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VERTEX EDGE CLOSED NEIGHBORHOOD (VECN) PRIME LABELING OF SOME m-FOLD SNAKE RELATED GRAPHS

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Abstract

Let G be a graph with vertex set $|V(G)| = p$ and edge set $|E(G)| = q$. Let $s \in V(G)$ then $N_V[s] = \{s, t \in V(G)/t$ is adjacent to $s\}$ and $N_E[s] = \{s \in V(G), e \in E(G)/e$ is incident with $s\}$. A one-one mapping $\phi : V(G) \cup E(G) \rightarrow \{1, 2, 3, \ldots, p+q\}$ is a VECN prime labeling if for each vertex $s \in V(G)$ with $d(s) \geq 1$ where $d(s)$ is the degree of s then $\gcd\{\phi(s), \phi(t) / t \in N_V[s]\} = 1$ and $\gcd\{\phi(s), \phi(e) / e \in N_E[s]\} = 1$. A graph which admits VECN prime labeling is called a VECN prime graph. In this paper, we investigate VECN prime labeling of an m-fold Quadrilateral snake $m(Q_n)$, m-fold Alternate Quadrilateral snake $m(AQ_n)$, and m-fold Irregular Triangular snake $m(IT_n)$.

1. Introduction

One of the Mathematical model for addressing circuit design, network,

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missile guidance and coding theory is graph labeling. In 1980's the concept of Prime labeling was introduced by Tout et al. [7]. The notion of Neighborhood prime labeling was introduced by S. K. Patel et al. [6]. Pandya and Shrimali [5] have introduced the idea of Vertex edge neighborhood prime labeling. Mukund Bapat [2] have introduced the notion of Closed neighborhood prime labeling. In this sequel we introduced VECN prime labeling and investigate VECN prime labeling of an m-fold Quadrilateral snake $m(Q_n)$ m-fold Alternate Quadrilateral snake $m(AQ_n)$ and m-fold Irregular Triangular snake $m(IT_n)$.

Definition 1.1 [7]**.** Let $G = (V(G), E(G))$ be a graph with n vertices. A function $\phi : V(G) \to \{1, 2, 3, \ldots, n\}$ is said to be a prime labeling, if it is bijective and for every pair of adjacent vertices u and v, $\gcd(\phi(u), \phi(v)) = 1$. A graph which admits prime labeling is called a prime graph.

Definition 1.2 [6]. Let $G = (V(G), E(G))$ be a graph with n vertices. A bijective function $\phi : V(G) \to \{1, 2, 3, \ldots, n\}$ is said to be a neighborhood prime labeling, if for every vertex $v \in V(G)$ with $\deg(v) \geq 1$, $\gcd\{\phi(u) : u \in N(v)\} = 1$. A graph which admits neighborhood prime labeling is called a neighborhood prime graph.

Definition 1.1. An m-fold quadrilateral snake $m(Q_n)$ of length $n-1$ is obtained from a path $\alpha_1, \alpha_2, \ldots, \alpha_n$ by joining α_i and α_{i+1} to new vertices β_{2i-1}^j and β_{2i}^j for $1 \leq j \leq m, 1 \leq i \leq n-1$. That is every edge of a path is replaced by an m-cycle C_4.

Definition 1.2. An m-fold alternate quadrilateral snake $m(AQ_n)$ is obtained from a path $\alpha_1, \alpha_2, \ldots, \alpha_n$ by joining α_{2i-1} and α_{2i} to new vertices β_{2i-1}^j and β_{2i}^j for $1 \leq j \leq m, 1 \leq i \leq \frac{n}{2}$. That is every alternate edge of a path is replaced by an m-cycle C_4.

Definition 1.3. An m-fold irregular triangular snake $m(IT_n)$ is the graph obtained from the path $\alpha_1, \alpha_2, \ldots, \alpha_n$ by joining α_i and α_{i+2} to new vertices β_i^j for $1 \leq j \leq m, 1 \leq i \leq n-2$.

