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DECISION MAKING PROBLEMS BASED ON MEASURES OF BIPOLAR FUZZY ROUGH SETS

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Abstract

In this paper, Hamming distance on bipolar fuzzy rough sets is defined. A similarity measure on bipolar fuzzy rough sets is defined using Hamming distance and a decision making method based on the similarity measure on bipolar fuzzy rough set is developed. Further, an information measure on bipolar fuzzy rough set is defined and a decision making method is developed. Illustrations are provided to explain the applications of these methods.

1. Introduction

The theory of rough sets was proposed by Pawlak [10, 11]. In [3, 4, 9], the concept of fuzzy rough sets were studied by replacing crisp binary relations with fuzzy relations on the universe. Zhang [13] introduced the concept of bipolar fuzzy sets as an extension of fuzzy sets. Muthuraj [8] defined bipolar fuzzy set and established some properties of bipolar fuzzy and anti fuzzy subgroup. The similarity measure based on distance between soft sets was introduced by Majumdar and Samanta [6]. The same authors [7] also defined similarity measure based on distance between intuitionistic fuzzy soft sets. Deli and Cagman [2] defined some types of distance between two IFSSs and proposed similarity measures between them. They constructed a decision making method and applied it to a medical diagnosis problem based on similarity measures of IFSSs. Anita Shanthi et al. [1] developed a decision

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making method based on similarity and information measures of interval valued intuitionistic fuzzy soft set of root type. A detailed study on distance and similarities in IFS was presented by Szmidt [12].

In this paper, similarity and information measures on bipolar fuzzy rough sets are defined. Decision making method based on this similarity and information measures on BFRSs are also developed. Further, illustrations are provided to explain the application of these methods.

2. Similarity Measure on Bipolar Fuzzy Rough Sets

In this section, Hamming distance between bipolar fuzzy rough sets is defined. Using this Hamming distance a similarity measure on bipolar fuzzy rough sets is defined and a decision making method based on this similarity measure is developed.

Definition 2.1. Let U be a nonempty finite universe and BFR a bipolar fuzzy equivalence relation defined on $U \times U$. The pair (U, BFR) is called a bipolar fuzzy approximation space. For any bipolar fuzzy set A, the lower and upper approximation with respect to (U, BFR) denoted by $BF\underline{R}(A)$ and $BF\overline{R}(A)$ are two bipolar fuzzy sets defined as:

$$BF\underline{R}(A) = \{x, \mu_{BF\underline{R}(A)}(x)/x \in U\},$$

$$BF\overline{R}(A) = \{x, \mu_{BF\overline{R}(A)}(x)/x \in U\},$$

where

$$\mu_{BF\underline{R}^n(A)}(x) = \bigwedge_{u \in U}\{(-1 - \mu_{BF\underline{R}}(x, u)) \vee \mu_A(x), x \in U\}.$$

$$\mu_{BF\underline{R}^p(A)}(x) = \bigwedge_{u \in U}\{(1 - \mu_{BF\underline{R}}(x, u)) \vee \mu_A(x), x \in U\}.$$

$$\mu_{BF\overline{R}^n(A)}(x), \mu_{BF\overline{R}^p(A)} = \bigvee_{u \in U}(\mu_{BF\overline{R}}(x, u) \wedge \mu_A(x)).$$

The pair $BFR(A) = (BF\underline{R}(A), BF\overline{R}(A))$ is called the BFR set of A with respect to (U, BFR).

Definition 2.2. Let $U = \{x_1, x_2, \ldots, x_k\}$ be an universal set, $C = \{c_1, c_2, \ldots, c_m\}$ be a set of criteria. $BFR(A)$ and $BFR(B)$ be two BFR

sets on U. Then the Hamming distance between $BFR(A)$ and $BFR(B)$ is defined

$$H_d((BFR(A), (BFR(B))))$$

$$= \frac{1}{mk}\left\{\sum_{j=1}^{k}(| \mu^n_{BF\underline{R}(A)}(x_j) - \mu^n_{BF\underline{R}(B)}(x_j) | + | \mu^p_{BF\underline{R}(A)}(x_j) - \mu^p_{BF\underline{R}(B)}(x_j) |\right.$$

$$\left. + | \mu^n_{BF\overline{R}(A)}(x_j) - \mu^n_{BF\overline{R}(B)}(x_j) | + | \mu^p_{BF\overline{R}(A)}(x_j) - \mu^p_{BF\overline{R}(B)}(x_j) |)\right\}.$$

