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NEUTROSOPHIC QUOTIENT MAPPINGS

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Abstract

In this paper, we have initiate the concept of neutrosophic quotient map, neutrosophic α-quotient map, neutrosophic strongly α quotient map and neutrosophic α^*- quotient map in neutrosophic topological spaces. Also we analyze their characterizations and investigate their properties.

1. Introduction

The concept of neutrosophy and neutrosophic set was introduced by Smarandache where each element has degree of membership, degree of nonmembership and indeterminacy function. Neutrosophic sets are gained significant attention in solving many real life problems that involve uncertainty, impreciseness, vagueness, incompleteness, inconsistent, and indeterminacy. As a consequence topological ideas have been defined and studied on neutrosophic sets, giving birth to Neutrosophic Topology.

Salama and Alblowi [6] introduced the new concept of neutrosophic topological space in 2012. The neutrosophic closed sets and neutrosophic continuous functions were introduced by Salama et al. [7] in 2014. Further the fundamental sets like semi-open sets, pre-open sets, α-open sets are introduced in neutrosophic topological spaces then theirs properties are well-read by Ishwarya et al. and Imran et al. [2, 3].

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In this paper, we introduce the concept of neutrosophic quotient map, neutrosophic α-quotient map, neutrosophic strongly α quotient map and neutrosophic α^*-quotient map in neutrosophic topological spaces and some properties are investigate and study in neutrosophic topological spaces.

2. Preliminaries

Definition 2.1. Neutrosophic set and its Operators

A neutrosophic set $\mathcal{S}$ is an object of the following form

$$\mathcal{A} = \{\langle s, \mathcal{P}_{\mathcal{A}}(s), \mathcal{Q}_{\mathcal{A}}(s), \mathcal{R}_{\mathcal{A}}(s) : s \in \mathcal{S}\rangle\}$$

Where $\mathcal{P}_{\mathcal{A}}(s), \mathcal{Q}_{\mathcal{A}}(s)$ and $\mathcal{R}_{\mathcal{A}}(s)$ denote the degree of membership, the degree of indeterminacy and the degree of non-membership for each element $s \in \mathcal{S}$, to the set $\mathcal{A}$ respectively.

Definition 2.2. 0_N may be defined as

$0_N = \{\langle x, 0, 0, 1\rangle : x \in X\}$

$0_N = \{\langle x, 0, 1, 1\rangle : x \in X\}$

$0_N = \{\langle x, 1, 0, 1\rangle : x \in X\}$

$0_N = \{\langle x, 0, 0, 0\rangle : x \in X\}$

1_N may be defined as

$0_N = \{\langle x, 1, 0, 0\rangle : x \in X\}$

$0_N = \{\langle x, 1, 0, 1\rangle : x \in X\}$

$0_N = \{\langle x, 1, 1, 0\rangle : x \in X\}$

$0_N = \{\langle x, 1, 1, 1\rangle : x \in X\}$

Definition 2.3. Let $\mathcal{A}$ and $\mathcal{B}$ be Neutrosophic sets of the form

$$\mathcal{A} = \{\langle s, \mathcal{P}_{\mathcal{A}}(s), \mathcal{Q}_{\mathcal{A}}(s), \mathcal{R}_{\mathcal{A}}(s) : s \in \mathcal{S}\rangle\}$$

and

$$\mathcal{B} = \{\langle s, \mathcal{P}_{\mathcal{B}}(s), \mathcal{Q}_{\mathcal{B}}(s), \mathcal{R}_{\mathcal{B}}(s) : s \in \mathcal{S}\rangle\}.$$

Then

(i) $\mathcal{A} \subseteq \mathcal{B}$ if and only if $\mathcal{P}_{\mathcal{A}}(s) \leq \mathcal{P}_{\mathcal{B}}(s)$, $\mathcal{Q}_{\mathcal{A}}(s)$, $\mathcal{Q}_{\mathcal{B}}(s)$ and $\mathcal{R}_{\mathcal{A}}(s) \geq \mathcal{R}_{\mathcal{B}}(s)$;

(ii) $\overline{\mathcal{A}} = \{\langle \mathcal{R}_{\mathcal{A}}(s), \mathcal{Q}_{\mathcal{A}}(s), \mathcal{P}_{\mathcal{A}}(s) : s \in \mathcal{S} \rangle\}$;