2. Main Results

Theorem 2.1. *An m-fold Quadrilateral snake* $m(Q_n)$ *is a VECN prime graph for all* $m \geq 1$ *and* $n \geq 2$.

Proof. Let $m(Q_n)$ be an m-fold $V[m(Q_n)] = \{u_i / 1 \leq i \leq n\} \cup \{v_i^j / 1 \leq i \leq 2(n-1), 1 \leq j \leq m\}$ quadrilateral snake graph with and $E[m(Q_n)] = \{e_i / 1 \leq i \leq n-1\} \cup \{e_i^j / 1 \leq i \leq 2n-2, 1 \leq j \leq m\} \cup \{e_k^j / 1 \leq k \leq n-1, 1 \leq j \leq m\}$ such that $e_i = \{u_i u_{i+1} / 1 \leq i \leq n-1\}$, $e_k^j = \{v_{2i-1}^j v_{2i}^j / 1 \leq i \leq n-1, 1 \leq j \leq m\}$ and $e_i^j = e_{2i-1}^j \cup e_{2i}^j$ where $e_{2i-1}^j = \{u_i v_{2i-1}^j / 1 \leq i \leq n-1, 1 \leq j \leq m\}$ and $e_{2i}^j = \{v_{2i}^j u_{i+1} / 1 \leq i \leq n-1, 1 \leq j \leq m\}$. Clearly, $p = n(2m+1) - 2m$, $q = (n-1)(3m+1)$ and $p + q = 5m(n-1) + 2n - 1$.

Define $\phi : V[m(Q_n)] \cup E[m(Q_n)] \to \{1, 2, 3, \ldots, 5m(n-1) + 2n - 1\}$ as follows

$\phi(u_i) = 2i - 1, 1 \leq i \leq n$

$\phi(e_i) = 4m(n-1) + 2i, 1 \leq i \leq n-1$

$\phi(v_i^j) = 2(n-1)(2j-1) + 2i + 1, 1 \leq i \leq 2n-2, 1 \leq j \leq m$

$\phi(e_i^j) = 4(j-1)(n-1) + 2i, 1 \leq i \leq 2n-2, 1 \leq j \leq m$

$\phi(e_k^j) = (4m+2)(n-1) + m(k-1) + j + 1, 1 \leq k \leq n-1, 1 \leq j \leq m$

We claim that ϕ is a VECN prime graph for all $m \geq 1$ and $n \geq 2$.

Let α be any vertex of $m(Q_n)$.

Case (i). Let $\alpha = \{u_i / 1 \leq i \leq n\}$ with $d(\alpha) \geq 2$.

Then $\gcd\{\phi(\alpha), \phi(\beta) / \beta \in N_V[\alpha]\} = 1$ and $\gcd\{\phi(\alpha), \phi(e) / e \in N_E[\alpha]\} = 1$.

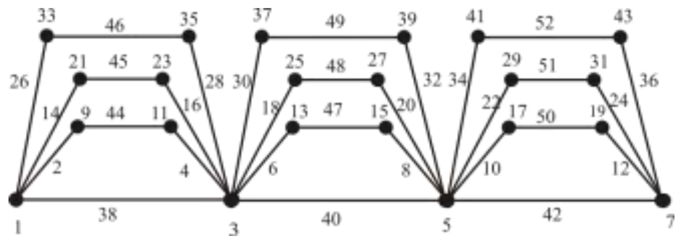
Case (ii). Let $\alpha = \{v_i^j / 1 \leq i \leq 2(n-1), 1 \leq j \leq m\}$ with $d(\alpha) = 2$.

Then $\gcd d\{\phi(\alpha), \phi(\beta) / \beta \in N_V[\alpha]\} = 1$ and $\gcd\{\phi(\alpha), \phi(e) / e \in N_E[\alpha]\} = 1$.

Thus ϕ admits VECN prime labeling.

Hence $m(Q_n)$ is a VECN prime graph for all $m \geq 1$ and $n \geq 2$.