Definition 2.3. Let $BFR(A)$ and $BFR(B)$ be two BFR sets on U. Then, by using the Hamming distance a similarity measure between $BFR(A)$ and $BFR(B)$ denoted by $S^m(BFR(A), BFR(B))$ is defined as

$$S^m(BFR(A), BFR(B)) = \frac{1}{1 + H_d(BFR(A), BFR(B))}.$$

2.1. Method. Assume that $BFR(A)$ is a BFR sets representing the collection of data from the previous records for the specific problem prepared by the experts and this can be considered as known value. Consider the $BFRS$, $BFR(B)$ as the data for the same problem collected currently. The decision maker wants to check whether $BFR(B)$ is similar to $BFR(A)$. For solving this decision making problem, the similarity measure between $BFR(A)$ and $BFR(B)$ using the Definitions are calculated. Two $BFRS$ sets are significantly similar if $S^m(BFR(A), BFR(B)) > \frac{1}{2}$.

2.2. Algorithm:

The algorithm for a decision making method based on similarity measure of *BFRSs* is given below:

Step 1: Construct a $BFRS$, $BFR(A)$ over U based expert's evaluation.

Step 2: Construct a $BFRS$, $BFR(B)$ over U based on available data.

Step 3: Compute the Hamming distance between $BFR(A)$ and $BFR(B)$ using Definitions 2.2.

Step 4: Calculate the similarity measure between $BFR(A)$ and $BFR(B)$ using Definitions 2.3.

Step 5: Conclude using the value of similarity measure.

2.3. Example

A person wants to invest on shares of company B. Let $U = \{x_1, x_2, x_3, x_4\}$ be four types of shares viz. shares outstanding, treasury shares, issued shares, shares authorized, depending on the three attributes $\{c_1, c_2, c_3\}$, where c_1 = income, c_2 = growth rate, c_3 = profit or loss percent. Based on these recommendations the person has to decide whether he can invest on the shares.

Step 1. A $BFRS$, $BFR(A)$ over U based on the data collected from the previous records of shares is given below.

	c_1	c_2	c_3
x_1	((-0.16, 0.09), (-0.2, 0.38))	((-0.16, 0.2), (-0.2, 0.38))	((-0.16, 0.38), (0.2, 0.38))
x_2	((-0.15, 0.28), (-0.15, 0.38))	((-0.17, 0.5), (-0.17, 0.5))	((-0.17, 0.4), (-0.17, 0.4))
x_3	((-0.2, 0.32), (-0.4, 0.42))	((-0.2, 0.4), (-0.8, 0.42))	((-0.2, 0.22), (- 0.8, 0.4))
x_4	((-0.4, 0.45), (-0.6, 0.45))	((-0.4, 0.45), (-0.6, 0.45))	((-0.4, 0.5), (-0.6, 0.5))

Step 2. A $BFRS$, $BFR(A)$ based on the current information of shares of company B is given in Table 1.

Step 3. Hamming distance between $BFR(A)$ and $BFR(B)$ calculated using Definition 2.2 is $H_d(BFRM(A), BFRM(B)) = 0.465$.

Step 4. Similarity measure between $BFR(A)$ and $BFR(B)$ calculated using Definition 2.3 is

$$S^m(BFRM(A), BFRM(B)) = \frac{1}{1 + H_d(BFRM(A), BFRM(B))} = 0.682.$$

Table 1.

	c_1	c_2	c_3
x_1	((– 0.12,0.16) (-0.3,0.3))	((– 0.11 , 0.3), (-0.3, 0.3))	((– 0.12 , 0.25), (0.3,0.3))
x_2	((– 0.02,0.38), (-0.17, 0.45))	((– 0.18, 0.4), (-0.18, 0.45))	((– 0.18,0.45),(-0.18,0.45))
x_3	((– 0.45,0.28),(-0.45, 0.28))	((– 0.45,0.38),(-0.45 , 0.38))	((– 0.35,0.4),(- 0.45,0.48))
x_4	((– 0.26,0.42),(-0.36, 0.42))	((– 0.08 , 0.3),(-0.22, 0.4))	((– 0.26,0.4),(- 0.26, 0.42))

Step 5. Similarity measure between $BFR(A)$ and $BFR(B)$ is $> \frac{1}{2}$. Since the two BFRSs are significantly similar it is concluded the person can purchase the shares of the company B.