(iii) $\mathcal{A} \cup \mathcal{B} = \{\langle s, \mathcal{P}_{\mathcal{A}}(s) \vee \mathcal{P}_{\mathcal{B}}(s), \mathcal{Q}_{\mathcal{A}}(s) \wedge \mathcal{Q}_{\mathcal{B}}(s), \mathcal{R}_{\mathcal{A}}(s) \wedge \mathcal{R}_{\mathcal{B}}(s) : s \in \mathcal{S} \rangle\}$;

(iv) $\mathcal{A} \cap \mathcal{B} = \{\langle s, \mathcal{P}_{\mathcal{A}}(s) \wedge \mathcal{P}_{\mathcal{B}}(s), \mathcal{Q}_{\mathcal{A}}(s) \vee \mathcal{Q}_{\mathcal{B}}(s), \mathcal{R}_{\mathcal{A}}(s) \vee \mathcal{R}_{\mathcal{B}}(s) : s \in \mathcal{S} \rangle\}$.

Definition 2.4. Neutrosophic topology A neutrosophic topology in a nonempty set $\mathcal{X}$ is a family $\mathfrak{J}$ of neutrosophic sets in $\mathcal{X}$ satisfying the following axioms:

(i) $0_N, 1_N \in \mathfrak{J}$,

(ii) $\mathcal{A} \cap \mathcal{B} \in \mathfrak{J}$ for any $\mathcal{A}, \mathcal{B} \in \mathfrak{J}$;

(iii) $\cup(\mathcal{A})_i$ for any arbitrary family $(\mathcal{A})_i : i \in j \subseteq \mathfrak{J}$.

Definition 2.5. A neutrosophic set $\mathcal{A}$ in a neutrosophic topological space (X,T) is called,

(i) a neutrosophic semi-open set if $\mathcal{A} \subseteq Ncl(\text{Nint}(\mathcal{A}))$.

(ii) a neutrosophic α-open set if $\mathcal{A} \subseteq \text{Nint}(Ncl(\text{Nint}(\mathcal{A})))$.

(iii) a neutrosophic pre-open set if $\mathcal{A} \subseteq \text{Nint}(\text{Nclt}(\mathcal{A}))$.

3. Neutrosophic Quotient Mappings

Definition 3.1. Let $d : (\mathcal{S}, \mathfrak{J}) \to (\mathcal{T}, \xi)$ be a surjective map. Then d is said to be neutrosophic-quotient map if d is neutrosophic continuous and $d^{-1}(\mathcal{A})$ is neutrosophic open in $(\mathcal{S}, \mathfrak{J})$ implies $\mathcal{A}$ is a neutrosophic open set in $(\mathcal{T}, \xi)$.

Definition 3.2. Let $d : (\mathcal{S}, \mathfrak{J}) \to (\mathcal{T}, \xi)$ be a surjective map. Then d is said to be

(i) neutrosophic α-quotient map if d is neutrosophic α-continuous and $d^{-1}(\mathcal{A})$ is neutrosophic open in $(\mathcal{S}, \mathfrak{J})$ implies $\mathcal{A}$ is a neutrosophic open set in $(\mathcal{T}, \xi)$.

(ii) neutrosophic semi-quotient map if d is neutrosophic semi-continuous and $d^{-1}(\mathcal{A})$ is neutrosophic open in $(\mathcal{S}, \mathfrak{J})$ implies $\mathcal{A}$ is neutrosophic semi-open in $(\mathcal{T}, \xi)$.

(iii) neutrosophic pre-quotient map: if d is neutrosophic pre-continuous and $d^{-1}(\mathcal{A})$ is neutrosophic open in $(\mathcal{S}, \mathfrak{J})$ implies $\mathcal{A}$ is neutrosophic pre-open in $(\mathcal{T}, \xi)$.

Theorem 3.3. *If $d : (\mathcal{S}, \mathfrak{J}) \to (\mathcal{T}, \xi)$ is surjective neutrosophic α-continuous and neutrosophic α-open, then d is a neutrosophic α-quotient map.*

Proof. Let $d^{-1}(\mathcal{A})$ be neutrosophic open in $(\mathcal{S}, \mathfrak{J})$. Then $d(d^{-1}(\mathcal{A}))$ is a neutrosophic α-open set. since d is a neutrosophic α-open set. Hence $\mathcal{A}$ is a neutrosophic α-open set, as dis surjective, $d(d^{-1}(\mathcal{A})) = \mathcal{A}$. Thus d is a neutrosophic α-quotient map.

