Advances and Applications in Mathematical Sciences
Volume 22, Issue 2, December 2022, Pages 435-443

PROPERTIES OF FUZZY CO-LOCALLY IRRESOLVABLE SETS

S. ANJALMOSE[1,*], A. VIRGIN RAJ[2] and M. KALAIMATHI[3]

[1]PG and Research
[2,3]Research Scholar, PG and Research
Department of Mathematics
St. Joseph's College of Arts and Science (Autonomous)
Affiliated to Thiruvalluvar University
Cuddalore-607001, Tamilnadu, India
E-mail: ansalmose@gmail.com
avirginraj@gmail.com
kalaimathi@sjctnc.edu.in

Abstract

In this paper we introduce a new concept of fuzzy co-locally irresolvable sets and their properties are discussed with suitable examples.

1. Introduction

The idea of fuzzy sets and fuzzy set operations were introduced by L. A. Zadeh [8]. The first notion of fuzzy topological space had been defined by C. L. Chang [3]. The concept of fuzzy locally closed and fuzzy co-locally closed sets were introduced and studied by the authors in [4]. The fuzzy co-locally somewhere dense set were introduced and studied by the authors Dr. S. Anjalmose and A. Virgin Raj [2]. The fuzzy resolvable set and fuzzy irresolvable sets were introduced and studied by the authors Dr. G. Thangaraj et al. [5] [6]. In this paper we introduce a concept of fuzzy co-locally irresolvable sets. Several properties are also discussed with suitable examples.

2020 Mathematics Subject Classification: 54A40, 03E72.
Keywords: Fuzzy open sets, fuzzy co-locally closed sets, fuzzy co-locally somewhere dense, fuzzy co-locally irresolvable sets.
*Corresponding author; E-mail: ansalmose@gmail.com
Received August 1, 2021; Accepted November 15, 2021

2. Preliminaries

Definition 2.1 [4]**.** A fuzzy subset η of a fuzzy topological space X is called fuzzy locally closed set if $\eta = (\gamma \wedge \zeta)$, where γ is a fuzzy open set and ζ is fuzzyclosed set. The complement of fuzzy locally closed set is called fuzzy locally open set.

Definition 2.2 [4]**.** A fuzzy subset η of a fuzzy topological space X is called fuzzy co-locally closed set if $\eta = (\gamma \vee \zeta)$, where γ is a fuzzyopen set and ζ is fuzzyclosed set. The complement of fuzzy co-locally closed set is called fuzzy co-locally open set.

Definition 2.3 [1]**.** A fuzzy set η in a fuzzy topological space (X, T) is called fuzzy locally dense if there exists no fuzzy locally closed set β in (X, T) such that $\eta < \beta < 1$.

Definition 2.4 [7]**.** A fuzzy set η in a fuzzy topological space (X, T) is called fuzzy somewhere dense if $cl(\eta) \neq 0$ in (X, T).

Definition 2.5 [2]**.** A fuzzy set η in a fuzzy topological space (X, T) is called fuzzy co-locally somewhere dense if $l_c - \operatorname{int} l_c - cl(\eta) \neq 0$ in (X, T).

Definition 2.6 [5]**.** A fuzzy set λ in a fuzzy topological space (X, T) is called a fuzzy resolvable if for each fuzzy closed set μ in (X, T), $cl(\mu \wedge \lambda) \wedge cl[\mu \wedge (1 - \lambda)]$ is a fuzzy nowhere dense in (X, T). That is, λ is a fuzzy resolvable set in (X, T) if $\operatorname{int} cl\{cl[(\mu \wedge \lambda)] \wedge cl(\mu \wedge (1 - \lambda))\} = 0$, where $1 - \mu \in T$.

Definition 2.7 [6]**.** Let (X, T) be a fuzzy topological space. A fuzzy set λ is called a fuzzy irresolvable set if for a fuzzy closed set μ in (X, T), $l_c - cl(\mu \wedge \lambda) \wedge l_c - cl[\mu \wedge (1 - \lambda)]$ is a fuzzy somewhere dense in (X, T). That is a fuzzy irresolvable set in (X, T) if $\operatorname{int} cl\{cl(\mu \wedge \lambda) \wedge cl[\mu \wedge (1 - \lambda)]\} \neq 0$, where $1 - \mu \in T$.

