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DEGREE AND DISTANCE-BASED TOPOLOGICAL DESCRIPTORS OF *k*-ROOTED HYPERTREE

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Abstract

The interconnection network, k-rooted hypertree is a combination of k-rooted complete binary tree and hypertree. In this paper, we have analyzed and evaluated the numerical expressions of distance-based and degree-based topological descriptors of n-rooted hypertree.

1. Introduction

Trees are connected graphs with no cycles which can be used as data structures in computer science. Trees are used as theoretical models in various fields such as operations research and theory of electrical and design networks, etc [10]. Also, trees are used as biological entities such as DNA sequences. A complete binary tree is the type of tree with exactly two children from each parent node. Hypertree is a combination of complete binary tree and hypercube. Hypertrees are isomorphic to the biological structure, dendrimers. The topological indices of hypertree help in the QSAR study of dendrimers in the topological properties of dendrimeric metalorganic

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networks containing very heavy atoms [7]. The 1-rooted complete binary tree is obtained by joining a node to the root node of the complete binary tree by an edge. The k-rooted complete binary tree, T_n^k is obtained by taking k distinct complete binary trees whose root node is joined to distinct vertices in P_k. We denote $i = \{1, 2, \ldots, n\}$ as $i \in [n]$.

In this article, we constructed a new interconnection network and studied their distance and degree based topological indices. We have introduced and elaborated on k-rooted hypertree and its properties in Sections 2 and 3. Section 4 elaborates on the terminologies used to find the distance and degree-based indices and we derived the analytical expressions for them.

2. *k*-Rooted Hypertree

The basic skeleton of k-rooted hypertree is a combination of hypertree and n-rooted complete binary tree. Each complete binary tree forms a hypertree in k-rooted hypertree.

A k-rooted hypertree with dimension n has $n+1$ levels, where $n \geq 2$. The 1-rooted hypertree of dimension n is obtained by joining a vertex to the root node of $HT(n)$. At Level 0, it has k vertices and $(k-1)$ horizontal edges. Level 1 contains k vertices. There are vertical edges connecting each vertex in level 0 to its corresponding vertex in level 1 as shown in Figure 1. Thus, the k-rooted hypertree is obtained by taking k vertex disjoint 1-rooted hypertree of dimension n and the roots, $u_1, u_2, \ldots, u_k$ are joined by edges $u_i - u_{i+1}$, where $1 \leq i \leq k-1$. We denote k-rooted hypertree of dimension n as HT_n^k.

At Level 0, each node is labeled as $(i, 0), i \in [k]$. From Level 1, each vertex (i, x) gives rise to children $(i, 2x)$ and $(i, 2x+1)$ where $i \in [k]$. The labeling of k-rooted hypertree with dimension n is shown in Figure 1.

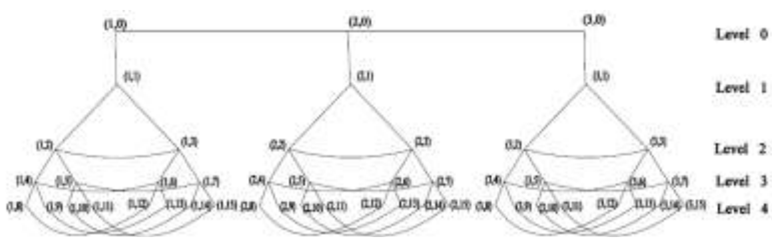

Figure 1. 3-rooted hypertree of dimension 3.

3. Properties

The k-rooted hypertree with dimension n, HT_n^k, has $2^n k$ vertices and $2k - 1 + k(3.2^{n-1} - 3)$ edges. It is a planar graph. HT_n^k has diameter $2n + k - 1$. The vertex and edge connectivity of HT_n^k is 1, if $k \geq 2$. It is neither Hamiltonian nor Eulerian but also not Pancyclic graph.

