Advances and Applications in Mathematical Sciences
Volume 21, Issue 4, February 2022, Pages 2243-2253

RECIPROCAL GD-DISTANCE OF PRODUCT GRAPHS

K. THANGA KANNIGA, V. MAHESWARI and K. PALANI

[1]Research Scholar, Reg. No. 19122012092012
[2]Assistant Professor, [3]Associate Professor
PG and Research, Department of Mathematics
A.P.C. Mahalaxmi College for Women
Thoothukudi-628002, Tamilnadu, India
(Affiliated to Manonmaniam Sundaranar University
Tirunelveli-627012, Tamilnadu, India)
E-mail: kannigaprasathkpk@gmail.com
mahiraj2005@gmail.com
palani@apcmcollege.ac.in

Abstract

In the mathematical field of graph theory, the distance between two vertices in a graph is the number of edges in a shortest path connecting them. This is also known as the geodesic distance. The reciprocal Gd-distance $(Rd^{Gd}(G))$ of a graph G is defined as, $Rd^{Gd}(G)\sum_{\{x, y\} \subseteq V(G)}[\deg x + \deg y + \frac{1}{d_G}]$. In this paper, we determine the exact value of the reciprocal Gd-distance of the tensor product and the strong product of two graphs.

1. Introduction

Throughout the article we consider nontrivial, finite, simple and connected graphs. Given an undirected graph $G = (V(G), E(G))$, with vertex set $V(G)$ and edge set $E(G)$. The first Zagreb index is defined as $M_1(G) = \sum_{uv \in E(G)}[d_G(u) + d_G(v)]$. The first Zagreb coindex is defined as $\overline{M_1}(G) = \sum_{xy \notin E(G)}[d_G(x) + d_G(y)]$.

2020 Mathematics Subject Classification: 05C12, 05C76.
Keywords: Gd-distance, Reciprocal Gd-distance, Harary index, Zagreb index, Zagreb coindex, Tensor product, Strong product.
Received November 7, 2021; Accepted December 21, 2021

Chartrand et al. introduced the concept of detour distance, $D(u, v)$, which is the length of the longest path between u and v. The Dd-distance was introduced by A. Anto Kinsley and P. Siva Ananthi. If u, v are vertices of a connected graph G Dd-length of a $u - v$ path is defined as $D^{Dd}(u, v) = D(u, v) + \deg(u) + \deg(v)$.

V. Maheswari et al. introduced the concept of Gd-distance by considering the length of the shortest path between u and v in addition to degree of end vertices. Set u, v be the vertices of a graph G, then the Gd-length of a $u - v$ path is defined as $d^{Gd}(u, v) = d(u, v) + \deg(u) + \deg(v)$. The Gd-distance of a connected graph G is defined as

$$d^{Gd}(G) = \sum_{\{x, y\} \subseteq V(G)} [d(x, y) + \deg x + \deg y].$$

The reciprocal Gd-distance for a connected graph G is defined as $Rd^{Gd}(G) = \sum_{\{x, y\} \subseteq V(G)} [\deg x + \deg y + \frac{1}{d_G(x, y)}]$. The Harary index of a graph G, denoted by $H(G)$, is defined as the sum of reciprocals of distances between all pairs of vertices of a connected graph. That is, $H(G) = \sum_{u, v \in V(G)} \frac{1}{d_G(u, v)}$. The reciprocal Gd-distance for a connected graph G can also be defined as $Rd^{Gd}(G) = H(G) + (n - 1) \sum_{y \in V(G)} \deg y$.

Graph products became an interesting area of research, and different types of products have been worked out in graph theory. The tensor product was introduced by Alfred North Whitehead and Bertrand Russell in their Principia Mathematica (1912). The tensor product $G \times H$ of graphs G and H is a graph such that the vertex set $G \times H$ is the Cartesian product of $V(G) \times V(H)$ and vertices (u, v) and (x, y) are adjacent in $G \times H$ if and only if u is adjacent to x and v is adjacent to y.

