^2)e^{-\theta x}$
	Quasi Akash Distribution	$\hat{\alpha} = 0.00100$ $\hat{\theta} = 0.09736$ (0.00803)	$\frac{\theta^2}{(\alpha\theta + 2)}(\alpha + \theta z^2)e^{-\theta z}$
	Lindley Distribution	$\hat{\theta} = 0.06299$ (0.00805)	$\frac{\theta^2}{(\theta + 1)}(1 + z)e^{-\theta z}$
Data set 9.2	Exponentiated Quasi Akash Distribution (EQAD)	$\hat{\alpha} = 0.70805$ (0.66888) $\hat{\theta} = 1.62918$ (0.1279) $\hat{\beta} = 5.0217$ (2.0260)	$f(z) = \beta\left[1 - \left[1 + \frac{\theta z(\theta z + 2)}{\alpha\theta + 2}\right]e^{-\theta z}\right]^{\beta - 1}$ $\frac{\theta^2}{\theta\alpha + 2}(\alpha + \theta z^2)e^{-\theta x}$
	Quasi Akash Distribution (QAD)	$\hat{\alpha} = 0.0010$ (0.3862)	$\frac{\theta^2}{(\alpha\theta + 2)}(\alpha + \theta z^2)e^{-\theta z}$

		$\hat{\theta} = 1.0864$ (0.13663)	
	Lindley Distribution (LD)	$\hat{\alpha} = 0.0010$ (0.0473) $\hat{\theta} = 1.4486$ (0.0982)	$\frac{\theta^2}{(\theta+1)}(1+z)e^{-\theta z}$

Table 9.5. Model comparison of proposed model and its related models for data set 9.1.

Distribution	$-\log L$	AIC	BIC	AICC	HQIC	Shanon entropy $H(X)$
Exponentiated Quasi Akash Distribution (EQAD)	104.082	214.165	218.467	215.0539	215.5673	3.357
Quasi Akash Distribution (QAD)	120.234	244.468	247.336	244.8967	245.403	3.878
Lindley Distribution (LD)	126.994	255.988	257.422	256.126	256.455	4.096

Table 9.6. Model comparison of proposed model and its related models for data set 9.2.

Distribution	$-\log L$	AIC	BIC	AICC	HQIC	Shanon entropy $H(X)$
Exponentiated Quasi Akash Distribution (EQAD)	90.9356	187.8713	194.4403	188.258	190.467	1.377
Quasi Akash Distribution (QAD)	102.245	208.4904	212.8697	208.6809	210.220	1.549
Lindley Distribution (LD)	122.384	246.7681	248.957	246.8306	247.6334	1.85

In order to compare the exponentiated quasi Akash distribution with quasi Akash distribution and Lindley distribution, We compute the criteria like AIC (Akaike information criterion), AICC (corrected Akaike information criterion), BIC (Bayesian information criterion) and HQIC which represent the loss of information resulting from fitting probability models to data. The better distribution corresponds to lesser AIC, AICC, BIC and HQIC values. Also we computed the Shannon's entropy $(H(Z))$ which represents the average uncertainty. The better model possesses lesser Shannon's entropy value.

$$AIC = 2k - 2\log L \qquad AICC = AIC + \frac{2k(k+1)}{n-k-1}$$

$$BIC = k\log n - 2\log L \qquad HQIC = 2k\log(\log(n)) + 2\log L$$

where k is the number of parameters in the statistical model, n is the sample size and $-2\log L$ is the maximized value of the log-likelihood function under the considered model. From Tables 9.5 and 9.6, it has been observed that the exponentiated quasi Akash distribution possesses the lesser AIC, AICC BIC, HQIC and $H(Z)$ values as compared to quasi Akash distribution and Lindley distribution for data sets 9.1 and 9.2 respectively. Hence we can conclude that the exponentiated quasi Akash distribution leads to a better fit than the quasi Akash distribution and Lindley distribution for data sets 9.1 and 9.2 respectively.

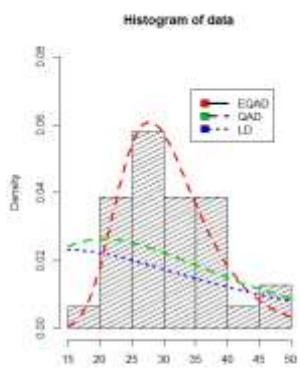

Figure 3. Graphical data set 1 fitted by proposed and related models.

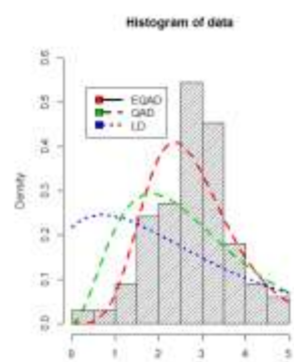

Figure 4. Graphical data set 2 fitted by proposed and related models.

10. Conclusion

In the present study, we have introduced a new exponentiated quasi Akash distribution. The subject distribution is generated by using the exponentiated technique and taking the two parameter quasi Akash distribution as the base distribution. Some mathematical properties along with reliability measures are discussed. Model is fitted to real life data for examining its significance.

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