Example 2.1. The VECN prime labeling of $3(Q_4)$ is in Figure 2.1



Theorem 2.2. *An m-fold Alternate Quadrilateral snake* $m(AQ_n)$ *is a VECN prime graph for all* $m \geq 1$ *and* $n \geq 4$.

Proof. Let $m(AQ_n)$ be an m-fold alternate quadrilateral snake graph with $V(m(AQ_n)) = \{u_i / 1 \leq i \leq n\} \cup \{v_i^j / 1 \leq i \leq n, 1 \leq j \leq m\}$ and $E(m(AQ_n)) = \{e_i^j / 1 \leq j \leq m, 1 \leq i \leq n\} \cup \{e_i / 1 \leq i \leq n-1\} \cup \{e_k^j / 1 \leq k \leq \frac{n}{2}, 1 \leq j \leq m\}$ such that $e_i = \{u_i u_{i+1} / 1 \leq i \leq n-1\}$, $e_k^j = \{v_{2i-1}^j v_{2i}^j / 1 \leq i \leq \frac{n}{2}, 1 \leq j \leq m\}$ and $e_i^j = e_{2i-1}^j \cup e_{2i}^j$ where $e_{2i-1}^j = \{u_{2i-1} v_{2i-1}^j / 1 \leq i \leq \frac{n}{2}, 1 \leq j \leq m\}$ and $e_{2i}^j = \{v_{2i}^j u_{2i} / 1 \leq i \leq \frac{n}{2}, 1 \leq j \leq m\}$.

Clearly $p = n(m+1)$, $q = \frac{n}{2}(3m+2) - 1$ and $p + q = \frac{n}{2}(5m+4) - 1$.

Define $\phi : V[m(AQ_n)] \cup E[m(AQ_n)] \to \{1, 2, 3, \ldots, \frac{n}{2}(5m+4) - 1\}$ as follows

$\phi(u_i) = 2i - 1, 1 \leq i \leq n$

$\phi(e_i) = 2(mn + i), 1 \leq i \leq n - 1$

$\phi(e_i^j) = 2n(\mathrm{j} - 1) + 2i, 1 \leq i \leq n, 1 \leq j \leq m$

$\phi(e_k^j) = 2n(m+1) + \frac{n}{2}(j-1) + k - 1, 1 \leq k \leq \frac{n}{2}, 1 \leq j \leq m$

$\phi(v_i^j) = 2nj + 2i - 1, 1 \le i \le n, 1 \le j \le m$

We claim that ϕ is a VECN prime graph for all $m \ge 1$ and $n \ge 4$.

Let α be any vertex of $m(AQ_n)$.

Case (i). Let $\alpha = \{u_i / 1 \le i \le n\}$ with $d(\alpha) \ge 2$.

Then $\gcd\{\phi(\alpha), \phi(\beta)/\beta \in N_V[\alpha]\} = 1$ and $\gcd\{\phi(\alpha), \phi(e)/e \in N_E[\alpha]\} = 1$.

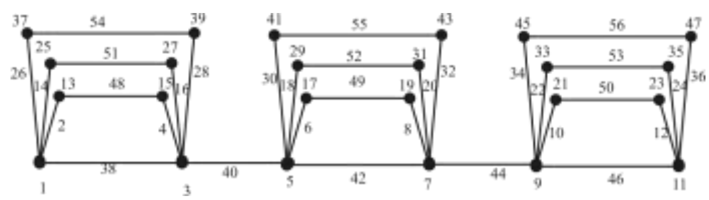
Case (ii). Let $\alpha = \{v_i^j / 1 \le i \le n, 1 \le j \le m\}$ with $d(\alpha) = 2$.

Then $\gcd\{\phi(\alpha), \phi(\beta)/\beta \in N_V[\alpha]\} = 1$ and $\gcd\{\phi(\alpha), \phi(e)/e \in N_E[\alpha]\} = 1$.

Thus ϕ admits VECN prime labeling.

Hence $m(AQ_n)$ is a VECN prime graph for all $m \ge 1$ and $n \ge 4$.