The strong product $G \boxtimes H$ of graphs G and H is a graph such that the vertex set of $G \boxtimes H$ is the Cartesian product $V(G) \times V(H)$; and distinct vertices (u, v) and (x, y) are adjacent in $G \boxtimes H$ if and only if:

(i) $u = x$ and v is adjacent to y, or

(ii) $v = y$ and u is adjacent to x, or

(iii) u is adjacent to x and v is adjacent to y.

In this paper, the exact formula for the reciprocal Gd-distance of the tensor product and the strong product of two graphs are obtained. The basic tools in our analysis are the following results.

Lemma 1.1 [6]. *Let G be a connected graph on $n \geq 2$ vertices. For any pair of vertices $x_{ij}, x_{kp} \in V(G \times K_r)$, $r \geq 3$.*

(i) *If $v_i v_k \in E(G)$, then*

$$d_{G\times K_r}(x_{ij}, x_{kp}) = \begin{cases} 1, & \text{if } j \neq p \\ 2, & \text{if } j = p \text{ and } v_i v_k \text{ is on a triangle of } G \\ 3, & \text{if } j = p \text{ and } v_i v_k \text{ is on a triangle of } G \end{cases}$$

(ii) *If $v_i v_k \notin E(G)$, then $d_{G\times K_r}(x_{ij}, x_{kp}) = d_G(v_i, v_k)$*

(iii) $d_{G\times K_r}(x_{ij}, x_{ip}) = 2.$

Lemma 1.2 [6]. *Let G be a nontrivial connected graph and $K = K_{r_0, r_1, \ldots, r_{n-1}}$. Let $H = G \times K$ and let $(u_i, v_j) \in V(H)$ and let $v_j \in V_j$. Then the degree of (u_i, v_j) is $d_H(u_i, v_j) = d_G(u_i) d_K(v_j) = d_G(u_i)(r - r_j)$.*

2. Main Results

2.1. Reciprocal Gd-distance of tensor product of graphs

In this section, we compute the reciprocal Gd-distance of $G \times K_r$.

Theorem 2.1.1. *Let G be a connected graph with $n \geq 2$ vertices and m edges. Then*

$$R^{Gd}(G \times K_r) = \frac{1}{2} r\{(r-1)[4m(r-1) + \frac{n}{2} + 2\overline{M_1}(G) + 2M_1(G)$$

$$+ 4(n-1)(r-1)\varepsilon(G) + 2H(G)] + \sum_{\substack{i,k=0 \\ i \neq k \\ u_i u_k \notin E(G)}}^{n-1} \frac{1}{d_G(u_i, u_k)}$$

$$+ \sum_{\substack{i,k=0 \\ i \neq k \\ u_i u_k \in E_1}}^{n-1} \frac{1}{2} + \sum_{\substack{i,k=0 \\ i \neq k \\ u_i u_k \in E_2}}^{n-1} \frac{1}{3}\},$$

where $r \geq 3$ *and* $\varepsilon(G) = |E(G)|$.

Proof. Set $V(G) = \{u_1, u_2, \ldots u_n\}$ and $V(K_r) = \{v_1, v_2, \ldots, v_r\}$.

Let x_{ij} denote the vertex (u_i, v_j) of $G \times K_r$.

The degree of the vertex x_{ij} in $G \times K_r$ is $d_G(u_i)d_{K_r}(v_j)$, that is $d_{G \times K_r}(x_{ij}) = (r-1)d_G(u_i)$. By the definition of reciprocal Gd-distance