3. Fuzzy co-locally resolvable and fuzzy co-locally irresolvable sets

Definition 3.1. A fuzzy set λ in a fuzzy topological space (X, T) is called a fuzzy co-locally resolvable if for each fuzzy co-locally closed set μ in

(X, T), $l_c - cl(\mu \wedge \lambda) \wedge l_c - cl[\mu \wedge (1-\lambda)]$ is a fuzzy co-locally nowhere dense in (X, T). That is, λ is a fuzzy co-locally resolvable set in (X, T) if $l_c - \operatorname{int} l_c - \{l_c - cl(\mu \wedge \lambda) \wedge l_c - cl(\mu \wedge (1-\lambda))\} = 0$, where $1 - \mu \in T$.

Definition 3.2. Let (X, T) be a fuzzy topological space. A fuzzy set λ is called a fuzzy co-locally irresolvable set if for a fuzzy co-locally closed set μ in (X, T), $l_c - cl(\mu \wedge \lambda) \wedge l_c - cl(\mu \wedge (1-\lambda))$ is a fuzzy co-locally somewhere dense in (X, T). That is, λ is a fuzzy co-locally irresolvable set in (X, T) if $l_c - \operatorname{int} l_c - cl\{l_c - cl(\mu \wedge \lambda) \wedge l_c - cl(\mu \wedge (1-\lambda))\} \neq 0$, where $1 - \mu \in T$.

Example 3.1. Let $X = \{\lambda, \mu, \gamma\}$. The fuzzy sets λ, μ and γ are defined on X as follows:

$$\lambda : X \to [0, 1] \text{ defined as } \lambda\left(\frac{a}{0}, \frac{b}{0.2}, \frac{c}{0.5}\right),$$

$$\mu : X \to [0, 1] \text{ defined as } \mu\left(\frac{a}{0.1}, \frac{b}{0.2}, \frac{c}{0.7}\right),$$

$$\gamma : X \to [0, 1] \text{ defined as } \gamma\left(\frac{a}{0.3}, \frac{b}{0.5}, \frac{c}{0.9}\right).$$

Then $T = \{0, \lambda, \mu, \gamma, 1\}$ is a fuzzy topology on X. Now the fuzzy sets $1 - \lambda, \alpha, \beta, \varepsilon, \zeta, \eta, \vartheta, \nu$ and σ are fuzzy co-locally closed in (X, T), since $\lambda \vee (1-\lambda) = 1 - \lambda$, $\lambda \vee (1-\mu) = \alpha$, $\lambda \vee (1-\gamma) = \beta$, $\mu \vee (1-\lambda) = \varepsilon$, $\mu \vee (1-\mu) = \zeta$, $\mu \vee (1-\gamma) = \eta$, $\gamma \vee (1-\lambda) = \vartheta$, $\gamma \vee (1-\mu) = \nu$ and $\gamma \vee (1-\gamma) = \sigma$. Therefore the fuzzy sets $\lambda, 1-\alpha, 1-\beta, 1-\varepsilon, 1-\zeta, 1-\eta, 1-\vartheta, 1-\nu$ and $1-\sigma$ are fuzzy co-locally open sets in (X, T). Now $l_c - \operatorname{int} l_c - cl[l_c - cl(\alpha \wedge \lambda) \wedge l_c - cl(\alpha \wedge (1-\lambda))] = l_c - \operatorname{int} l_c - cl[l_c - cl(\lambda) \wedge l_c - cl(\alpha)]$ $= l_c - \operatorname{int} l_c - cl(\beta) = 1 - \beta \neq 0$, where $1 - \mu \in T$. Hence λ is a fuzzy co-locally irresolvable set in (X, T). $1 - \mu \in T$.

Proposition 3.1. If λ is a fuzzy co-locally resolvable set in a fuzzy topological space (X, T), then for each fuzzy co-locally closed set μ in (X, T), $[\lambda \wedge (1-\lambda) \wedge \mu]$ is a fuzzy co-locally nowhere dense set in (X, T).

