Advances and Applications in Mathematical Sciences
Volume 21, Issue 2, December 2021, Pages 993-999

A STUDY ON SHOCK WAVE

R. RAMYA, R. G. BALAMURUGAN, A. KIRUTHIKA and A. SARANYA

Department of Mathematics
Dhanalakshmi Srinivasan College of Arts
and Science For Women (Autonomous)
Perambalur, India
E-mail: ramya25071987@gmail.com

Abstract

This article introduces the reader to the exhilarating world of shock waves. with their unique ability to enhance the pressure and temperature instantaneously in any medium of propagation, shock waves are now being used for many innovative applications in the industry. The origin of shock wave, their characteristics, laboratory tools used in their study along with a few interesting industrial applications of shock waves developed at the Indian institute of science are described.

Introduction

In shock wave is a type of propagating disturbance that moves faster than the local speed of sound in the medium. Like an ordinary wave a shock wave carries energy and can propagates through a medium but is characterized by an abrupt nearly discontinuous change in pressure, temperature and density of the medium for the purpose of comparison flows also known as a Prandtl – Meyer expansion fan.

Definition 1.1. A shock wave is said to be a normal shock, the plane of which is perpendicular to the direction of flow.

Definition 1.2. A shock wave is said to be an oblique shock the plane of which is inclimed at an angle to the direction of flow.

2010 Mathematics Subject Classification: 76L05.

Keywords: Shock wave, oblique shock, normal shock.

Received November 2, 2021; Accepted November 15, 2021

Definition 1.3. The strength of a shock wave P is defined as the ratio of rise in pressure to upstream pressure.

$$P = \frac{(p_2 - p_1)}{p_1}.$$

Shock Waves:

Let us consider a subsonic and supersonic flow past a body as shown in Figure, in both the cases, the body acts as an obstruction to the flow and thus there is a change in energy and momentum of the flow.

The changes in flow properties are communicated through pressure waves moving at speed of sound everywhere in the flow field.

(i.e. both upstream and downstream).

As shown in figure, if the incoming stream is subsonic

i.e. $M_\infty < 1; V_\infty < a_\infty,$

The sound waves propagate faster than the flow speed and warn the medium about the presence of the body. So, the streamlines approaching the body begin to adjust themselves far upstream and the flow properties change the pattern Gradually in the vicinity of the body. In contrast, when the flow is supersonic, figure

i.e. $M_\infty > 1; V_\infty > a_\infty,$

The sound waves overtake the speed of the body and these weak pressure waves merge themselves ahead of the body leading to compression in the vicinity of the body. In other words, the flow medium gets compressed at a very short distance ahead of the body in a very thin region that may be comparable to the mean free path of the molecules in the medium. Since, these compression waves propagate upstream, so they tend to Merge as shock wave. Ahead of the shock wave, the flow has no idea of presence of the body and immediately behind the shock; the flow is subsonic as shown in figure The thermodynamic definition of a shock wave may be written as "the instantaneous compression of the gas".

The energy for compressing the medium, through a shock wave is obtained from the kinetic energy of the flow upstream the shock wave.

The reduction in kinetic energy is accounted as heating of the gas to a static temperature above that corresponding to the isentropic compression value. Consequently, in flowing through the shock wave, the gas experiences a decrease in its available energy and accordingly, an increase in entropy.

So, the compression through a shock wave is considered as an irreversible process.

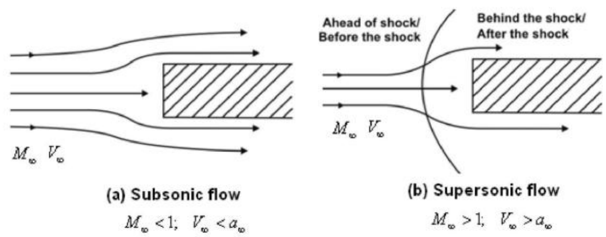


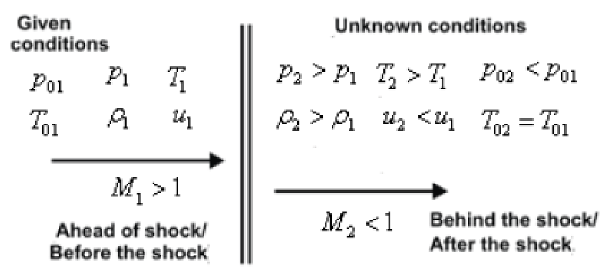
Figure Shock wave phenomena.