$R^{Gd}(G \times K_r)$

$$= \frac{1}{2} \sum_{x_{ij}, x_{kp} \in V(G \times K_r)} [d_{G \times K_r}(x_{ij}) + d_{G \times K_r}(x_{kp}) + \frac{1}{d_{G \times K_r}(x_{ij}, x_{kp})}]$$

$$= \frac{1}{2} \{\sum_{i=0}^{n-1} \sum_{\substack{j,p=0 \\ j \neq p}}^{r-1} [d_{G \times K_r}(x_{ij}) + d_{G \times K_r}(x_{ip}) + \frac{1}{d_{G \times K_r}(x_{ij}, x_{ip})}$$

$$+ \sum_{\substack{i,k=0 \\ i \neq k}}^{n-1} \sum_{j=0}^{r-1} [d_{G \times K_r}(x_{ij}) + d_{G \times K_r}(x_{kj}) + \frac{1}{d_{G \times K_r}(x_{ij}, x_{kj})}$$

$$+ \sum_{\substack{i,k=0 \\ i \neq k}}^{n-1} \sum_{j=0}^{r-1} [d_{G \times K_r}(x_{ij}) + d_{G \times K_r}(x_{kp}) + \frac{1}{d_{G \times K_r}(x_{ij}, x_{kp})}]\}$$

$$= \frac{1}{2}(A_1 + A_2 + A_3), \ldots (1)$$

where A_1 to A_3 are the sums of the above terms, in order. We shall calculate A_1 to A_3 of (1) separately. First we compute A_1

$$A_1 = \sum_{i=0}^{n-1} \sum_{\substack{j,p=0 \\ j \neq p}}^{r-1} [d_{G \times K_r}(x_{ij}) + d_{G \times K_r}(x_{ip}) + \frac{1}{d_{G \times K_r}(x_{ij}, x_{ip})}]$$

For this initially we compute

$$A_1' = \sum_{\substack{j,\, p=0 \\ j \neq p}}^{r-1} [d_{G \times K_r}(x_{ij}) + d_{G \times K_r}(x_{ip}) + \frac{1}{d_{G \times K_r}(x_{ij},\, x_{ip})}]$$

$$= \sum_{\substack{j,\, p=0 \\ j \neq p}}^{r-1} [d_G(u_i)(r-1) + d_G(u_i)(r-1) + \frac{1}{2}] \text{ by lemmas 1.1 and 1.2.}$$

$$= 2(r-1)d_G(u_i)2\binom{r}{2} + \frac{1}{2}2\binom{r}{2}$$

$$= 2r(r-1)^2 d_G(u_i) + \frac{r(r-1)}{2}$$

Now $A_1 = \sum_{i=0}^{n-1}[2r(r-1)^2 d_G(u_i) + \frac{r(r-1)}{2}]$

$$= 4mr(r-1)^2 + \frac{nr(r-1)}{2} \qquad (2)$$

Next we compute A_2

$$A_2 = \sum_{i,\, k=0}^{n-1} \sum_{j=0}^{r-1} [d_{G \times K_r}(x_{ij}) + d_{G \times K_r}(x_{kj}) + \frac{1}{d_{G \times K_r}(x_{ij},\, x_{kj})}]$$

For this initially we compute

$$A_2' = \sum_{\substack{i,\, k=0 \\ i \neq k}}^{n-1} [d_{G \times K_r}(x_{ij}) + d_{G \times K_r}(x_{kj}) + \frac{1}{d_{G \times K_r}(x_{ij},\, x_{kj})}]$$

Let $E_1 = \{uv \in E(G) \mid uv$ is on a C_3 in $G\}$ and $E_2 = E(G) - E_1$

Now $A_2' \sum_{i,\, k=0}^{n-1} [d_{G \times K_r}(x_{ij}) + d_{G \times K_r}(x_{kj}) + \frac{1}{d_{G \times K_r}(x_{ij},\, x_{kj})}]$

$$= \sum_{\substack{i,\, k=0 \\ i \neq k \\ u_i u_k \notin E(G)}}^{n-1} [d_G(u_i)(r-1) + d_G(u_K)(r-1) + \frac{1}{d_G(u_i,\, u_k)}]$$

$$+ \sum_{\substack{i,\, k=0 \\ i \neq k \\ u_i u_k \in E_1}}^{n-1} [d_G(u_i)(r-1) + d_G(u_K)(r-1) + \frac{1}{2}]$$

$$+\sum_{\substack{i,k=0\\ i\neq k\\ u_iu_k\in E_2}}^{n-1}[d_G(u_i)(r-1)+d_G(u_K)(r-1)+\frac{1}{3}],$$ by lemmas 1.1 and 1.2.