Theorem 3.4. *If $d : (\mathcal{S}, \mathfrak{J}) \to (\mathcal{T}, \xi)$ be a neutrosophic open surjective neutrosophic α-irresolute and $e : (\mathcal{T}, \xi) \to (\mathcal{V}, \omega)$ be a neutrosophic α-quotient map. Then $e \circ d$ is a neutrosophic α-quotient map.*

Proof. Let $\mathcal{A}$ be any neutrosophic open set in $(\mathcal{V}, \omega)$. Then $e^{-1}(\mathcal{A})$ is a neutrosophic α-open, since e is a neutrosophic α-quotient map. And also since d is neutrosophic α-irresolute, $d^{-1}(e^{-1}(\mathcal{A}))$ is a neutrosophic α-open set. Hence $(e \circ d)^{-1}(\mathcal{A})$ is a neutrosophic α-open set implies $e \circ d$ is a neutrosophic α-open set. Hence $e \circ d$ is a neutrosophic α-continuous. Also, assume that $(e \circ d)^{-1}(\mathcal{A})$ be neutrosophic open in $(\mathcal{S}, \mathfrak{J})$ for $T \subseteq \mathcal{A}$, that is $d^{-1}(e^{-1}(\mathcal{A}))$ is neutrosophic open in $(\mathcal{S}, \mathfrak{J})$. Since $d : (\mathcal{S}, \mathfrak{J}) \to (\mathcal{T}, \xi)$ is neutrosophic open, $d(d^{-1}(e^{-1}(\mathcal{A})))$ is neutrosophic open in $(\mathcal{T}, \xi)$. It follows that $e^{-1}(\mathcal{A})$ is neutrosophic open in $(\mathcal{T}, \xi)$, d is surjective. Since e is a neutrosophic α-quotient map, $\mathcal{A}$ is a neutrosophic α-quotient map, $\mathcal{A}$ is a neutrosophic α-open set. Thus $e \circ d$ is a neutrosophic α-quotient map.

Corollary 3.5. *Let $d : (\mathcal{S}, \mathfrak{J}) \to (\mathcal{T}, \xi)$ be an onto neutrosophic open neutrosophic irresolute (respectively, neutrosophic pre-irresolute) map and $e : (\mathcal{T}, \xi) \to (\mathcal{V}, \omega)$ be a neutrosophic semi-quotient (respectively, neutrosophic pre-quotient) map. Then $e \circ d$ is a neutrosophic semi-quotient (respectively, neutrosophic pre-quotient) map.*

Theorem 3.6. *The function $d : (\mathcal{S}, \mathfrak{J}) \to (\mathcal{T}, \xi)$ is a neutrosophic α-quotient if and only if it is a neutrosophic semi-quotient map and a neutrosophic pre-quotient map.*

Proof. Let d be a neutrosophic α-quotient map. To prove that d is neutrosophic semi-quotient map. Since d is a neutrosophic α-quotient map, $d^{-1}(\mathcal{A})$ is a neutrosophic α-open set, hence it is neutrosophic semi-open and neutrosophic pre-open in $(\mathcal{S}, \mathfrak{J})$. That is, $\mathcal{A}$ is any neutrosophic open set in $(\mathcal{T}, \xi)$ implies $d^{-1}(\mathcal{A})$ is neutrosophic semi-open in $(\mathcal{S}, \mathfrak{J})$. Hence d is neutrosophic semi-continuous. Let $d^{-1}(\mathcal{A})$ be a neutrosophic open set in $(\mathcal{S}, \mathfrak{J})$. Since d is a neutrosophic α-quotient map, $\mathcal{A}$ is a neutrosophic α-open set in $(\mathcal{T}, \xi)$, which is neutrosophic semi-open and neutrosophic pre-open in $(\mathcal{T}, \xi)$, that is, $d^{-1}(\mathcal{A})$ is neutrosophic-open in $(\mathcal{S}, \mathfrak{J})$ implies $\mathcal{A}$ is neutrosophic semi open in $(\mathcal{S}, \mathfrak{J})$. Hence d is neutrosophic semi-quotient map. Similarly we can prove that d is a neutrosophic pre-quotient map.