Normal Shock Waves

A normal shock wave is one of the situations where the flow properties change drastically in one direction. The shock wave stands perpendicular to the flow as shown in Figure The quantitative analysis of the changes across a normal shock wave involves the determination of flow properties.

All conditions of are known ahead of the shock and the unknown flow properties are to be determined after the shock. There is no heat added or taken away as the flow traverses across the normal shock.

Hence, the flow across the shock wave is adiabatic $(q = 0)$



The basic one dimensional compressible flow equations can be written as below;

$$\rho_1 u_1 = \rho_2 u_2; p_1 + \rho_2 u_1^2 = \rho_2 + \rho_2 u_2^2; h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2}$$

For a calorically perfect gas, thermodynamic relations can be used,

$$p = \rho RT; h = c_p T; a = \sqrt{\gamma p/\rho}$$

The continuity and momentum equations can be simplified to obtain,

$$\frac{a_1^2}{\gamma u_1} - \frac{a_2^2}{\gamma u_2} = u_2 - u_1$$

Since, $a^* = \sqrt{\gamma RT^*}$ and $M^* \frac{v}{a^*}$, the energy equation is written as,

$$\frac{a^2}{\gamma - 1} + \frac{u^2}{2} = \frac{\gamma + 1}{2(\gamma - 1)} a^{*2} \Rightarrow a^2 = \frac{\gamma + 1}{2} a^{*2} - \frac{\gamma - 1}{2} u^2$$

Both a_1^2 and a_2^2 can now be expressed as,

$$a_1^2 = \frac{\gamma + 1}{2} (a^*)^2 - \frac{\gamma - 1}{2} u_1^2; a_2^2 = \frac{\gamma + 1}{2} (a^*)^2 - \frac{\gamma - 1}{2} u_2^2$$

Substitute the above two equations and solve for a^{*^2}

$$a^{*^2} = u_1 u_2 \Rightarrow M_2^* = \frac{1}{M_2^*}$$

Recall the relation for M and M^* and substitute in above equation,

$$M^{*2} = \frac{(\gamma + 1) M^2}{2 + (\gamma + 1) M^2}$$

Substitute the above two equations, and solve for M_2

$$M_2^2 = \frac{1 + \left(\frac{\gamma - 1}{2}\right) M_1^2}{\gamma M_1^2 - \left(\frac{\gamma - 1}{2}\right)}$$

Using continuity equation and Prandtl relation, we can write,

$$\frac{\rho_2}{\rho_1} = \frac{u_1}{u_2} = \frac{u_1^2}{u_1 u_2} = \frac{u_1^2}{a^{*2}} = (M_1^*)^2$$

Substitute equation and solve for density and velocity ratio across the normal shock.

$$\frac{\rho_2}{\rho_1} = \frac{u_1}{u_2} = \frac{(\gamma+1)M_1^2}{2+(\gamma-1)M_1^2}$$

The pressure ratio can be obtained by the combination of momentum and continuity equations i.e.

$$p_2 - p_1 = \rho_1 u_1 (u_1 - u_2) = \rho_1 u_1^2 \left(1 - \frac{u_2}{u_1}\right); \Rightarrow \frac{p_2 - p_1}{p_1} = \gamma M_1^2 \left(1 - \frac{u_2}{u_1}\right)$$

Substituting the ratio $\left(\frac{u_1}{u_2}\right)$ from equations and simplifying for the pressure ratio across the normal shock, we get,

$$\frac{p_2}{p_1} = 1 + \frac{2\gamma}{\gamma+1}(M_1^2 - 1)$$

For a calorically perfect gas, equation of state relation can be used to obtain the temperature ratio across the normal shock

$$\frac{h_2}{h_1} = \frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)\left(\frac{\rho_1}{\rho_2}\right) = \left[1 + \frac{2\gamma}{\gamma+1}(M_1^2 - 1)\right]\left[\frac{2+(\gamma-1)M_1^2}{(\gamma+1)M_1^2}\right]$$

Thus, the upstream Mach number is the powerful tool to dictating the shock wave properties.