$$=(r-1)\sum_{\substack{i,k=0\\ i\neq k\\ u_iu_k\notin E(G)}}^{n-1}[d_G(u_i)+d_G(u_k)]+2(r-1)M_1(G)$$

$$+\sum_{\substack{i,k=0\\ i\neq k\\ u_iu_k\notin E(G)}}^{n-1}\frac{1}{d_G(u_i,\, u_k)}+\sum_{\substack{i,k=0\\ i\neq k\\ u_iu_k\in E_1}}^{n-1}\frac{1}{2}+\sum_{\substack{i,k=0\\ i\neq k\\ u_iu_k\in E_2}}^{n-1}\frac{1}{3}$$

Now,

$$A_2=r(r-1)\sum_{\substack{i,k=0\\ i\neq k\\ u_iu_k\notin E(G)}}^{n-1}[d_G(u_i)+d_G(u_k)]+2r(r-1)M_1(G)$$

$$+r\sum_{\substack{i,k=0\\ i\neq k\\ u_iu_k\notin E(G)}}^{n-1}\frac{1}{d_G(u_i,u_k)}+r\sum_{\substack{i,k=0\\ i\neq k\\ u_iu_k\in E_1}}^{n-1}\frac{1}{2}+r\sum_{\substack{i,k=0\\ i\neq k\\ u_iu_j\in E_{2n}}}^{n-1}\frac{1}{3}\ldots(3)$$

Now we compute A_3

$$A_3\sum_{\substack{i,k=0\\ i\neq k}}^{n-1}\sum_{\substack{j,p=0\\ j\neq k}}^{r-1}[d_{G\times K_r}(x_{ij})+d_{G\times K_r}(x_{kp})+\frac{1}{d_{G\times K_r}(x_{ij},x_{kp})}]$$

For this initially we compute

$$A_3'\sum_{\substack{i,k=0\\ i\neq 0}}^{n-1}[d_{G\times K_r}(x_{ij})+d_{G\times K_r}(x_{kp})+\frac{1}{d_GK_r(x_{ij},x_{kp})}]$$

$$\sum_{\substack{i,k=0\\ i\neq k}}^{n-1}[d_G(u_i)(r-1)+d_G(u_k)(r-1)+\frac{1}{d_G(u_i,u_k)}],$$ by lemmas 1.1 and 1.2.

$$=(r-1)\sum_{\substack{i,k=0\\ i\neq k}}^{n-1}[d_G(u_i)+d_G(u_k)]+\sum_{\substack{i,k=0\\ i\neq k}}^{n-1}\frac{1}{d_G(u_i,u_k)}$$

$$=4(n-1)(r-1)\varepsilon(G)+2H(G)$$

Now

$$A_3 = \sum_{\substack{j, p=0 \\ j \neq p}}^{n-1} [4(n-1)(r-1)\varepsilon(G) + 2H(G)]$$

$$= 4r(r-1)^2(n-1)\varepsilon(G) + 2r(r-1)H(G) \quad (4)$$

Using (2), (3) and (4) in (1) we have

$$= \frac{1}{2}[A_1 + A_2 + A_3]$$

$$= \frac{1}{2}\{4mr(r-1)^2 + \frac{nr(r-1)}{2}G + r(r-1)\sum_{\substack{i, k=0 \\ i \neq k \\ u_i u_k \notin E(G)}}^{n-1} [d_G(u_i) + d_G(u_k)]$$

$$+ 2r(r-1)M_1(G) + r\sum_{\substack{i, k=0 \\ i \neq k \\ u_i u_k \notin E(G)}}^{n-1} \frac{1}{d_G(u_i, u_k)} + r\sum_{\substack{i, k=0 \\ i \neq k \\ u_i u_k \in E_1}}^{n-1} \frac{1}{2} + r\sum_{\substack{i, k=0 \\ i \neq k \\ u_i u_k \in E_2}}^{n-1} \frac{1}{3}$$