For the above example, the $BFRS$, $BFR(C)$, describing the information of shares of company C are as follows.

	c_1	c_2	c_3
x_1	((– 0.35,0.62) (-0.65,0.7))	((– 0.35 , 0.62), (-0.65, 0.7))	((– 0.35, 0.62), (0.65,0.64))
x_2	((– 0.28, 0.3),(-0.72 , 0.42))	((– 0.28, 0.3), (-0.72 , 0.42))	((– 0.28,0.3),(-0.72,0.42))
x_3	((– 0.44, 0.57),(-0.56 , 0.6))	((– 0.44, 0.57),(-0.56 , 0.6))	((– 0.44,0.57),(- 0.56,0.6))
x_4	((– 0.26,0.73),(-0.74 , 0.86))	((– 0.26 , 0.73),(-0.74, 0.86))	((– 0.26,0.73),(-0.74,0.76))

The Hamming distance between $BFR(A)$ and $BFR(B)$ is 1.0075 and similarity measure between $BFR(A)$ and $BFR(C)$ is $S^m(BFR(A), BFR(C)) = 0.4981$. It is concluded the person will not purchase the shares of company C.

3. Information Measure on Bipolar Fuzzy Rough Sets

In this section, an information measure on bipolar fuzzy rough sets is defined and a method for solving decision making problem is proposed. An algorithm based on information measure is developed for this purpose and an example is provided to illustrate the working of this algorithm.

3.1. Description of the problem

Let $U = \{A_1, A_2, A_m\}$ be the set of m alternatives which has to be ranked based on the criteria $C = \{c_1, c_2, \ldots, c_k\}$. Assume that each alternative A_i is expressed as on $BFRS$.

Let $(\mu^{n}{}_{BF\underline{R}(A_i)}(c_j), \mu^{n}_{BF\overline{R}(A_i)}(c_j))$ and $(\mu^{p}{}_{BF\underline{R}(A_i)}(c_j), \mu^{p}_{BF\overline{R}(A_i)}(c_j))$ denote the negative and positive membership degrees for each alternative A_i assigned with reference to a criterion c_j for all $i \in \{1, 2, \ldots, m\}$ and $j \in \{1, 2, \ldots, k\}$. Then the problem is to rank the alternatives.

Definition 3.1. Let the universal set $U = \{A_1, A_2, \ldots, A_m\}$ and the set of criteria $C = \{c_1, c_2, \ldots, c_k\}$. For any $BFRS$, $BFR(A)$ an information measure to indicate the degree of fuzziness of $BFR(A)$ is defined as

$$I_m(BFR(A)) =$$

$$\frac{1}{k}\sum_{j=1}^{k}\left| \frac{\min\{\mu^n_{BF\underline{R}(A)}(c_j), \mu^n_{BF\overline{R}(A)}(c_j)\} + \min\{\mu^p_{BF\underline{R}(A)}(c_j), \mu^p_{BF\overline{R}(A)}(c_j)\}}{\max\{\mu^n_{BF\underline{R}(A)}(c_j), \mu^n_{BF\overline{R}(A)}(c_j)\} + \max\{\mu^p_{BF\underline{R}(A)}(c_j), \mu^p_{BF\overline{R}(A)}(c_j)\}} \right|.$$

3.2. Method

Each alternative A_i is represented by a $BFRS$, one corresponding to each criterion c_j. Then the $BFRS$ representing the alternatives are converted to a single real number $I_m(BFRA_i)$ using the information measure. The objects are ranked by comparing the values of $I_m(BFRA_i)$. The better choice for the decision maker is to choose the object for which the information measure is the least.

3.3. Algorithm

The algorithm for ranking the alternatives of decision making problem based on bipolar fuzzy rough sets is given below:

Step 1: Construct bipolar fuzzy rough sets $BFR(A_i)$ over U based on previous records and data of the specific problem.