Proof. Let λ be a fuzzy co-locally resolvable set in (X, T). Then for each fuzzy co-locally closed set in (X, T), $l_c - \operatorname{int} l_c - cl[l_c - cl\{\mu \wedge \lambda\} \wedge l_c - cl \{\mu \wedge (1-\lambda)\}] = 0$, in (X, T). Now, $[l_c - cl(\mu \wedge \lambda) \wedge l_c - cl(\mu \wedge (1-\lambda))] \geq l_c - cl[(\mu \wedge \lambda) \wedge (\mu \wedge (1-\lambda))]$ in (X, T) and then $[l_c - cl(\mu \wedge \lambda) \wedge l_c - cl(\mu \wedge (1-\lambda))] \geq l_c - cl[\mu \wedge \lambda \wedge (1-\lambda)]$. Then $l_c - \operatorname{int} l_c - cl[l_c - cl(u \wedge \lambda) \wedge cl(\mu \wedge (1-\lambda))] \geq l_c - \operatorname{int} l_c - cl\, l_c - cl[\mu \wedge \lambda \wedge (1-\lambda)] = l_c - \operatorname{int} l_c - cl[\mu \wedge \lambda \wedge (1-\lambda)]$ and thus $0 \geq l_c - \operatorname{int} l_c - cl[\mu \wedge \lambda \wedge (1-\lambda)]$. That is, $l_c - \operatorname{int} l_c - cl[\mu \wedge \lambda \wedge (1-\lambda)]$. Hence, for each fuzzy co-locally closed set μ in (X, T), $[\mu \wedge \lambda \wedge (1-\lambda)]$ is a fuzzy co-locally nowhere set in (X, T).

Proposition 3.2. *If* λ *is a fuzzy co-locally resolvable set in a fuzzy topological space* (X, T), *then is also a fuzzy co-locally resolvable set in* (X, T).

Proof. Let λ be a fuzzy co-locally resolvable set in (X, T). Then, for each fuzzy co-locally closed set μ in (X, T), $l_c - \operatorname{int} l_c - cl[l_c - cl\{\mu \wedge \lambda\} \wedge l_c - cl\{\mu \wedge (1-\lambda)\}] = 0, \ldots (1)$.

Now $cl\{\mu \wedge (1-\lambda)\} = 0$, $l_c - \operatorname{int} l_c - cl[l_c - cl\{\mu \wedge (1-\lambda)\} \wedge l_c - cl\{\mu \wedge (1-[1-\lambda])\}] = l_c - \operatorname{int} l_c - cl[l_c - cl\{\mu \wedge (l_c - cl\{\mu \wedge (1-\lambda)\} \wedge l_c - cl\{\mu \wedge \lambda\}] 0$, from (1). Hence $1 - \lambda$ is a fuzzy co-locally resolvable set in (X, T).

Proposition 3.3. *If* λ *is a fuzzy set in a fuzzy topological space* (X, T) *in which fuzzy co-locally closed set* μ *and* $l_c - \operatorname{int}(\mu) = 0$, *then* λ *is a fuzzy co-locally resolvable set in* (X, T).

Proof. Let λ be a non-zero fuzzy set defined on X in (X, T). Then, for a fuzzy co-locally closed setμin (X, T), by hypothesis, $l_c - \operatorname{int}(\mu) = 0$, in (X, T). Now $l_c - \operatorname{int} l_c - cl\{l_c - cl(\lambda \wedge \mu) \wedge l_c - cl([1-\lambda] \wedge \mu)\} \leq l_c - \operatorname{int} l_c - cl\{l_c - cl(\lambda) \wedge l_c - cl(\mu) \wedge l_c - cl[1-\lambda]) \wedge l_c - cl(\mu)\} = l_c - \operatorname{int} l_c - cl\{[l_c - cl(\lambda) \wedge \mu] \wedge [l_c - cl(1-\lambda) \wedge \mu]\} = l_c - \operatorname{int} l_c - cl\{[l_c - cl(\lambda) \wedge l_c - cl(1-\lambda)] \wedge \mu\} \leq l_c - \operatorname{int}\{l_c - cl\, l_c - cl(\lambda) \wedge l_c - cl\, l_c - cl(1-\lambda) \wedge l_c - cl(\mu)\} = l_c - \operatorname{int}\{[l_c - cl(\lambda) \wedge l_c\, cl(1-\lambda)] \wedge \mu\} = l_c - \operatorname{int}\{bd(\lambda) \wedge \mu\} = l_c - \operatorname{int}\{bd(\lambda)\} \wedge l_c - \operatorname{int}\{\mu\} = l_c - \operatorname{int}\{bd(\lambda)\} \wedge 0 = 0$. Thus, for a fuzzy co-locally closed set μ, $l_c - \operatorname{int} l_c - cl\{l_c - cl(\lambda \wedge \mu) \wedge l_c - cl([1-\lambda] \wedge \mu)\} = 0$ and hence λ is a fuzzy co-locally resolvable set in (X, T).