4. Terminologies

The topological index is a numeric quantity that characterizes the topological properties of a graph which is invariant under graph automorphism. For a graph $G(V, E)$, the degree of a vertex $a \in V(G)$ is the number of edges incident to the vertex a. The distance between two vertices $c, d \in V(G)$, $d_G(c, d)$, is the length of the shortest path between the two vertices c and d. The open neighborhood of a vertex $a \in V(G)$, $N_G(a) = \{b : b \in V(G), d(a, b) = 1\}$. Define $N_r(rs \mid G) = \{p \in V(G) : d_G(r, p) < d_G(s, p)\}$ and $M_r(rs \mid G) = \{pq \in E(G) : d_G(pq, r) < d_G(pq, s)\}$. The cardinality of $N_r(rs \mid G)$ and $M_r(rs \mid G)$ is denoted by $n_r(rs \mid G)$ and $m_r(rs \mid G)$ respectively. Table 1 and Table 2 present distance and degree-based topological indices.

For the definition of strength weighted graph, we refer to [1]. The Djoković-Winkler relation of a graph G is defined as follows: an edge $c = uv$ is in relation θ with another edge $d = wz$ if $d(u, w) + d(v, z) \neq d(u, z) + d(w, v)$. The properties of θ relation are reflexive and symmetric and also its transitive closure is an equivalence class. The edges partition into θ^* classes, whose partition set be defined as $\{D_i; 1 \leq i \leq k\}$. For any $i \in [k]$, the quotient graph G/D_i is a graph with connected components of $G - D_i$ as vertices, where two vertices are adjacent if at least one of the vertex in the component c_i is adjacent to at least one vertex in component c_j. A partition $C = \{C_i; 1 \leq i \leq r\}$ of $E(G)$ is coarser than $\{D_i; 1 \leq i \leq s\}$ if C_i is the union of one or more sets in D_i. We refer to Theorem 3.3 in [6], Theorem 2.1 and Theorem 2.3 in [5], and Theorem 1 in [1] for calculating various topological indices of k-rooted hypertree. We have used the same results for calculating Mostar and Edge Mostar indices.

Table 1. Distance-based topological indices.

Topological indices	Mathematical expressions
Wiener	$W(G) = \sum_{p,\, q \subseteq V(G)} w_v(p) w_v(q) d(p, q)$
Szeged	$Sz(G) = \sum_{e=cd \in E(G)} s_e(e) n_c(e \mid G) n_d(e \mid G)$
Edge-Szeged	$Sz_e(G) = \sum_{e=cd \in E(G)} s_e(e) m_c(e \mid G) m_d(e \mid G)$
Edge-vertex-Szeged	$Sz_{ev}(G) = \frac{1}{2} \sum_{e=cd \in E(G)} s_e(e) [n_c(e \mid G) m_d(e \mid G)$ $+ n_d(e \mid G) m_c(e \mid G)]$
Mostar	$Mo(G) = \sum_{e=cd \in E(G)} s_e(e) \lvert n_c(e \mid G) - n_d(e \mid G) \rvert$
Edge Mostar	$Mo_e(G) = \sum_{e=cd \in E(G)} s_e(e) \lvert m_c(e \mid G) - m_d(e \mid G) \rvert$
Padmakar Ivan	$PI(G) = \sum_{e=cd \in E(G)} s_e(e) [m_c(e \mid G) + m_d(e \mid G)]$

Table 2. Some degree-based topological indices.

Topological Zagreb	Mathematical expressions
Second Zagreb	$M_2(G) = \sum_{e=cd \in E(G)} \deg_G(c) \deg_G(d)$
Randić	$R(G) = \sum_{e=cd \in E(G)} \frac{1}{\sqrt{\deg_G(c) \deg_G(d)}}$
Atom Bond Connectivity	$ABC(G) = \sum_{e=cd \in E(G)} \sqrt{\frac{\deg_G(c) + \deg_G(d) - 2}{\deg_G(c) \deg_G(d)}}$

Harmonic	$H(G) = \sum_{e=cd\in E(G)} \frac{2}{\deg_G(c) + \deg_G(d)}$
Sum Connectivity	$SC(G) = \sum_{e=cd\in E(G)} \frac{1}{\sqrt{\deg_G(c) + \deg_G(d)}}$
Geometric Arithmetic	$GA(G) = \sum_{e=cd\in E(G)} \frac{2\sqrt{\deg_G(c) + \deg_G(d)}}{\deg_G(c)\deg_G(d)}$