Example 2.2. The VECN prime labeling of $3(AQ_6)$ is in Figure 2.2



Theorem 2.3. *An m-fold irregular Triangular snake* $m(IT_n)$ *is a VECN prime graph for all* $m \ge 1$ *and* $n \ge 3$.

Proof. Let $m(IT_n)$ be an m-fold irregular triangular snake graph with $V[m(IT_n)] = \{u_i / 1 \le i \le n\} \cup \{v_i^j / 1 \le i \le n-2, 1 \le j \le m\}$ and $E[m(IT_n)] = \{e_i / 1 \le i \le n-1\} \cup \{e_i^j / 1 \le i \le 2(n-2), 1 \le j \le m\}$ such that $e_i = \{u_i u_{i+1} / 1 \le i \le n-1\}$, and $e_i^j = e_{2i-1}^j \cup e_{2i}^j$ where $e_{2i-1}^j = \{u_i v_i^j / 1 \le i \le n-2, 1 \le j \le m\}$, and $e_{2i}^j = \{v_i^j u_{i+2} / 1 \le i \le n-2, 1 \le j \le m\}$. Clearly, $p = n(m+1) - 2m$, $q = n(2m+1) - (4m+1)$ and $p + q = 3m(n-2) + 2n - 1$.

Define $\phi : V[m(IT_n)] \cup E[m(IT_n)] \to \{1, 2, 3, \ldots, 3m(n-2) + 2n - 1\}$ as follows

$\phi(u_i) = n - i + 1, 1 \le i \le n$

$\phi(v_i^j) = (n-2)j + 2 + i, 1 \le i \le n-2, 1 \le j \le m$

$\phi(e_i) = (3m+1)n - 6m + i, 1 \le i \le n-1$

$$\phi(e_i^j) = \begin{cases} m(n-3+i) + n + 2j - 1, 1 \le j \le m, & i = 1, 3, \ldots, 2n-5 \\ m(n-4+i) + n + 2j, 1 \le j \le m, & i = 2, 4, \ldots, 2n-4 \end{cases}$$

We claim that ϕ is a VECN prime graph for all $m \ge 1$ and $n \ge 3$.

Let α be zany vertex of $m(IT_n)$.

Case (i). Let $\alpha = \{u_i/1 \le i \le n\}$ with $d(\alpha) \ge 2$.

Then $\gcd\{\phi(\alpha), \phi(\beta)/\beta \in N_V[\alpha]\} = 1$ and $\gcd\{\phi(\alpha), \phi(e)/e \in N_E[\alpha]\} = 1$.

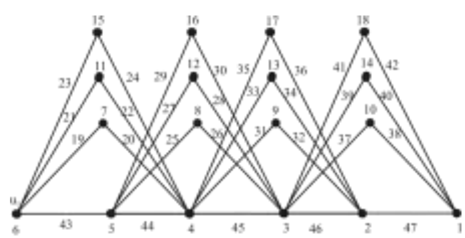
Case (ii). Let $\alpha = \{v_i^j/1 \le i \le n-2, 1 \le j \le m\}$ with $d(\alpha) = 2$.

Then $\gcd\{\phi(\alpha), \phi(\beta)/\beta \in N_V[\alpha]\} = 1$ and $\gcd\{\phi(\alpha), \phi(e)/e \in N_E[\alpha]\} = 1$.

Thus ϕ admits VECN prime labeling.

Hence $m(IT_n)$ is a VECN prime graph for all $m \ge 1$ and $n \ge 3$.

Example 2.3. The VECN prime labeling of $3(IT_6)$ is in Figure 2.3



3. Conclusion

This paper examines the VECN prime labeling of m-fold quadrilateral snake $m(Q_n)$, m-fold alternate quadrilateral snake $m(AQ_n)$, and m-fold irregular triangular snake $m(IT_n)$.

4. Future work

The concept of VECN prime labeling inspires us to propose a new type of labeling namely vertex edge closed k-neighborhood prime labeling and to further prove some results satisfying the conditions of vertex edge closed k-neighborhood prime labeling.

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