Proposition 3.4. *If* λ *is a fuzzy co-locally resolvable set in a fuzzy topological space* (X, T), *then for each fuzzy co-locally closed set* μ *in* (X, T), $l_c - \operatorname{int}[\lambda \wedge (1-\lambda)] \leq l_c - cl(1-\mu)$, *in* (X, T).

Proof. Let λ be a fuzzy co-locally resolvable set in (X, T). Then, by proposition 3.1, for each fuzzy co-locally closed set μ in (X, T), $l_c - \operatorname{int} l_c - cl$ $[\mu \wedge \lambda \wedge (1-\lambda)] = 0$, in (X, T). But $l_c - \operatorname{int}[\mu \wedge \lambda \wedge (1-\lambda)] \leq l_c - \operatorname{int} l_c -$ $cl[\mu \wedge \lambda \wedge (1-\lambda)]$, implies that $l_c - \operatorname{int}[\mu \wedge \lambda \wedge (1-\lambda)] = 0$. Since $l_c - \operatorname{int}[\mu \wedge \lambda \wedge (1-\lambda)] = l_c - \operatorname{int}(\mu) \wedge l_c - \operatorname{int}(\lambda \wedge (1-\lambda))$, $l_c - \operatorname{int}(\mu) \wedge l_c - \operatorname{int}$ $(\lambda \wedge (1-\lambda)) = 0$ Then, $l_c - \operatorname{int}(\lambda \wedge (1-\lambda)) \leq 1 - [l_c - \operatorname{int}(\mu)]$ and hence, for each fuzzy co-locally closed set μ in (X, T), $l_c - \operatorname{int}[\lambda \wedge (1-\lambda)] \leq l_c - cl(1-\mu)$, in (X, T).

Proposition 3.5. *If* λ *is a fuzzy open and fuzzy co-locally dense set in a fuzzy topological space* (X, T), *then* $(1-\lambda)$ *is a fuzzy co-locally resolvable set in* (X, T).

Proof. Let λ be a fuzzy co-locally open and fuzzy co-locally dense set in (X, T). Since λ is fuzzy co-locally open in (X, T), $(1-\lambda)$ is fuzzy co-locally closed in (X, T). Also, since λ is fuzzy co-locally dense in (X, T), $l_c - cl(\lambda) = 1$. Then, $l_c - \operatorname{int}(1-\lambda) = 1 - [l_c - cl(\lambda)] = 1 - 1 = 0$. Hence $(1-\lambda)$ is a fuzzy co-locally closed set with $l_c - \operatorname{int}(1-\lambda) = 0$, in (X, T). Then, by proposition 3.3, $(1-\lambda)$ is a fuzzy co-locally resolvable set in (X, T).

Proposition 3.6. *If* $\lambda \leq \mu$, *for each fuzzy co-locally closed set* μ *with* $l_c - \operatorname{int}(\mu) = 0$ *in a fuzzy topological space* (X, T), *then* λ *is a fuzzy co-locally resolvable set but not a fuzzy co-locally open set in* (X, T).

Proof. Let λ be a fuzzy set defined on X and μ is a fuzzy co-locally closed set such that $l_c - \operatorname{int}(\mu) = 0$ in (X, T). Then, by proposition 3.3, λ is a fuzzy co-locally resolvable set in (X, T). Since $\lambda \leq \mu$, $l_c - \operatorname{int}(\lambda) \leq l_c - \operatorname{int}(\mu)$ and $l_c - \operatorname{int}(\mu) = 0$, implies that $l_c - \operatorname{int}(\lambda) = 0$ and hence λ is not a fuzzy co-locally open set in (X, T).