Theorem 1. *If* $n \geq 2$, *then* $k \geq 3$,

1. $W(HT_n^k) = \frac{k(2\times 2^{2n}k^2 - 35\times 2^{2n} + 12\times 2^n k) + k(48\times 2^n \times n - 12\times 2^n - 12\times 2^{2n}k + 12\times 2^{2n}kn + 48)}{12}$

2. $Sz(HT_n^k) = \frac{k(3\times 2^{3n} - 118\times 2^{2n} + 4\times 2^{2n}k^2) + k(120\times 2^n k + 192\times 2^n n - 108\times 2^n - 96\times 2^{2n}k) + k(48\times 2^{2n}kn + 240)}{24}$

3. $Sz_e(HT_n^k) = \frac{107k}{6} - \frac{45.2^{2n}k^2}{4} + \frac{83.2^{2n}k^3}{8} - \frac{19.2^n k}{2} - 10k^2 + \frac{k^3}{6}$

$$- \frac{87.2^{2n}k}{8} + 2^{3n-3}k + 45.2^{n-1}k^2 - 2^{n-1}k^3 + 21.2^n kn - 3.2^n k^2 n$$

$$+ 9.2^{2n-1}k^2 n - 1$$

4. $Mo(HT_n^k) = 2^{2n}k^2 - 8k + 7.2^n k - 2^n k^2 - 4.2^n kn$

$$+ \sum_{j=1}^{k-1} |\, 2^n j + 2^n(j-k)\,|$$

5. $Mo_e(HT_n^k) = 3.2^{2n-1}k^2 - 12k + 11.2^n k - 5.2^{n-1}k^2 - 6.2^n kn$

$$+ \frac{\sum_{j=1}^{k-1} |\,(3.2^n - 2)(2j-k)\,|}{2}$$

6. $PI(HT_n^k) = 11k + \frac{3.2^{2n}k^2}{2} - \frac{15.2^n k}{2} + \frac{2^{2n}k}{2} - 2^n k^2 + 2$

Proof. We determine the θ^* classes of HT_n^k. The θ^* classes of HT_n^k from Level 1 is as follows: $B_i = \{((i, n-1), (i, j+2^{j-1}-1)), ((i, 2^{j-2}-1), (i, 2^{j-1}-1)), ((i, 2^{j-1}-1), (i, 2^{j-1}+2^{j-2}-1))\}$, where $i \in [k]$, $j \in [2^{n-1}-1]$, $A(j, l) = \{((i, 2^{j-1}(2l-1)-1), (i, 2^{j-1}(2l-2)-1)), ((i, 2^{n-1}+2^{j-1}(2l-1)-1), (i, 2^{n-1}+2^{j-1}(2l-2)-1)\}$ where $i \in [k]$, $j \in [n-2]$, $l \in [2^{n-j-1}]$, l is even, and $A(j, l) = \{((i, 2^{j-1}(2l-1)-1), (i, l2^{j}-1)), ((i, 2^{n-1}+2^{j-1}(2l-1)-1), (i, 2^{n-1}+l2^{j}-1))\}$ where $i \in [k]$, $j \in [n-2]$, $l \in [2^{n-j-1}]$, l is odd. Denote $D_i = (i, 0)-(i+1, 0); i \in [k-1]$ and $E_j = (j, 0)-(j, 1); j \in [k]$ as equivalence classes. $F_1, F_2, \ldots, F_{n-1}$ are coarser than the θ^* partition of hypertree. Let $F_j = \bigcup_{i=1}^{n-2} \bigcup_{l=1}^{2^{n-j-1}} A(j, l)$, $1 \le i \le k$, $1 \le i \le n-2$, $F_{n-1} = \bigcup_{i=1}^{k} B_i$, $F_n = \bigcup_{i=1}^{k-1} D_i$ and $F_{n+1} = \bigcup_{i=1}^{k} E_i$ are coarser than the θ^* partitions $A(j, l)$, B_i, D_j, and E_j. Here, the quotient graphs are reduced using Theorem 2.1 and Theorem 2.3 in [5].

