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MONOPHONIC DISTANCE LAPLACIAN ENERGY OF GRAPHS

R. DIANA and T. BINU SELIN

Research Scholar, Reg. No. 20113162092015
Department of Mathematics
Scott Christian College (Autonomous)
Nagercoil-629 003, TamilNadu, India
(Affiliated to Manonmaniam Sundaranar University
Abishekapatti, Tirunelveli-627 012, TamilNadu, India)
E-mail: dianajino@gmail.com

Assistant Professor
Department of Mathematics
Scott Christian College (Autonomous)
Nagercoil-629 003, TamilNadu, India
E-mail: binuselin@gmail.com

Abstract

Let G be a simple connected graph of order n, v_i its vertex. Let $\delta_1^L, \delta_2^L, \ldots, \delta_n^L$ be the eigenvalues of the distance Laplacian matrix D^L of G. The distance Laplacian energy of G is $LE_D(G) = \sum_{i=1}^{n} | \delta_i^L - \frac{1}{n} \sum_{i=1}^{n} D_i |$, where D_i is the sum of the distance between v_i and other vertices of G was already studied. Here we defined the monophonic distance Laplacian energy as $LE_M(G) = \sum_{i=1}^{n} | \mu_i^L - \frac{1}{n} \sum_{j=1}^{n} MT_G(v_j) |$, where $MT_G(v_j)$ is the j^{th} row sum of monophonic distance matrix $M(G)$, and $\mu_1^L \leq \mu_2^L, \ldots, \leq \mu_n^L$ be the eigenvalues of the monophonic distance Laplacian matrix $M^L(G)$.

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1. Introduction

I. Gutman established the concept of graph energy in 1978 [7]. Consider the graph G, which has n vertices and m edges. Let $A = (a_{ij})$ be the adjacency matrix of the graph. The energy $E(G)$ of G is defined as $E(G) = \sum_{i=1}^{n} | \lambda_i |$ [2, 7]. In the year 2008, I. Gutman and others established the concept of graph distance energy [4]. Jieshan Vang, Lihuayou and I. Gutman introduced the distance Laplacian energy of a graph in the year 2013 [10]. The monophonic number of a graph was introduced by A. P. Santhakumaran and others in 2014 [12]. We offer the monophonic distance Laplacian energy of a graph as a new idea based on these.

2. Definitions and Examples

Definition 2.1. The monophonic distance matrix G is defined as

$$M = M(G) = (d_{m_{ij}})_{n\times n}, \text{ where } d_{mij} = \begin{cases} d_m(v_i, v_j) & \text{if } i \neq j \\ 0 & \text{otherwise} \end{cases}$$

Here $d_m(v_i, v_j)$ is the monophonic distance of v_i to v_j.

Definition 2.2. The monophonic transmission $MT_G(v)$ of a vertex v as $\sum_{u\in V(G)} d_m(u, v)$ and monophonic transmission matrix $MT(G)$ is the diagonal matrix $diag\,[(MT_G(v_1), MT_G(v_2), \ldots, MT_G(v_n)]$. The connected graph G and its monophonic distance Laplacian matrix defined as $M^L(G) = MT(G) - M(G)$. The eigenvalues of monophonic distance Laplacian matrix $M^L(G)$ are denoted by $\mu_1^L \leq \mu_2^L, \ldots, \leq \mu_n^L$ and $Spec_M{}^L(G)$. Since the monophonic distance Laplacian matrix is symmetric and its eigenvalues are real, it can be ordered as. The monophonic distance Laplacian energy of a graph is defined as $LE_M(G) = \sum_{i=1}^{n} | \mu_i^L - \frac{1}{n} \sum_{j=1}^{n} MT_G(v_j) |$.

Example 2.3. The monophonic distance Laplacian energy of $LE_M(G) = 48$.