$$+ 4r(r-1)^2(n-1)\varepsilon(G) + 2r(r-1)H(G)\}$$

$$= \frac{1}{2}r\{(r-1)[4m(r-1) + \frac{n}{2} + 2\overline{M_1}(G) + 2M_1(G) + 4(n-1)(r-1)\varepsilon(G) + 2H(G)]$$

$$+ \sum_{\substack{i, k=0 \\ i \neq k \\ u_i u_k \notin E(G)}}^{n-1} \frac{1}{d_G(u_i, u_k)} + \sum_{\substack{i, k=0 \\ i \neq k \\ u_i u_k \in E_1}}^{n-1} \frac{1}{2} + \sum_{\substack{i, k=0 \\ i \neq k \\ u_i u_k \in E_2}}^{n-1} \frac{1}{3}\}.$$

Thus

$$R^{Gd}(G \times K_r) = \frac{1}{2}r\{(r-1) + \frac{n}{2} + 2\overline{M_1}(G) + 2M_1(G) + 4(n-1)(r-1)\varepsilon(G)$$

$$+ 2H(G)] + \sum_{\substack{i, k=0 \\ i \neq k \\ u_i u_k \notin E(G)}}^{n-1} \frac{1}{d_G(u_i, u_k)} + \sum_{\substack{i, k=0 \\ i \neq k \\ u_i u_k \in E_1}}^{n-1} \frac{1}{2} + \sum_{\substack{i, k=0 \\ i \neq k \\ u_i u_k \in E_2}}^{n-1} \frac{1}{3}\}, \quad \text{where}$$

$r \geq 3$.

2.2. Reciprocal Gd-distance of Strong Product of Graphs. In this section we compute the reciprocal Gd-distance of the strong product $G_1 \boxtimes G_2$ of the graphs G_1 and G_2.

Theorem 2.2.1. *Let G be a connected graph with n vertices and m edges.*

Then $Rd^{Gd}(G \boxtimes K_r) = r[2mnr^2 - 2mr + n^2r^2 - \frac{1}{2}nr + \frac{1}{2}n - n^2r + rH(G)]$.

Proof. Let $V(G) = \{u_1, u_2, \ldots, u_n\}$ and $V(K_r) = \{v_1, v_2, \ldots, v_r\}$. The degree of the vertex x_{ij} in $G \boxtimes K_r$ is $d_G(u_i) + d_{K_r}(v_j) + d_G(u_i)d_{K_r}(v_j)$, that is $d_{G \boxtimes K_r}(x_{ij}) = rd_G(u_i) + (r-1)$.

For any pair of vertices $x_{ij}, x_{kp} \in V(G \boxtimes K_r)$, $d_{G \boxtimes K_r}(x_{ij}, x_{ip}) = 1$ and $d_{G \boxtimes K_r}(x_{ij}, x_{kp}) = d_G(u_i, u_k)$.

By the definition of reciprocal Gd-distance

$$Rd^{Gd}(G \boxtimes K_r)$$

$$= \frac{1}{2}\sum_{x_{ij}, x_{kp} \in V(G \boxtimes K_r)} [d_{G \boxtimes K_r}(x_{ij}) + d_{G \boxtimes K_r}(x_{kp}) + \frac{1}{d_{G \boxtimes K_r}(x_{ij}, x_{kp})}]$$

$$= \frac{1}{2}\sum_{i=0}^{n-1}\sum_{\substack{j, p=0 \\ j \neq p}}^{n-1} [d_{G \boxtimes K_r}(x_{ij}) + d_{G \boxtimes K_r}(x_{ip}) + \frac{1}{d_{G \boxtimes K_r}(x_{ij}, x_{kp})}]$$