Conversely, let d be a neutrosophic semi-quotient map and a pre-quotient map. Let $\mathcal{A}$ be any neutrosophic open set in $(\mathcal{T}, \xi)$. Since d is both a neutrosophic semi-quotient and a neutrosophic pre-quotient map, $d^{-1}(\mathcal{A})$ is both neutrosophic semi-open and neutrosophic pre-open in $(\mathcal{S}, \mathfrak{J})$, so that $d^{-1}(\mathcal{A})$ is neutrosophicα-open set. Hence d is neutrosophic α-continuous. Since d is a neutrosophic semi-quotient map and a pre-quotient map, $\mathcal{A}$ is neutrosophic semi-open and neutrosophic pre-open in $(\mathcal{T}, \xi)$ so that $\mathcal{A}$ is neutrosophic α-open in $(\mathcal{T}, \xi)$. Thus d is a neutrosophic α-quotient map.

4. Neutrosophic Strongly Quotient Mappings

Definition 4.1. Let $d : (\mathcal{S}, \mathfrak{J}) \to (\mathcal{T}, \xi)$ be a surjective map. Then d is said to be

(i) neutrosophic strongly α-quotient map provided a set of $(\mathcal{T}, \xi)$ is neutrosophic open in $(\mathcal{T}, \xi)$ if and only if $d^{-1}(\mathcal{A})$ is a neutrosophicα-open in $(\mathcal{S}, \mathfrak{J})$.

(ii) neutrosophic strongly semi-quotient map provided a set $\mathcal{A}$ of $(\mathcal{T}, \xi)$ is neutrosophic open in $(\mathcal{T}, \xi)$ if and only if $d^{-1}(\mathcal{A})$ is a neutrosophic semi-open in $(\mathcal{S}, \mathfrak{J})$.

(iii) neutrosophic strongly pre-quotient map provided a set $\mathcal{A}$ of $(\mathcal{T}, \xi)$ is neutrosophic open in $(\mathcal{T}, \xi)$ if and only if $d^{-1}(\mathcal{A})$ is a neutrosophic pre-open in $(\mathcal{S}, \mathfrak{J})$.

Theorem 4.2. *If a function $d : (\mathcal{S}, \mathfrak{J}) \to (\mathcal{T}, \xi)$ is neutrosophic strongly semi-quotient and neutrosophic strongly pre-quotient, then d is neutrosophic strongly α-quotient.*

Proof. Let $\mathcal{A}$ be a neutrosophic open set in $(\mathcal{T}, \xi)$. Since d is neutrosophic strongly semi-quotient and neutrosophic strongly pre-quotient, $d^{-1}(\mathcal{A})$ is neutrosophic semi-open as well as neutrosophic pre-open. So, $d^{-1}(\mathcal{A})$ is neutrosophicα-open. Let $d^{-1}(\mathcal{A})$ be a neutrosophicα-open set in $(\mathcal{S}, \mathfrak{J})$. So, $d^{-1}(\mathcal{A})$ is semi-open in $(\mathcal{S}, \mathfrak{J})$. Since d is neutrosophic strongly semiquotient, $\mathcal{A}$ is neutrosophic open in $(\mathcal{T}, \xi)$. Hence it follows that $\mathcal{A}$ is neutrosophic open in $(\mathcal{T}, \xi)$ if and only if $d^{-1}(\mathcal{A})$ is neutrosophic α-open in $(\mathcal{S}, \mathfrak{J})$. So d is a neutrosophic strongly α-quotient map.

5. Neutrosophic α^*-Quotient Mappings

Definition 5.1. Let $d : (\mathcal{S}, \mathfrak{J}) \to (\mathcal{T}, \xi)$ be a surjective map. Then d is said to be

(i) neutrosophicα∗-quotient map if d is neutrosophic α-irresolute and $d^{-1}(\mathcal{A})$ is neutrosophicα-open in $(\mathcal{S}, \mathfrak{J})$ implies $\mathcal{A}$ is a neutrosophic open set in $(\mathcal{T}, \xi)$.

(ii) neutrosophic semi ∗-quotient map if d is neutrosophic semi-irresolute and $d^{-1}(\mathcal{A})$ is neutrosophic semi-open in $(\mathcal{S}, \mathfrak{J})$ implies $\mathcal{A}$ is a neutrosophic open set in $(\mathcal{T}, \xi)$.

(iii) neutrosophic pre ∗-quotient map if d is neutrosophic pre-irresolute and $d^{-1}(\mathcal{A})$ is neutrosophic pre-open in $(\mathcal{S}, \mathfrak{J})$ implies $\mathcal{A}$ is a neutrosophic open set in $(\mathcal{T}, \xi)$.