Step 2: Calculate the information measure $I_m(BFRA_i)$ using Definition 3.1.

Step 3: Compare the values of $I_m(BFRA_i)$ and conclude.

3.4. Example

A customer has to decide the laptop he wants to buy among the four types of laptops (alternatives). $BFRA_1, BFRA_2, BFRA_3, BFRA_4$ based on the criteria $C = \{c_1, c_2, c_3\}$ where c_1 = battery life, c_2 = memory capacity, c_3 = cost of the laptops, respectively.

Step 1. Based on the information obtained from the dealers of laptop regarding the benefits, the *BFR* sets based on the four criteria are given below

	c_1	c_2	c_3
$BFRA_1$	((-0.16,0.09)(-0.2,0.38))	((-0.16,0.38)(-0.2,0.38))	((-0.16,0.2)(-0.2,0.38))
$BFRA_2$	((-0.15,0.28)(-0.15,0.38))	((-0.17,0.5)(-0.17,0.5))	((-0.17,0.4)(-0.17,0.4))
$BFRA_3$	((-0.2,0.32)(-0.4,0.42))	((-0.2,0.4)(-0.8,0.42))	((-0.2,0.22)(-0.8,0.4))
$BFRA_4$	((-0.4,0.55)(-0.6,0.6))	((-0.4,0.55)(-0.6,0.6))	((-0.4,0.45)(-0.6,0.5))

Step 2: The values of $I_m(BFR(A_j))$ are estimated as follows:

$$I_m(BFR(A_1)) = 0.3295,\ I_m(BFR(A_2)) = 0.6413,$$

$$I_m(BFR(A_3)) = 0.5515,\ I_m(BFR(A_4)) = 0.375.$$

Step 3. The better alternative is the one which has least value for information measure. The information measures obtained are $I_m(BFR(A_1)) < I_m(BFR(A_4)) < I_m(BFR(A_3)) < I_m(BFR(A_2))$. Therefore the ranking of the alternatives is $BFRA_1 \succ BFRA_4 \succ BFRA_3 \succ BFRA_2$. i.e., Alternative $BFRA_1$ is the best Laptop when compared with the others.

References

[1] S. Anita Shanthi and J. Vadivel Naidu, Similarity measure of interval Valued intuitionistic fuzzy soft set of root type in decision making, The Journal of Soft Computing and Decision Support Systems 2(4) (2015), 42-47.

[2] I. Deli and N. Cagman, Similarity measures of intuitionistic fuzzy soft sets and their decision making, axXiv:1301.0456 2 (2013), 1-15.

[3] D. Dubois and H. Prade, Rough fuzzy set and fuzzy rough sets, International Journal of General Systems 17 (1990), 191-209.

[4] D. Dubois and H. Prade, Putting fuzzy sets and rough sets together, Intelligent Decision Support, Kluwer Academic, Dordrecht (1992), 203-232.

[5] George J. Klir and Tina A. Folger, Fuzzy sets, Uncertainty and Information, Prentice Hall of India, (2005).

[6] P. Majumdar and S. K. Samanta, Similarity measure of soft sets, New Mathematics and Natural Computation 4(1) (2008), 1-12.

[7] P. Majumdar and S. K. Samanta, on distance based similarity measure between intuitionistic fuzzy soft sets, Anusandhan 12(22) (2010), 41-50.

[8] R. Muthuraj and M. Sridharan, Some properties of Bipolar fuzzy cosets of a bipolar fuzzy and Bipolar anti fuzzy HX subgroup, IOSR Journal of Mathematics 9(6) (2014), 128-137.

[9] S. Nanda, Fuzzy rough sets, Fuzzy Sets and Systems 45 (1992), 157-160.

[10] Z. Pawlak, Rough sets, International Journal of Computer and Information Sciences 11 (1982), 341-356.

[11] Z. Pawlak, Rough Sets, Theoretical Aspects of Reasoning About Data, Kluwer Academic Publishers, Boston, (1991).

[12] E. Szmidt, Distance and Similarities in Intuitionistic Fuzzy Sets, Springer, (2014).

[13] W. R. Zhang, Bipolar fuzzy sets, IEEE International Conference on Fuzzy Sets (1994), 305-309.