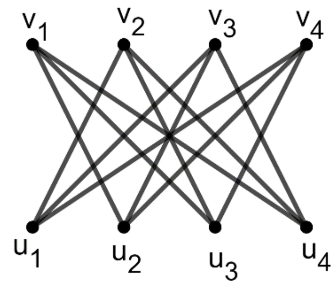

Figure 1.

3. Main Results

Theorem 3.1. *The monophonic distance Laplacian energy of* K_n *is* $LE_M(K_n) = 2(n-1)$, $n \geq 2$.

Proof. The monophonic distance matrix of K_n is $M(K_n) = [J_n - I_n]$, where J_n is the matrix with all entries 1's of order n and I_n is the identity matrix of order n.

The monophonic transmission matrix of K_n is $MT(K_n) = [(n-1)I_n]$.

The monophonic distance Laplacian matrix of K_n is $M^L(K_n) = [nI_n - J_n]$.

The M^L –spectrum of K_n is $Spec_{M^L}(K_n) = \begin{bmatrix} 0 & n \\ 1 & n-1 \end{bmatrix}$

The monophonic distance Laplacian energy of K_n is

$$LE_M(K_n) = \sum_{i=1}^{n} | \mu_i^L - \frac{1}{n} \sum_{j=1}^{n} MT_G(v_j) |$$

$$= \sum_{i=1}^{n} | \mu_i^L - \frac{n(n-1)}{n} |$$

$$= 2(n-1).$$

Theorem 3.2. *The monophonic distance Laplacian energy of* $K_{n,\,n}$, $LE_M(K_{n,n}) = 8(n-1)$.

Proof. The monophonic distance matrix of $K_{n,n}$ is

$$M(K_{n,n}) = \begin{bmatrix} 2(J_n - I_n) & J_n \\ J_n & 2(J_n - I_n) \end{bmatrix}$$

The monophonic transmission matrix of $K_{n,n}$ is

$$MT(K_{n,n}) = \begin{bmatrix} (3n-2)I_n & 0 \\ 0 & (3n-2)I_n \end{bmatrix}$$

The monophonic distance Laplacian matrix of $K_{n,n}$ is

$$M^L(K_{n,n}) = \begin{bmatrix} 3nI_n - 2J_n & -J_n \\ -J_n & 3nI_n - 2J_n \end{bmatrix}$$

The M^L $-$spectrum of $K_{n,n}$ is

$$Spec_{M^L}(K_{n,n}) = \begin{bmatrix} 0 & 2n & 3n \\ 1 & 1 & 2n-2 \end{bmatrix}$$

The monophonic distance Laplacian energy of $K_{n,n}$ is

$$LE_M(K_{n,n}) = \sum_{i=1}^{2n} |\, \mu_i^L - (3n-2)\,|$$

$$= 8(n-1)$$

Theorem 3.3. *The monophonic distance Laplacian energy of* $H_{n,n}$ *is* $LE_M(H_{n,n}) = 16(n-1)$.

Proof. The M^L $-$spectrum of $H_{n,n}$ is

$$Spec_{M}{}^{L}(H_{n,n}) = \begin{bmatrix} 0 & 2n+4 & 5n & 5n+4 \\ 1 & 1 & n-1 & n-1 \end{bmatrix}$$

The monophonic distance Laplacian energy of $H_{n,n}$ is

$$LE_M(H_{n,n}) = \sum_{i=1}^{2n} |\, \mu_i^L - (5n-2)\,|$$

$$= 16(n-1).$$

Theorem 3.4. *The monophonic distance Laplacian energy of* $K_{n\times 2}$ *is* $LE_M(K_{n\times 2}) = 4n$.