Definition 5.2. A function $d : (\mathcal{S}, \mathfrak{J}) \to (\mathcal{T}, \xi)$ is called a neutrosophic strongly α-open map if the image of every neutrosophic α-open set in $(\mathcal{S}, \mathfrak{J})$ is a neutrosophic α-open set in $(\mathcal{T}, \xi)$.

Theorem 5.3. *Let $d : (\mathcal{S}, \mathfrak{J}) \to (\mathcal{T}, \xi)$ be an onto neutrosophic strongly α-open and a neutrosophic α-irresolute map. Let $e : (\mathcal{T}, \xi) \to (\mathcal{V}, \omega)$ be a neutrosophic α^*-quotient map. Then $e \circ d$ is a neutrosophic α^*-quotient map.*

Proof. We claim that $e \circ d$ is neutrosophicα-irresolute. Let $\mathcal{A}$ be a neutrosophic α-open set in $(\mathcal{V}, \omega)$. Then $e^{-1}(\mathcal{A})$ is a neutrosophic α-open set in $(\mathcal{T}, \xi)$ as e is a neutrosophic α^*-quotient map. Since d is neutrosophic α-irresolute $d^{-1}(e^{-1}(\mathcal{A})) = (e \circ d)^{-1}(\mathcal{A})$ is a neutrosophicα-open set in $(\mathcal{S}, \mathfrak{J})$. So, $e \circ d$ is a neutrosophic α-irresolute map. Suppose $(e \circ d)^{-1}(\mathcal{A}) = d^{-1}(e^{-1}(\mathcal{A}))$ is a neutrosophic α-open set in $(\mathcal{S}, \mathfrak{J})$. Since d is neutrosophic strongly α-open, $d^{-1}(d^{-1}(e^{-1}(\mathcal{A})))$ is a neutrosophic α-open set in $(\mathcal{S}, \mathfrak{J})$. Since d is an onto map $d(d^{-1}(e^{-1}(\mathcal{A}))) = e^{-1}(\mathcal{A})$. So $e^{-1}(\mathcal{A})$ is a neutrosophic α-open set in $(\mathcal{T}, \xi)$. This implies that $\mathcal{A}$ is a neutrosophic open set in $(\mathcal{V}, \omega)$ as e is a neutrosophicα∗-quotient map. Hence $e \circ d$ is a neutrosophic α^*-quotient map.

Theorem 5.4. *If a function* $d : (\mathcal{S}, \mathfrak{J}) \to (\mathcal{T}, \xi)$ *is neutrosophic semi* $*$*-quotient and neutrosophic pre* $*$*-quotient then d is neutrosophic* α^**-quotient.*

Proof. Let $\mathcal{A}$ be a neutrosophic open set in $(\mathcal{T}, \xi)$. Since d is neutrosophic strongly semi-quotient and neutrosophic strongly pre-quotient, $d^{-1}(\mathcal{A})$ is neutrosophic semi-open as well as neutrosophic pre-open. So, $d^{-1}(\mathcal{A})$ is neutrosophicα-irresolute. Let $d^{-1}(\mathcal{A})$ be a neutrosophicα-open set in $(\mathcal{S}, \mathfrak{J})$, since d is neutrosophic semi$*$-quotient map and neutrosophicpre-$*$-quotient map, $\mathcal{A}$ is neutrosophic open set in V. Hence d is a neutrosophic α^*-quotient map.

6. Comparisons

Theorem 6.1. *Let* $(\mathcal{S}, \mathfrak{J})$ *and* $(\mathcal{T}, \xi)$ *be neutrosophic topological spaces. If* $d : (\mathcal{S}, \mathfrak{J}^{\alpha})^{-} \to (\mathcal{T}, \xi^{\alpha})$ *is a neutrosophic quotient map then* $d : (\mathcal{S}, \mathfrak{J}) \to (\mathcal{T}, \xi)$ *is a neutrosophic* α*-quotient map.*