In general, $HT_n^k/E_i, 1 \le i \le n-2$ is isomorphic to $K_{1, 2^{n-i-1}k}$. The quotient graph HT_n^k/E_i has one vertex with vertex weight and vertex strength as $[2^{n-i}k, 3.2^{n-i-1}k-k-1]$ and $2^{n-i-1}k$ vertices with vertex weight and vertex strength as $[2^{i+1}-2, 3.2^{i}-5]$ as shown in Figure 2. The quotient graph HT_n^k/E_{n-1} is isomorphic to k times K_3 identified by one vertex in each K_3 as shown in Figure 3a). The reduced graph of HT_n^k/E_{n-1} is isomorphic to $K_{1, k}$ and is further reduced as shown in Figure 3c).

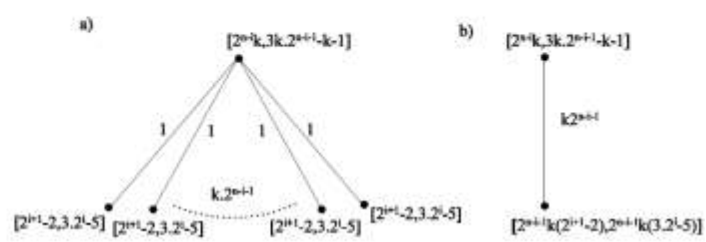

Figure 2. (a) HT_n^k/E_i, where $1 \le i \le n-2$, (b) Reduced graph of HT_n^k/E_i.

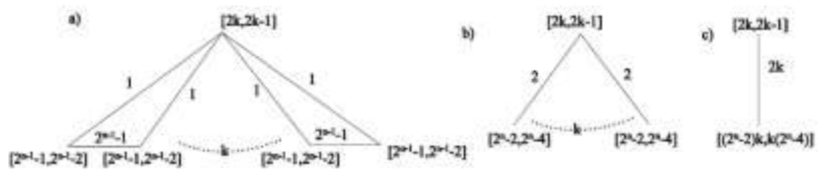

Figure 3. (a) HT_n^k/E_{n-1}, (b) Applying Theorem 2.3 in [5], (c) Reduced graph of HT_n^k/E_{n-1}.

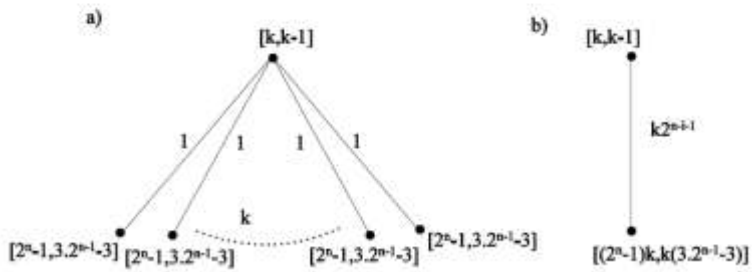

Figure 4. (a) HT_n^k/E_n, (b) Reduced graph of HT_n^k/E_n.

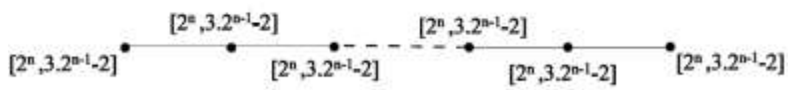

Figure 5. HT_n^k/E_{n+1}.