Proposition 3.7. *If* λ *is a fuzzy set in a fuzzy topological space* (X, T) *in which fuzzy co-locally open sets are fuzzy co-locally dense* sets, *then* λ *is a fuzzy co-locally resolvable set in* (X, T).

Proof. Let λ be a non-zero fuzzy set in (X, T). If μ is a non-zero fuzzy co-locally closed set in (X, T), then $1-\mu$ is a fuzzy co-locally open set in (X, T) and by hypothesis, $l_c - cl(1-\mu) = 1$, in (X, T). Now $1-[l_c - \text{int}(\mu)] = l_c - cl(1-\mu) = 1$, implies that $l_c - \text{int}(\mu) = 0$, in (X, T). Then, by proposition 3. λ is a fuzzy co-locally resolvable set in (X, T).

Proposition 3.8. *If* λ *is a fuzzy set in a fuzzy topological space* (X, T) *in which* $l_c - \text{int}\{bd(\lambda)\} = 0$, *then* λ *is a fuzzy co-locally resolvable set in* (X, T).

Proof. Let λ be a non-zero fuzzy set defined on X in (X, T). By hypothesis, $l_c - \text{int}\{bd(l)\} = 0$, in (X, T). Now, for a fuzzy co-locally closed set μ in (X, T), as in the proof of proposition 3.1, $l_c - \text{int}\, l_c - cl\{l_c - cl(\lambda \wedge \mu) \wedge l_c - cl([1-\lambda] \wedge \mu)\} \leq l_c - \text{int}\{bd(\lambda)\} \wedge l_c - \text{int}\{\mu\} = 0 \wedge l_c\, \text{int}\{\mu\} = 0$ and thus, for a fuzzy co-locally closed set μ in (X, T), $l_c - \text{int}\, l_c - cl\{l_c - cl(\lambda \wedge \mu) \wedge l_c - cl([1-\lambda] \wedge \mu)\} = 0$, implies that λ is a fuzzy co-locally resolvable set in (X, T).

Proposition 3.9. *If* λ *is a fuzzy co-locally irresolvable set in a fuzzy topological space* (X, T), *then* $bd(\lambda)$ *is a fuzzy co-locally somewhere dense set in* (X, T).

Proof. Let λ be a fuzzy co-locally irresolvable set in (X, T). Then, for a fuzzy co-locally closed set μ in (X, T), $l_c - \text{int}\, l_c - cl\{l_c - cl(\lambda \wedge \mu) \wedge l_c - cl(1-\lambda) \wedge \mu\} \neq 0$, in (X, T). Now $l_c - \text{int}\, l_c - cl\{l_c - cl(\lambda \wedge \mu) \wedge l_c - cl[(1-\lambda) \wedge \mu]\} \leq l_c - \text{int}\, l_c - cl\{[l_c - cl(\lambda) \wedge l_c - cl(\mu)] \wedge [l_c - cl(1-\lambda) \wedge l_c - cl(\mu)]\} = l_c - \text{int}\, l_c - cl\{[l_c - cl(\lambda) \wedge \mu] \wedge [l_c - cl(1-\lambda) \wedge \mu]\} = l_c - \text{int}\, l_c - cl\{[l_c - cl(\lambda) \wedge l_c - cl(1-\lambda)] \wedge \mu\} = l_c - \text{int}\, l_c - cl[bd(\lambda)]$. Thus, $l_c - \text{int}\, l_c - cl\{(\lambda \wedge \mu) \wedge l_c - cl[1-\lambda] \wedge \mu\} \neq 0$, implies that $l_c - \text{int}\, l_c - cl[bd(\lambda)] \neq 0$, in (X, T). Thus, $bd(\lambda)$ is a fuzzy co-locally somewhere dense in (X, T).

Proposition 3.10. *If* λ *is a fuzzy co-locally irresolvable set in a fuzzy topological space* (X, T), *then* λ *and* $1-\lambda$ *are fuzzy co-locally somewhere dense sets in* (X, T).