Proof. Let $\mathcal{A} \in \xi$ So $\mathcal{A} \in \xi^{\alpha}$. Since d is a neutrosophic quotient map, $d^{-1}(\mathcal{A}) \in \mathfrak{J}^{\alpha}$. Hence it is proved that when $\mathcal{A}$ is a neutrosophic open set in $(\mathcal{T}, \xi)$, then $d^{-1}(\mathcal{A})$ is a neutrosophic α-open set in $(\mathcal{S}, \mathfrak{J})$. So d is a neutrosophic α-continuous map. Suppose $d^{-1}(\mathcal{A})$ is neutrosophic open in $(\mathcal{S}, \mathfrak{J})$ then $d^{-1}(\mathcal{A}) \in \mathfrak{J}^{\alpha}$. Since d is a neutrosophic quotient map, $\mathcal{A} \in \mathfrak{J}^{\alpha}$ and so $\mathcal{A}$ is a neutrosophic α-open set in $(\mathcal{T}, \xi)$. Hence $d : (\mathcal{S}, \mathfrak{J}) \to (\mathcal{T}, \xi)$ is a neutrosophic α-quotient map.

Definition 6.2. Let $(\mathcal{S}, \mathfrak{J})$ and $(\mathcal{T}, \xi)$ be neutrosophic topological spaces. A function $d : (\mathcal{S}, \mathfrak{J}) \to (\mathcal{T}, \xi)$ is called neutrosophic quasi α-open if the image of every neutrosophic α-open set in $(\mathcal{S}, \mathfrak{J})$ is neutrosophic open in $(\mathcal{T}, \xi)$.

Theorem 6.3. *Let* $(\mathcal{S}, \mathfrak{J})$ *and* $(\mathcal{T}, \xi)$ *be neutrosophic topological spaces. If* $d : (\mathcal{S}, \mathfrak{J}) \to (\mathcal{T}, \xi)$ *is neutrosophic quasi* α*-open then* $d : (\mathcal{S}, \mathfrak{J}) \to (\mathcal{T}, \xi)$ *is neutrosophic strongly* α*-open.*

Proof. Let $\mathcal{A}$ be a neutrosophic α-open set in $(\mathcal{S}, \mathfrak{J})$. So $\mathcal{A} \in \mathfrak{J}^{\alpha}$. Since $d : (\mathcal{S}, \mathfrak{J}^{\alpha})^{-} \to (\mathcal{T}, \xi^{\alpha})$ is neutrosophic quasi α-open, $d(\mathcal{A})$ is neutrosophic open in $(\mathcal{T}, \xi^{\alpha})$. So $d(\mathcal{A})$ is a neutrosophic α-open set in $(\mathcal{T}, \xi)$. Hence it follows that $d : (\mathcal{S}, \mathfrak{J}) \to (\mathcal{T}, \xi)$ is a neutrosophic strongly α-open map.

Theorem 6.4. *Every neutrosophic quotient map is a neutrosophic α-quotient map.*

Proof. Let $\mathcal{A}$ be a neutrosophic open set in $(\mathcal{T}, \xi)$. Since d is a neutrosophic quotient map, $d^{-1}(\mathcal{A})$ is neutrosophic open in $(\mathcal{S}, \mathfrak{J})$ and so, $d^{-1}(\mathcal{A})$ is a neutrosophic α-open set in $(\mathcal{S}, \mathfrak{J})$. So d is a neutrosophic α-continuous map. Let $d^{-1}(\mathcal{A})$ be neutrosophic open in $(\mathcal{S}, \mathfrak{J})$. Since d is a neutrosophic quotient map, $\mathcal{A}$ is a neutrosophic open set in $(\mathcal{T}, \xi)$ and so $\mathcal{A}$ is a neutrosophic α-open set in $(\mathcal{T}, \xi)$. Hence d is a neutrosophic α-quotient map.

Example 6.5. Let $\mathcal{S} = \{p, q, r\} = \{0_N, D_1, D_2, D_3, D_4, 1_N\}$ be a neutrosophic topology on $(\mathcal{S}, \mathfrak{J})$

$D_1 = \langle s, (0.4, 0.3, 0.3), (0.4, 0.3, 0.3), (0.5, 0.7, 0.7)\rangle$

$D_2 = \langle s, (0.3, 0.4, 0.4), (0.6, 0.5, 0.5), (0.5, 0.5, 0.5)\rangle$

$D_3 = \langle s, (0.4, 0.4, 0.4), (0.4, 0.3, 0.3), (0.5, 0.5, 0.5)\rangle$

$D_4 = \langle s, (0.3, 0.3, 0.3), (0.6, 0.5, 0.5), (0.5, 0.7, 0.7)\rangle$, and let $T = \{p, q, r\}$, $\xi = \{0_N, F_1, F_2, F_3, F_4, 1_N\}$ be a neutrosophic topology on (T, ξ)