The "stagnation properties" across the normal shock can be computed as follows;

$$\frac{p_{02}}{p_{01}} = \frac{(p_{02}/p_2)}{(p_{01}/p_1)}\left(\frac{p_2}{p_1}\right)$$

Here, the ratios $\left(\frac{p_{01}}{p_1}\right)$ and $\left(\frac{p_{02}}{p_2}\right)$ can be obtained from the isentropic relation for the regions '1 and 2' respectively.

Knowing the upstream Mach number M_1, gives the downstream Mach number M_2.

Further, equation can be used to obtain the static pressure ratio $\left(\frac{p_2}{p_1}\right)$.

After substitution of these ratios reduces to,

$$\frac{p_{02}}{p_{01}} = \frac{\left(1 + \frac{\gamma - 1}{2} M_2^2\right)^{\frac{\gamma}{\gamma-1}}}{\left(1 + \frac{\gamma - 1}{2} M_1^2\right)^{\frac{\gamma}{\gamma-1}}} \left[1 + \frac{2\gamma}{\gamma + 1}\left(M_1^2 - 1\right)\right]$$

Many a times, another significant pressure ratio is important for a normal shock which is normally called as Rayleigh Piton Tube relation.

$$\frac{P_{02}}{P_1} = \left(\frac{P_{02}}{P_2}\right)\left(\frac{P_2}{p_1}\right) \Rightarrow \frac{P_{02}}{P_1} = \left(1 + \frac{\gamma - 1}{2} M_2^2\right)^{\frac{\gamma}{\gamma-1}}\left[1 + \frac{2\gamma}{\gamma + 1}(M_1^2 - 1\right]$$

Recall the energy equation for a calorically perfect gas:

$$C_P T_1 + \frac{u_1^2}{2} = c_p T_2 + \frac{u_2^2}{2} \Rightarrow c_p T_{01} = c_p T_{01}.$$

Thus, the stagnation temperatures do not change across a normal shock.

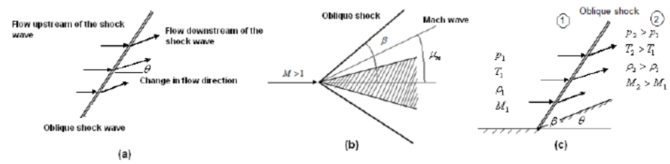
Oblique Shock Wave The normal shock waves are straight in which the flow before and after the wave is normal to the shock. It is considered as a special case in the general family of oblique shock waves that occur in supersonic flow. In general, oblique shock waves are straight but inclined at an angle to the upstream flow and produce a change in flow direction as shown in Figure (a). An infinitely weak oblique shock may be defined as a Mach wave (Figure b). By definition, an oblique shock generally occurs, when a supersonic flow is 'turned into itself' as shown in Figure (c).

Here, a supersonic flow is allowed to pass over a surface, which is inclined at an angle to the horizontal.

The flow streamlines are deflected upwards and aligned along the surface. Since, the upstream flow is supersonic;

The streamlines are adjusted in the downstream an oblique shock wave angle with the horizontal such that they are parallel to the surface in the downstream.

All the streamlines experience same deflection angle across the oblique shock.



Conclusion

The use of more versatile system, such as modified electromagnetic shock wave generators and piezoelectric shock wave systems composed of various individually driven piezoelectric segments, as well as multichannel electrical discharges source will help improve the existing clinical treatments and develop novel biomedical applications for shock waves. Progress in understanding of the detailed phenomena involved in shock wave interactions with living tissue and cells will certainly be followed by an increasing number of biomedical uses and by safer and more efficient clinical shock therapies.

References

[1] John D. Anderson Jr., Fundamentals of Aerodynamics, (3 rd ed.) (2010).

[2] L. D. landau, and E. M. lifshitz, fluid mechanics, volume 6 of course of theoretical physics. course of theoretical physical /by LD landau and EM lifshitz 6 (1987).

[3] A.H. Shapiro, The dynamics and thermodynamics of compressible Fluid flow, 1(voi.454). Ronald press, New York, (1953).

[4] R. courant, and K.O. Fredric's, Supersonic flow and shock waves (21) Springer science and business media, (1999).

[5] H. W. liepman, and A. Roshko, Elements of gas dynamics, John willey and sons (1957).