$$W(HT_n^k/E_i) = 2^{n-i}k^2(2^n - 2^{n-i}) + 2(k_2^{(2^{n-i-1})})(2^{i+1} - 2)^2$$

$$= 2^{2n-i}k^2 - 2^{2n-2i}k^2 + k(2^{n-i-1})(2^{i+1} - 2)^2 (2^{n-i-1}k - 1)$$

$$\sum_{i=1}^{n-2} W(HT_n^k/E_i) =$$

$$\frac{k}{3}(12.2^n k - 3.2^{2n} + 12.2^n n - 18.2^n - 9.2^{2n}k + 3.2^{2n}kn + 24)$$

$$W(HT_n^k/E_{n-1}) = 2k^2(2^n - 2) + k(k-1)(2^n - 2)^2 + k(2^{n-1} - 1)^3$$

$$W(HT_n^k/E_n) = (2^n - 1)(k^2 + k(k-1)(2^n - 1))$$

$$W(HT_n^k/E_{n+1}) = \frac{2^{2n-1}(k^3 - k)}{3}$$

$$W(HT_n^k) = \frac{k(2^{2n+1}k^2 - 35.2^{2n} + 12.2^n k + 48.2^n n - 12.2^n - 12.2^{2n} k)}{12}$$

$$+ \frac{k(12.2^{2n} kn + 48)}{12}$$

$$Sz(HT_n^k / E_i) = 2^{n-i} k((2^{n-i-1} k - 1)(2^{i+1} - 2) + 2^{n-i} k)(2^{i+1} - 2)$$

$$\sum_{i=1}^{n-2} Sz(HT_n^k / E_i) = 2^{n+1} k(4k + 4n + 2^{3-n} - 3.2^n k - 2^n + 2^n + kn - 6)$$

$$Sz(HT_n^k / E_{n-1}) = 2k(2k(2^{n-1} - 1) + 2(k-1)(2^{n-1} - 1)^2) + k(2^{n-1} - 1)^3$$

$$= k(2^{n-1} - 1)^3 + 2k(2k + (2k+2)(2^{n-1} - 1))(2^{n-1} - 1)$$

$$Sz(HT_n^k / E_n) = k(k + (k-1)(2^{n-1} - 1))(2^n - 1)$$

$$Sz(HT_n^k / E_{n+1}) = \frac{1}{3} k(k^2 - 1)2^{2n-1}$$

$$Sz(HT_n^k) =$$

$$\frac{k(3.2^{3n} - 118.2^{2n} + 4.2^{2n} k^2 + 120.2^n k + 192.2^n n - 108.2^n - 96.2^{2n} k)}{24}$$

$$+ \frac{k(48.2^{2n} kn + 240)}{24}$$

$$Sz_e(HT_n^k / E_i) = -2^{n-i} k(3.2^i - 5)(k - (2^{n-i-1} k - 1)(3.2^i - 3)$$

$$- 3.2^{n-i-1} k + 1)$$

$$\sum_{i=1}^{n-2} Sz_e(HT_n^k / E_i) = 2^{n-1} k(42n - 6kn + 82k - 5.2^{3-n} k + 5.2^{4-n}$$

$$- 33.2^n k - 9.2^n + 9.2^n kn - 68)$$

$$Sz_e(HT_n^k / E_{n-1}) = k(2^{n-1} - 1)^3 + 2k(2^n - 3)(4k + 2(2k-2)(2^{n-1} - 2)$$

$$+ (2^{n-1} - 1)(k-1) - 2)$$

$$Sz_e(HT_n^k/E_n) = k(3.2^n - 2)(3.2^{n-1} - 3)(k-1)$$

$$Sz_e(HT_n^k/E_{n+1}) =$$

$$\frac{(k-1)(28k + 92^{2n}k^2 - 48.2^n k + 4k^2 + 9.2^{2n}k - 12.2^n k^2 + 24)}{24}$$

$$Sz_e(HT_n^k) = \frac{107k}{6} - \frac{45.2^{2n}k^2}{4} + \frac{3.2^{2n}k^3}{8} - \frac{19.2^n k}{2} - 10k^2 + \frac{k^3}{6}$$

$$- \frac{87.2^{2n}k}{8} + \frac{2^{3n}k}{8} + \frac{45.2^n k^2}{2} - \frac{2^n k^3}{2} + 21.2^n kn - 3.2^n k^2 n + \frac{9.2^{2n}k^2 n}{2} - 1$$

$$Sz_{ev}(HT_n^k/E_i) = 2^{n-i-1}k(2k - 12.2^{2i} - 2^{i+1}k - 2^{n+3}k + 26.2^i$$