Proof. Let λ be a fuzzy co-locally irresolvable set in (X, T). Then, for a fuzzy co-locally closed set μ in (X, T), $l_c - \operatorname{int} l_c - cl\{l_c - cl(\lambda \wedge \mu) \wedge l_c - cl(1-\lambda) \wedge \mu\} \neq 0$, in (X, T). Now $l_c - \operatorname{int} l_c - cl\{l_c - cl(\lambda \wedge \mu) \wedge l_c - cl([1-\lambda] \wedge \mu)\} \leq l_c - \operatorname{int} l_c - cl\{[l_c - cl(\lambda) \wedge l_c - cl(\mu)] \wedge [l_c - cl(1-\lambda) \wedge l_c - cl(\mu)]\} = l_c - \operatorname{int} l_c - cl\{[l_c - cl(\lambda) \wedge \mu] \wedge [l_c - cl(1-\lambda) \wedge \mu]\} = l_c - \operatorname{int} l_c - cl\{[l_c - cl(\lambda) \wedge l_c - cl(1-\lambda)] \wedge \mu\} \leq l_c - \operatorname{int}\{l_c - cl\, l_c - cl(\lambda) \wedge l_c - cl\, l_c - cl(1-\lambda) \wedge l_c - cl(\mu)\} = l_c - \operatorname{int}\{l_c - cl(\lambda) \wedge l_c - cl(1-\lambda)] \wedge \mu\} l_c - \operatorname{int} l_c - cl(\lambda) \wedge l_c - \operatorname{int} l_c - cl(1-\lambda)] \wedge l_c - \operatorname{int}(\mu)\}$. Thus $l_c - \operatorname{int} l_c - cl\{l_c - cl(\lambda \wedge \mu) \wedge l_c - cl[(1-\lambda) \wedge \mu]\} \neq 0$, implies that $\{[l_c - \operatorname{int} l_c - cl(\lambda)] \wedge [l_c - \operatorname{int} l_c - cl (1-\lambda)] \wedge [l_c - \operatorname{int}(\mu)]\} \neq 0$, in (X, T) and thus $l_c - \operatorname{int} l_c - cl(\lambda) \neq 0$, $l_c - \operatorname{int} l_c - cl(1-\lambda) \neq 0$ and $l_c - \operatorname{int}(\mu) \neq 0$. Hence λ and $1-\lambda$ are fuzzy co-locally somewhere dense sets in (X, T).

Proposition 3.11. *If a fuzzy co-locally open set* λ *is a fuzzy co-locally irresolvable set in a fuzzy topological space* (X, T), *then* λ *is not a fuzzy co-locally dense set in* (X, T).

Proof. Let λ be a fuzzy co-locally open set in (X, T). By hypothesis, λ is a fuzzy co-locally irresolvable set in (X, T). Then, by proposition 3.10, $1-\lambda$ is a fuzzy co-locally somewhere dense set in (X, T) and then $l_c - \operatorname{int} l_c - cl(1-\lambda) \neq 0$, in (X, T). Now $l_c - \operatorname{int} l_c - cl(1-\lambda) = (1 - l_c - cl\, l_c - \operatorname{int}(\lambda) = 1 - l_c - cl(\lambda)$ and then $1 - l_c - cl(\lambda) \neq 0$. This implies that $l_c - cl(\lambda) \neq 1$, in (X, T). Hence λ is not a fuzzy co-locally dense set in (X, T).

Proposition 3.12. *If* λ *is a fuzzy co-locally closed set in a fuzzy topological space* (X, T) *in which fuzzy co-locally open sets are fuzzy co-locally irresolvable sets*, *then* $l_c - \operatorname{int}(\lambda) \neq 0$, *in* (X, T).

Proof. Let λ be a fuzzy co-locally closed set in (X, T). Then, $1-\lambda$ is a fuzzy co-locally open set in (X, T). Then, by proposition 3.11, $l_c - cl(1-\lambda) \neq 1$, and thus $1 - [l_c - \operatorname{int}(\lambda)] \neq 1$, in (X, T). Hence

$l_c - \text{int}(\lambda) \neq 0$, in (X, T).