$$+ \sum_{\substack{i, k=0 \\ i \neq k}}^{n-1}\sum_{j=0}^{n-1} [d_{G \boxtimes K_r}(x_{ij}) + d_{G \boxtimes K_r}(x_{kj}) + \frac{1}{d_{G \boxtimes K_r}(x_{ij}, x_{kj})}]$$

$$+ \sum_{\substack{i, k=0 \\ i \neq k}}^{n-1}\sum_{\substack{j, p=0 \\ j \neq p}}^{n-1} [d_{G \boxtimes K_r}(x_{ij}) + d_{G \boxtimes K_r}(x_{kp}) + \frac{1}{d_{G \boxtimes K_r}(x_{ij}, x_{kp})}]$$

$$= \frac{1}{2}(A_1 + A_2 + A_3) \tag{5}$$

where $A_1 + A_2 + A_3$ are the sums of the above expression, in order.

First we calculate

$$A_1 = \sum_{i=0}^{n-1}\sum_{\substack{j, p=0 \\ j \neq p}}^{r-1} [d_{G \boxtimes K_r}(x_{ij}) + d_{G \boxtimes K_r}(x_{ip}) + \frac{1}{d_{G \boxtimes K_r}(x_{ij}, x_{ip})}]$$

For this initially we compute

$$A_1' = \sum_{\substack{j, p=0 \\ j \neq p}}^{r-1} [d_{G \boxtimes K_r}(x_{ij}) + d_{G \boxtimes K_r}(x_{ip}) + \frac{1}{d_{G \boxtimes K_r}(x_{ij}, x_{ip})}]$$

$$= \sum_{\substack{j, p=0 \\ j \neq p}}^{r-1} [2rd_G(u_i) + 2(r-1) + 1]$$

$$= 2r^2(r-1)d_G(u_i) + 2r(r-1)^2 + r(r-1)$$

$$A_1 = \sum_{i=0}^{n-1} [2r^2(r-1)d_G(u_i) + 2r(r-1)^2 + r(r-1)]$$

$$= 4mr^2(r-1) + 2nr(r-1)^2 + nr(r-1)$$

$$= r(r-1)[4mr + 2nr - n] \tag{6}$$

Now

$$A_2 = \sum_{\substack{i, k=0 \\ i \neq k}}^{n-1} \sum_{j=0}^{r-1} [d_{G \boxtimes K_r}(x_{ij}) + d_{G \boxtimes K_r}(x_{kj}) + \frac{1}{d_{G \boxtimes K_r}(x_{ij}, x_{kj})}]$$

First we compute

$$A_2' = \sum_{\substack{i, k=0 \\ i \neq k}}^{n-1} [d_{G \boxtimes K_r}(x_{ij}) + d_{G \boxtimes K_r}(x_{kj}) + \frac{1}{d_{G \boxtimes K_r}(x_{ij}, x_{kj})}]$$

$$= \sum_{\substack{i, k=0 \\ i \neq k}}^{n-1} [rd_G(u_i) + (r-1) + rd_G(u_k) + \frac{1}{d_G(u_i, u_k)}]$$

$$= 2mr(n-1) + 2mr(n-1) + 2n(n-1)(r-1) + 2H(G)$$

$$= 4mr(n-1) + 2n(n-1)(r-1) + 2H(G)$$

Now

$$A_2 = \sum_{j=0}^{n-1} [4mr(n-1) + 2n(n-1)(r-1) + 2H(G)]$$

$$= 4mr^2(n-1) + 2nr(n-1)(r-1) + 2rH(G)$$

$$= 2r[2mr(n-1) + n(n-1)(r-1) + H(G)] \tag{7}$$

Eventually we calculate

$$A_3 = \sum_{\substack{i, k=0 \\ i \neq p}}^{n-1} \sum_{\substack{j, p=0 \\ j \neq p}}^{n-1} [d_{G \boxtimes K_r}(x_{ij}) + d_{G \boxtimes K_r}(x_{kp}) + \frac{1}{d_{G \boxtimes K_r}(x_{ij}, x_{kp})}]$$