$$+ 6.2^{i+n}k14)$$

$$\sum_{i=1}^{n-2} Sz_{ev}(HT_n^k/E_i) = 2^n k(13n - kn + 19k - 2^{2-n}k - 7.2^{2-n} - 3.2^n$$

$$- 5.2^{n+1}k + 3.2^n k + 3.2^n kn - 21)$$

$$Sz_{ev}(HT_n^k/E_{n-1}) = \frac{k}{8}(8k - 20.2^{2n} + 2^{2n} - 40.2^n k + 76.2^n + 14.2^{2n}k - 80)$$

$$Sz_{ev}(HT_n^k/E_n) = \frac{k}{2}((2^n - 1)(3.2^{n-1} - 3)$$

$$+ (k + (2^n - 1)(k-1))(2k + (k-1)(3.2^{n-1} - 3) - 2)))$$

$$Sz_{ev}(HT_n^k/E_{n+1}) = 3.2^{2n-3}k^3 - 3.2^{2n-3}k^2 + 2^{n-1}k - 2^{n-2}k^2 - 2^{n-2}k^3$$

$$Mo(HT_n^k/E_i) = 2^{n-i}k((2^{n-i-1}k - 1)(2^{i+1} - 2) - 2^{i+1} + 2^{n-i}k + 2)$$

$$\sum_{i=1}^{n-2} Mo(HT_n^k/E_i) = k(12.2^n - 2^{n+2}k - 2^{n+2}n + 2^{2n}k - 16)$$

$$Mo(HT_n^k/E_{n-1}) = 2k(2k + (2k-2)(2^{n-1} - 1) - 2^{n-1} + 1)$$

$$Mo(HT_n^k/E_i) = k(k + (2^n - 1)(k-1) - 2^n + 1)$$

$$Mo(HT_n^k/E_{n+1}) = \sum_{j=1}^{k-1} | 2^n j + 2^n (j-k) |$$

$$Mo(HT_n^k) = 2^{2n} k^2 - 8k + 7.2^n k - 2^n k^2 - 4.2^n kn$$

$$+ \sum_{j=1}^{k-1} | 2^n j + 2^n (j-k) |$$

$$Mo_e(HT_n^k/E_i) = 2^{n-i} k(2^{n-i-1} k - 1)(3.2^i - 3) - k + 3.2^{n-i-1} k - 3.2^i + 4)$$

$$\sum_{i=1}^{n-2} Mo_e(HT_n^k/E_i) = \frac{k}{2}(8k - 14.2^n k - 12.2^n n + 38.2^n + 3.2^{2n} k - 56)$$

$$Mo_e(HT_n^k/E_{n-1}) = k(3.2^n k - 2k - 5.2^n + 12)$$

$$Mo_e(HT_n^k/E_n) = k(2k + (k-1)(3.2^{n-1} - 3) - 3.2^{n-1} + 1)$$

$$Mo_e(HT_n^k/E_{n+1}) = \sum_{j=1}^{k-1} \left| \frac{(3.2^n - 2)(2j-k)}{2} \right|$$

$$Mo_e(HT_n^k) = 3.2^{n-1} k^2 - 12k + 11.2^n k - 5.2^{n-1} k^2 - 6.2^n kn$$

$$+ \frac{1}{2} \sum_{j=1}^{k-1} | (3.2^n - 2)(2j-k) |$$

$$PI(HT_n^k/E_i) = 2^{n-i} k(3.2^{n-1} k - k - 3)$$

$$\sum_{i=1}^{n-2} PI(HT_n^k/E_i) = k(4 - 2^n)(k - 3.2^{n-1} k + 3)$$

$$PI(HT_n^k/E_{n-1}) = \frac{k}{2}(2^{2n} - 4k + 6.2^n k - 6.2^n + 4)$$

$$PI(HT_n^k/E_n) = k(2k + (k-1)(3.2^{n-1} - 3) + 3.2^{n-1} - 5)$$

$$PI(HT_n^k) = 11k + 3.2^{2n-1} k^2 - 15.2^{n-1} k + 2^{2n-1} k - 2^{2n-1} k - 2^n k^2 + 2$$