$F_1 = \langle t, (0.5, 0.5, 0.5), (0.4, 0.3, 0.3), (0.4, 0.4, 0.4)\rangle$

$F_2 = \langle t, (0.4, 0.4, 0.4), (0.3, 0.3, 0.3), (0.5, 0.5, 0.5)\rangle$

$F_3 = \langle t, (0.5, 0.5, 0.5), (0.3, 0.3, 0.3), (0.4, 0.3, 0.3)\rangle$

$F_4 = \langle t, (0.4\ 0.4, 0.4), (0.4, 0.3, 0.3), (0.5, 0.5, 0.5)\rangle$. Define $d : (\mathcal{S}, \mathfrak{J}) \to (\mathcal{T}, \xi)$ by $d(p) = p$, $d(q) = q$, $d(r) = r$.

N_α - open set of $(\mathcal{S}, \mathfrak{J}) = \langle s, (0.4\ 0.4, 0.4), (0.3, 0.3, 0.3), (0.5, 0.5, 0.5)\rangle$.

N_α - open set of $(\mathcal{T}, \xi) = \langle s, (0.4\ 0.4, 0.4), (0.4, 0.3, 0.3), (0.5, 0.5, 0.5)\rangle$.

Here d is neutrosophic α-quotient map but not a neutrosophic quotient map. Since $d^{-1}(F_2) = F_2$ is not a neutrosophic open in $(\mathcal{S}, \mathfrak{J})$.

Theorem 6.6. *A neutrosophic* α^**-quotient map is a neutrosophic strongly* α*-quotient map.*

Proof. Let $d : (\mathcal{S}, \mathfrak{J}) \to (\mathcal{T}, \xi)$ be a neutrosophic α^*-quotient map. Suppose $\mathcal{A}$ is a neutrosophic open set in $(\mathcal{T}, \xi)$. Then $d^{-1}(\mathcal{A})$ is a neutrosophicα-open set in $(\mathcal{S}, \mathfrak{J})$ as $\mathcal{A}$ is neutrosophic α-open in $(\mathcal{T}, \xi)$ and d is neutrosophic α irresolute. Suppose $d^{-1}(\mathcal{A})$ is a neutrosophic α-open set in $(\mathcal{S}, \mathfrak{J})$ then $\mathcal{A}$ is a neutrosophic open set in $(\mathcal{T}, \xi)$ as d is a neutrosophic α^*-quotient map. Hence d is a neutrosophic strongly α-quotient map.

Example 6.7. Let $S = \{p, q, r\} = \{0_N, D_1, D_2, D_3, D_4, 1_N\}$ be a neutrosophic topology on $(\mathcal{S}, \mathfrak{J})$,

$D_1 = \langle s, (0.4, 0.3, 0.3), (0.4, 0.3, 0.3), (0.5, 0.7, 0.7)\rangle$

$D_2 = \langle s, (0.3, 0.4, 0.4), (0.6, 0.5, 0.5), (0.5, 0.5, 0.5)\rangle$

$D_3 = \langle s, (0.4, 0.4, 0.4), (0.4, 0.3, 0.3), (0.5, 0.5, 0.5)\rangle$

$D_4 = \langle s, (0.3, 0.3, 0.3), (0.6, 0.5, 0.5), (0.5, 0.7, 0.7)\rangle$, and

let $T = \{p, q, r\}, \xi = \{0_N, F_1, F_2, F_3, F_4, 1_N\}$ be a neutrosophic topology on (T, ξ),

$F_1 = \langle t, (0.5, 0.5, 0.5), (0.4, 0.3, 0.3), (0.4, 0.4, 0.4)\rangle$

$F_2 = \langle t, (0.4, 0.4, 0.4), (0.3, 0.3, 0.3), (0.5, 0.5, 0.5)\rangle$

$F_3 = \langle t, (0.5, 0.5, 0.5), (0.3, 0.3, 0.3), (0.4, 0.3, 0.3)\rangle$

$F_4 = \langle t, (0.4\ 0.4, 0.4), (0.4, 0.3, 0.3), (0.5, 0.5, 0.5)\rangle$.