Proof. Monophonic distance matrix of

$$M(K_{n\times 2}) = \begin{bmatrix} 0 & 1 & \dots & 1 & 2 \\ 1 & 0 & \dots & 2 & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 1 & 2 & \dots & 0 & 1 \\ 2 & 1 & \dots & 1 & 0 \end{bmatrix}$$

The M^L −spectrum of $K_{n\times 2}$ is

$$Spec_{M^L}(K_{n\times 2}) = \begin{bmatrix} 0 & 2n & 2n+2 \\ 1 & n-1 & n \end{bmatrix}$$

The monophonic distance Laplacian energy of $K_{n\times 2}$ is

$$LE_M(K_{n\times 2}) = \sum_{i=1}^{2n} |\, \mu_i^L - \frac{1}{n}\sum_{j=1}^{2n} MT_G(v_j)\,|$$

$$= \sum_{i=1}^{2n} |\, \mu_i^L - 2n \,|$$

$$= 4n$$

Theorem 3.5.

$$LE_M(S_n) = \begin{cases} \dfrac{4n^2}{n+1}, & for\ n = 1, 2 \\ \dfrac{6n^2 - 4n - 2}{n+1}, & for\ n > 3 \end{cases}$$

Proof. The monophonic distance Laplacian matrix of S_n is

$$M^L(S_n) = \begin{bmatrix} 2n-1 & -2 & \dots & -2 & -1 \\ -1 & 2n-1 & \dots & -2 & -1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -2 & -2 & \dots & 2n-1 & -1 \\ -1 & -1 & \dots & -1 & n \end{bmatrix}$$

The monophonic distance Laplacian energy of S_n is

Case (i) for $n = 1, 2$

$$LE_M(S_n) = \sum_{i=1}^{n+1} | \mu_i^L - \frac{1}{n+1} \sum_{j=1}^{n+1} MT_G(v_j) |$$

$$= \sum_{i=1}^{n+1} | \mu_i^L - \frac{2n^2}{n+1} |$$

$$= \frac{4n^2}{n+1}$$

Case (i) for $n > 3$

$$LE_M(S_n) = \sum_{i=1}^{n+1} | \mu_i^L - \frac{2n^2}{n+1} |$$

$$= \frac{6n^2 - 4n - 2}{n+1}$$

Theorem 3.6. *The monophonic distance Laplacian energy of friendship graph* F_n *is* $\frac{24n^2 - 16n - 2}{2n+1}$.

Proof. The M^L – spectrum of F_n is

$$Spec_{M^L}(F_n) = \begin{bmatrix} 0 & 2n+1 & 4n-1 & 4n+1 \\ 1 & 1 & n & n-1 \end{bmatrix}$$

The monophonic distance Laplacian energy of F_n is

$$LE_M(F_n) = \sum_{i=1}^{2n+1} | \mu_i^L - \frac{1}{2n+1} \sum_{j=1}^{2n+1} MT_G(v_j) |$$

$$= \left| 0 - \frac{8n^2 - 2n}{2n+1} \right| + \left| 2n + 1 - \frac{8n^2 - 2n}{2n+1} \right| + n \left| 4n - 1 - \frac{8n^2 - 2n}{2n+1} \right|$$

$$+ (n-1) \left| 4n + 1 - \frac{8n^2 - 2n}{2n+1} \right|$$

$$LE_M(F_n) = \frac{24n^2 - 16n - 2}{2n+1}$$

Theorem 3.7. $LE_M(P_2 \vee K_n) = LE_M(P_2) + LE_M(K_n) + 2$. *For* $n \geq 2$.

Proof. The monophonic distance Laplacian spectrum of is $\begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix}$

The monophonic distance Laplacian spectrum of K_n is $\begin{bmatrix} 0 & n \\ 1 & n-1 \end{bmatrix}$ we have $P_2 \vee K_n$ graph with $n+2$ vertices. Therefore $LE_M(P_2 \vee K_n) = \sum_{i=1}^{2} | \mu_i^L - 1 | + \sum_{i=1}^{n} | \mu_i^L - (n-1) | + 2$ Therefore $LE_M(P_2 \vee K_n) = LE(P_2) + LE_M(K_n) + 2$.

Conclusion

In this paper we obtained monophonic distance Laplacian energy of some standard graphs. We have planned to extend the results for various kinds of graph operations.

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