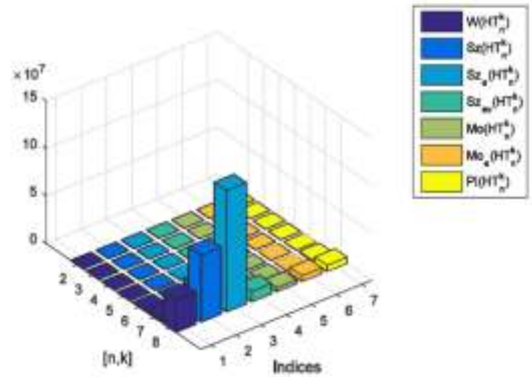

Figure 6. Comparative analysis of HT_n^k based on distance-based topological indices.

The comparative analysis of HT_n^k based on distance-based topological indices is shown in Figure 6.

Theorem 2. *If* $n \geq 2$ *and* $k \geq 3$, *then*

1. $M_2(HT_n^k) = 17 \times 2^n k - 41k - 15$

2. $R(HT_n^k) = 1.9571 \times 2^{n-2} k - 0.5974k - 0.1835$

3. $ABC(HT_n^k) = 3.9582 \times 2^{n-2} k - 0.6686k - 1.6475$

4. $H(HT_n^k) = 1.9166 \times 2^{n-2} k + 1.3047k - 1.4499$

5. $SC(HT_n^k) = 2.3775 \times 2^{n-2} k - 0.1565k - 0.3303$

6. $GA(HT_n^k) = 5.8856 \times 2^{n-2} k - 1.0407k - 0.0404$

Proof.

Table 3. Partition of edges.

Degree of the end vertices	Number of edges
$(\deg_G(a), \deg_G(b))$ where $ab \in E(G)$	
(2, 3)	$k + 2$
(3, 4)	$2k$

(4, 4)	$k(3.2^{n-2}-5)$
(4, 2)	$k2^{n-1}$
(2, 2)	$k2^{n-2}$
(3, 3)	$k-3$

The n-rooted hypertree has $2k-1+k(3.2^{n-1}-3)$ edges. Table 3 is referred to find the degree indices.

$$M_2(HT_n^k) = \sum_{e_{2,3}} 2\times 3 + \sum_{e_{3,4}} 3\times 4 + \sum_{e_{4,4}} 4\times 4 + \sum_{e_{4,2}} 4\times 2$$

$$+\sum_{e_{2,2}} 2\times 2 + \sum_{e_{3,3}} 3\times 3$$

$$= (k+2)6 + 2k\times 12 + k(3.2^{n-2}-5)\times 16 + (2^{n-1}k)\times 8 + 2^{n-2}k\times 4$$

$$+(k-3)\times 9$$

$$= 17\times 2^n k - 41k - 15$$

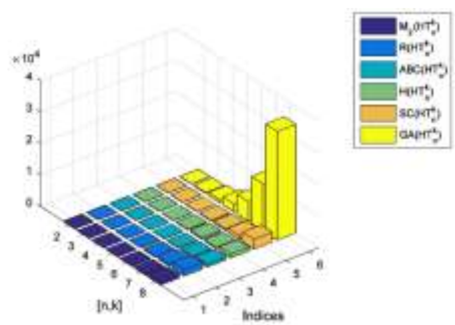

Figure 7. Comparative analysis of HT_n^k based on degree-based topological indices.