Define $d : (\mathcal{S}, \mathfrak{J}) \to (\mathcal{T}, \xi)$ by $d(p) = p$, $d(q) = q$, $d(r) = r$.

N_α- open set of $(\mathcal{S}, \mathfrak{J}) = \langle s, (0.4\ 0.4, 0.4), (0.3, 0.3, 0.3), (0.5, 0.5, 0.5)\rangle$.

N_α- open set of $(\mathcal{T}, \xi) = \langle s, (0.4\ 0.4, 0.4), (0.4, 0.3, 0.3), (0.5, 0.5, 0.5)\rangle$. Here d is neutrosophic strongly α-quotient map but not a neutrosophic α^*- quotient map. Since d is not neutrosophic α-irresolute.

Theorem 6.8. *Every neutrosophic strongly* α*-quotient map is a neutrosophic* α*-quotient map*.

Proof. Let $\mathcal{A}$ be a neutrosophic open set in $(\mathcal{T}, \xi)$. Since d is neutrosophic strongly α-quotient, $d^{-1}(\mathcal{A})$ is a neutrosophicα-open set in $(\mathcal{S}, \mathfrak{J})$. Let $d^{-1}(\mathcal{A})$ be neutrosophic open in $(\mathcal{S}, \mathfrak{J})$, then $d^{-1}(\mathcal{A})$ is a neutrosophic α-open set in $(\mathcal{S}, \mathfrak{J})$. Hence d is a neutrosophic α-quotient map.

Example 6.9. Let $S = \{p, q, r\} = \{0_N, D_1, D_2, D_3, D_4, 1_N\}$ be a neutrosophic topology on $(\mathcal{S}, \mathfrak{J})$,

$D_1 = \langle s, (0.3, 0.2, 0.2), (0.3, 0.2, 0.2), (0.4, 0.6, 0.6)\rangle$

$D_2 = \langle s, (0.2, 0.3, 0.3), (0.5, 0.4, 0.4), (0.4, 0.4, 0.4)\rangle$

$D_3 = \langle s, (0.3, 0.3, 0.3), (0.3, 0.2, 0.2), (0.4, 0.4, 0.4)\rangle$

$D_4 = \langle s, (0.2, 0.2, 0.2), (0.5, 0.4, 0.4), (0.4, 0.6, 0.6)\rangle$, and

let $T = \{p, q, r\}$, $\xi = \{0_N, F_1, F_2, F_3, F_4, 1_N\}$ be a neutrosophic topology on (T, ξ),

$F_1 = \langle t, (0.4, 0.4, 0.4), (0.3, 0.2, 0.2), (0.3, 0.3, 0.3)\rangle$

$F_2 = \langle t, (0.3, 0.3, 0.3), (0.2, 0.2, 0.2), (0.4, 0.4, 0.4)\rangle$

$F_3 = \langle t, (0.4, 0.4, 0.4), (0.2, 0.2, 0.2), (0.3, 0.2, 0.2)\rangle$

$F_4 = \langle t, (0.3, 0.3, 0.3), (0.3, 0.2, 0.2), (0.4, 0.4, 0.4)\rangle$.

Define $d : (\mathcal{S}, \mathfrak{J}) \to (\mathcal{T}, \xi)$ by $d(p) = p$, $d(q) = q$, $d(r) = r$.

N_α - open set of $(\mathcal{S}, \mathfrak{J}) = \langle s, (0.3, 0.3, 0.3), (0.2, 0.2, 0.2), (0.4, 0.4, 0.4)\rangle$.

N_α - open set of $(\mathcal{T}, \xi) = \langle s, (0.3\ 0.3, 0.3), (0.3, 0.2, 0.2), (0.4, 0.4, 0.4)\rangle$. Here d is neutrosophic α-quotient map but not a neutrosophic strongly αquotient map. Since $d^{-1}(F_1) = F_1$ is not a neutrosophicα-open in $(\mathcal{S}, \mathfrak{J})$.

Remark 6.10. The following table shows the relationships of neutrosophic quotient map with other sorts of neutrosophic quotient maps. The symbol T in a cell means that a set implies the other maps and the symbol F means that a set does not imply the other sets.

Functions	A	B	C	D
A	-	T	T	F
B	F	-	T	F
C	F	F	-	F
D	F	F	T	-

(A). Neutrosophic α^*-quotient map (B). Neutrosophic strongly α-quotient map (C). Neutrosophic α-quotient map (D). Neutrosophic quotient map.

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