Proposition 3.13. *If λ is a fuzzy set defined on X in a fuzzy topological space (X, T) in which fuzzy co-locally open sets are fuzzy co-locally irresolvable sets, then $1-\lambda$ is a fuzzy co-locally somewhere dense set in (X, T).*

Proof. Let λ be a fuzzy set defined on X in (X, T). If $l_c - \text{int}(\lambda)$ is a non - zero fuzzy co-locally open set in (X, T), then by hypothesis, $l_c - \text{int}(\lambda)$ is a fuzzy co-locally irresolvable set in (X, T). By proposition 3.11, $l_c - cl[l_c - \text{int}(\lambda)] \neq 1$, in (X, T). This implies that $1 - \{l_c - cl[l_c - \text{int}(\lambda)]\} \neq 0$ and thus $l_c - \text{int}\, l_c - cl(1-\lambda) \neq 0$, in (X, T). Hence $1-\lambda$ is a fuzzy co-locally somewhere dense set in (X, T).

Proposition 3.14. *If λ is a fuzzy co-locally closed set in a fuzzy topological space (X, T) in which fuzzy co-locally open sets are fuzzy co-locally irresolvable sets, then λ is a fuzzy co-locally somewhere dense set in (X, T).*

Proof. Let λ be a fuzzy co-locally closed set in (X, T) in which fuzzy co-locally open sets are fuzzy co-locally irresolvable sets. Then, by proposition 3.12, $l_c - \text{int}(\lambda) \neq 0$, in (X, T). Now $l_c - \text{int}\, l_c - cl(\lambda) = l_c - \text{int}(\lambda) \neq 0$, in (X, T). Hence, λ is a fuzzy co-locally somewhere dense set in (X, T).

Proposition 3.15. *If λ is a fuzzy co-locally closed set in a fuzzy topological space (X, T) in which fuzzy co-locally open sets are fuzzy co-locally irresolvable sets, and if $\lambda \leq \mu$, then μ is a fuzzy co-locally somewhere dense set in (X, T).*

Proof. Let λ be a fuzzy co-locally closed set in (X, T) in which fuzzy co-locally open sets are fuzzy co-locally irresolvable sets. Then, by proposition 3.14, λ is a fuzzy co-locally somewhere dense set in (X, T) and thus $l_c \,\text{int}\, l_c - cl(\lambda) \neq 0$. Now $\lambda \leq \mu$ implies that $l_c - \text{int}\, l_c - cl(\lambda) \leq l_c - \text{int}\, l_c - cl(\mu)$. Then $l_c - \text{int}\, l_c - cl(\mu) \neq 0$. Hence μ is a fuzzy co-locally somewhere dense set in (X, T).

References

[1] S. Anjalmose and A. Saravanan, Fuzzy locally dense sets and fuzzy locally nowhere dense sets, International Journal of research and analytical Reviews 6(2) (May-2019), 292-299.

[2] S. Anjalmose and A. Virgin Raj, Fuzzy Co-locally dense, Co-locally somewhere dense, Co-locally cs dense sets, GIS Science Journal 8(6) (2021), 1065-1077.

[3] C. L. Chang, Fuzzy Topological Spaces, J. Math. Anal. Appl. 24 (1968), 182-190.

[4] P. K. Gain, R. P. Chakraborty and M. Pal, Characterization of some fuzzy subsets of fuzzy ideal topological spaces and decomposition of fuzzy continuity, International Journal of Fuzzy Mathematics and Systems 2(2) (2012), 149-161.

[5] G. Thangaraj, B. Mathivathani and P. Sathya, On fuzzy Resolvable sets and fuzzy resolvable functions, Advances in fuzzy Mathematics 12(6) (2017), 1171-1181.

[6] G. Thangaraj and S. Lokeshwari, Fuzzy irresolvable sets and fuzzy open Hereditarily Irresolvable spaces, Bulletin of Pure and Applied Science 38E(1) (2019), 369-377.

[7] G. Thangaraj and S. Senthil, On somewhere fuzzy continuous functions, Annals of Fuzzy Mathematics and Informatics 15(2) (2018), 181-198.

[8] L. A. Zadeh, Fuzzy sets, Information and Control 8 (1965), 338-353.