$$R(HT_n^k) = \sum_{e_{2,3}} \frac{1}{\sqrt{6}} + \sum_{e_{3,4}} \frac{1}{\sqrt{12}} + \sum_{e_{4,4}} \frac{1}{4} + \sum_{e_{4,2}} \frac{1}{\sqrt{8}}$$

$$+\sum_{e_{2,2}} \frac{1}{\sqrt{4}} + \sum_{e_{3,3}} \frac{1}{3}$$

$$= \frac{k+2}{\sqrt{6}} + \frac{2k}{2\sqrt{3}} + \frac{k(3.2^{n-2}-5)}{4} + \frac{2^{n-1}k}{2\sqrt{2}} + \frac{2^{n-2}k}{2} + \frac{k-3}{3}$$

$$= 1.9571 \times 2^{n-2}k - 0.5974k - 0.1835$$

$$ABC(HT_n^k) = \sum_{e_{2,3}} \sqrt{\frac{3}{6}} + \sum_{e_{3,4}} \sqrt{\frac{5}{12}} + \sum_{e_{4,4}} \sqrt{\frac{6}{16}}$$

$$+ \sum_{e_{4,2}} \sqrt{\frac{4}{8}} + \sum_{e_{2,2}} \sqrt{\frac{2}{4}} + \sum_{e_{3,3}} \sqrt{\frac{4}{9}}$$

$$= \frac{k+2}{\sqrt{2}} + 2k\sqrt{\frac{5}{12}} + k(3.2^{n-2}-5)\sqrt{\frac{3}{8}} + \frac{2^{n-1}k}{\sqrt{2}} + \frac{2^{n-2}k}{\sqrt{2}}$$

$$+ \frac{2(k-3)}{3} = 3.9582 \times 2^{n-2}k - 0.6686 - 1.6475$$

$$H(HT_n^k) = \sum_{e_{2,3}} \frac{2}{5} + \sum_{e_{3,4}} \frac{2}{7} + \sum_{e_{4,4}} \frac{2}{8} + \sum_{e_{4,2}} \frac{2}{6}$$

$$+ \sum_{e_{2,2}} \frac{2}{4} + \sum_{e_{3,3}} \frac{2}{6}$$

$$= 0.4(k+2) + 0.2857 \times 2k + 0.25k(3.2^{n-2}-5)$$

$$+ 0.3333 \times 2^{n-1}k + 0.5k2^{n-2} + 0.3333(k-3)$$

$$= 1.9166 \times 2^{n-2}k + 1.3047k - 1.4499$$

$$SC(HT_n^k) = \sum_{e_{2,3}} \frac{1}{\sqrt{5}} + \sum_{e_{3,4}} \frac{1}{\sqrt{7}} + \sum_{e_{4,4}} \frac{1}{\sqrt{8}}$$

$$+ \sum_{e_{4,2}} \frac{1}{6} + \sum_{e_{2,2}} \frac{1}{\sqrt{4}} + \sum_{e_{3,3}} \frac{1}{\sqrt{6}}$$

$$= \frac{k+2}{\sqrt{5}} + \frac{2k}{\sqrt{7}} + \frac{k(3.2^{n-2}-5)}{\sqrt{8}} + \frac{2^{n-1}k}{\sqrt{6}} + \frac{2^{n-2}k}{2} + \frac{k-3}{\sqrt{6}}$$

$$= 2.3775 \times 2^{n-2}k - 0.1565k - 0.3303$$

$$GA(HT_n^k) = \sum_{e_{2,3}} \frac{2\sqrt{6}}{5} + \sum_{e_{3,4}} \frac{2\sqrt{12}}{7} + \sum_{e_{4,4}} \frac{2\sqrt{16}}{8}$$

$$+ \sum_{e_{4,2}} \frac{2\sqrt{8}}{6} \sum_{e_{2,2}} \frac{2\sqrt{4}}{4} + \sum_{e_{3,3}} \frac{2\sqrt{9}}{6}$$

$$= (k+2)\frac{2\sqrt{6}}{5} + 2k\frac{2\sqrt{12}}{7} + k(3 \times 2^{n-2} - 5) + 2^{n-1}k\frac{2\sqrt{2}}{3} + 2^{n-2}k + k - 3$$

$$= 5.8856 \times 2^{n-2}k - 1.0407k - 0.0404$$

Figure 7 shows the comparative analysis of degree-based indices of HT_n^k.

5. Conclusion

In this paper, we have introduced an interconnection network and discussed its distance and degree-based indices. The topological indices characterise the topological properties of k-rooted hypertree. The k-rooted hypertree can be used in computer algorithms and implementation.

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