First we compute

$$A_3' = \sum_{\substack{j,\, p=0 \\ j \neq p}}^{r-1} [d_{G \boxtimes K_r}(x_{ij}) + d_{G \boxtimes K_r}(x_{kp}) + \frac{1}{d_{G \boxtimes K_r}(x_{ij}, x_{kp})}]$$

$$= \sum_{j,\, p=0}^{r-1} [rd_G(u_i) + (r-1) + rd_G(u_k) + (r-1) + \frac{1}{d_G(u_i, u_k)}]$$

$$= r^2(r-1)d_G(u_j) + r(r-1)^2 + r^2(r-1)d_G(u_k) + r(r-1)^2$$

$$+ r(r-1)\frac{1}{d_G(u_i, u_k)}$$

Now $A_3 = \sum_{\substack{i,\, k=0 \\ i \neq}}^{r-1} [r^2(r-1)d_G(u_i) + r^2(r-1)d_G(u_k) + 2r(r-1)^2$

$$+ r(r-1)\frac{1}{d_G(u_i,\, u_k)}]$$

$$= 2mr^2(r-1)(n-1) + 2mr^2(r-1)(n-1) + 2r(r-1)^2 n(n-1)$$

$$+ 2r(r-1)H(G)$$

$$= 2r(r-1)[2mr(n-1) + n(n-1)(r-1) + H(G)] \tag{8}$$

(8) Using (6), (7) and (8) in (5), we have

$$A = \frac{1}{2}(A_1 + A_2 + A_3)$$

$$= \frac{1}{2}\{r(r-1)[4mr + 2nr - n]$$

$$+ 2r[2mr(n-1) + n(n-1)(n-1)(r-1) + H(G)]$$

$$+ 2r(r-1)[2mr(n-1) + n(n-1)(r-1) + H(G)]\}$$

$$= \frac{1}{2}[-nr^2 - 4mr^2 + nr + 4mnr^3 + 2n^2r^3 - 2n^2r^2 + 2r^2H(G)]$$

$$= r[2mnr^2 - 2mr + n^2r^2 - \frac{1}{2}nr + \frac{1}{2}n - n^2r + rH(G)]$$

Thus

$$Rd^{Gd}(G \boxtimes K_r) = r[2mnr^2 - 2mr + n^2r^2 - \frac{1}{2}nr + \frac{1}{2}n - n^2r + rH(G)]$$

References

[1] A. Anto Kinsley and P. Siva Ananthi, Dd-distance in graphs, Imperial Journal of Interdisciplinary Research (IJIR) (2017), 2454-1362.

[2] V. Maheswari, M. Joice Mabel, Gd-distance in graphs, International Journal of Mathematical Archive- 9(2), 2018.

[3] V. Maheswari, K. Thanga Kanniga and K. Bala Deepa Arasi, On Gd-distance of a graph, International Journal of Advanced Science and Technology 28(16) (2019), 1324-1330.

[4] K. Pattabiraman and M. Vijayaraghavan, Reciprocal degree distance of product graphs, Discrete Appl. Math. 179 (2014), 201-213.

[5] D. Plavsic, S. Nikolic, N. Trinajstic and Z. Mihalic, On the harary index for the characterization of chemical graphs, J. Math. Chem. 12 (1993), 235-250.

[6] V. Sheeba Agnes, Degree distance of tensor product and strong product of graphs, Published by Faculty of Sciences and Mathematics, Filomat 28(10) (2014), 2185-2198.

[7] K. Thanga Kanniga, V. Maheswari and K. Bala Deepa Arasi, Gd-distance of product graphs, Advances in Mathematics; Scientific Journal ISSN:1857-8365(printed);1857-8438 (electronic) 9(5) (2020), 2643-2654.

[8] K. Thanga kanniga, V. Maheswari and K. Palani, Gd-distance of wreath product of graphs, Journal of Physics: Conference Series 1947 (2021).