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PORTFOLIO MANAGEMENT, CLASSICAL AND ROBUST STATISTICS A LITERATURE REVIEW

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Abstract

This study reviews the existing studies on portfolio management, classical and robust statistics in detail. This commences with the origin of the modern portfolio theory and several other models which progressed after the traditional method. In the next section, major difficulties in the classical statistics have been discussed. Robust Statistics, Robust estimation, classical Robust Covariance matrix estimators, advantages of using good robust estimators as well as the features of robust optimization are also examined in the following section. Moreover, the authors discusses that the robust portfolio optimization can also be applied to portfolio allocation models which are different from the mean-variance framework. Finally, the application of the robust statistics in modern portfolio theory are also dealt with practical considerations.

1. History and Development of Modern Portfolio Theory

Harry Markowitz was the pioneer for “The Modern portfolio theory”. He introduced Modern portfolio theory in 1952. His work earned him a share of Nobel prize in Economics. This theory generally referred as Mean-variance (MV) portfolio Analysis. Markowitz’s mean-variance analysis determined the foundation for modern portfolio optimization theory. Michaud described that most modern finance textbooks consider mean-variance efficiency is the method optimal portfolio construction and asset allocation [17]. This became

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the standard in the field of investment management. He developed mean-variance (MV) analysis in the context of selecting a portfolio of common stocks. MV analysis has been progressively applied to asset allocation. Asset allocation refers that it is the procedure of diversifying the investment and it is more suitable of mean-variance analysis than portfolio selection.

Markowitz selection model is the foundation of the current theory of asset allocation. Since, Markowitz proposed his model, a lot of portfolio selection models have been developed in order to progress this model and improve portfolio theory. The Mean variance model's drawbacks led researchers to develop alternative portfolio selection models. While emphasizing MV Optimization, Michaud marked this as "It remains one of the outstanding puzzles of modern finance" and he termed this puzzle as "the Markowitz optimization Enigma" [17]. Markowitz also summarizes that, even though in theory Markowitz efficiency is a suitable framework for portfolio optimality, in practice it is to say error-maximized and investment irrelevant portfolios and suggests resmpling mean returns and the covariance matrix Σ of the asset from a confidence region around a nominal set of parameters. Even though researchers in finance content with the theory and for themselves MV analysis is significant to many asset pricing theories, but practitioners have reported there are several hindrances in implementing mean-variance analysis. Fabozzi et al. described that MPT has great impact in the practice of Portfolio management and it provides a framework to construct and select portfolios based on the expected performance of the investments and the risk appetite of the investor [7]. Yet, its major difficulty lies in the estimation Minimax model was introduced by Young [25]. Similarly, the Classic Mean-variance framework of Markowitz [14], mean absolute deviation (Konno and Yamazaki [11]), minimax (Young [32]), Value at Risk (Ahn et al. [1]; Basak and Shapiro [2]) developed by Academicians later. Classical statistics plays an vital role in the development and exploration of new ideas, theories and practicalities in modern portfolio theory. The usage of classical statistics makes the academicians and the portfolio managers to prove new and versatile models in a scientific manner. But the application of classical statistics in modern portfolio theory is having some handicaps that may exist in the estimation and in testing procedures. Nevertheless it has some disadvantages which is briefly discussed below in the next section.

2. Problems in Classical Statistics

Maronna [15] stated that while collecting data extensively, it often consists one or more atypical observations and it is called “Outliers”. Outliers are observations that are deviated from the majority or bulk of the data, or from the general pattern of the data. Without any noticeable gross errors, high Quality data obtained with greatest care. But real data normally contains gross errors. The most common definition for outlier is “an observation lying far from the rest of the data although this is not sufficient to identify an anomaly”. An remote observation is an outlier only if it is judged inconsistent with the remainder of the data. The purpose of statistical methods is to introduce some objectivity in the identification and treatment of the “strange” points. An extreme observation is a point fax from the rest of the data. This is an outlier if the distance is considered “unusual”. Regarding the treatment of outliers, rejection of strange points is not always the optimal solution. The most common criticism from the “antagonists” of robust methods is that blind deletion could result in a loss of some relevant information. If these points are generated from errors in, they should be eliminated from the data. In other cases, when the anomalies derive from a different probability distribution or a different deterministic model, there are various approaches for treating the outliers other than plain deletion. A first approach consists in applying a transformation, when possible and appropriate, in order to adapt the model to the furthest points. A second approach is to reduce the importance of outliers through down-weighting. There are variety of models in detecting single outliers, which when applied to groups of contaminated data, it leads to a problem “Masking”. The masking effect allows high tolerance of “bad data”, on the other hand, if too many outliers in the test is specified “swamping” will occur. Hence, to overcome these inadequacies and the classical statistics needs an alternative estimator which gives a birth to the new discipline of statistics called “Robust Statistics” and the first birth of this discipline was introduced by Rousseeuw and Leroy [20].

3. The Genesis of Robust Statistics

A. Robust statistics

Huber [9] defines the word robust as, insensitive to slight variations from statistical assumptions. The word 'Robustness' has diverse meanings and widely used in many fields of scientific research. Generally scientific experiments are constructed based on the framework which determines the validity of the results. Sometimes these initial assumptions are too restrictive and do not match what happens in reality. In general, the results are termed 'robust', if they are not affected by changes in the initial framework. According to Hampel [8], whose paper highly condensed first introduction to robust statistics stated that, robust statistics is the stability theory of Statistical procedures. It scrutinizes systematically the effect of deviation from modeling assumptions on known procedures. Robust method allows to find both Location and the dispersion of a multivariate cloud based on robustness criteria and to detect groups of outliers at the same time.

B. Robust estimation

Mean variance portfolios developed using sample mean and covariance matrix of asset returns which carry out poorly out of sample because of estimation error. These estimation errors is an significant drawback in mean-variance approach. In order to overcome the outcome of estimation error recently Robust Estimation is proposed. There are several estimators to overcome these difficulties. Introducing robust estimation with portfolio optimization is relatively recent when compared with Markowitz groundbreaking paper. Although, Robust estimation of a location parameter introduced by Huber [9], the first of Multi-variate robust estimators is a generalization of the maximum likelihood estimators for location and scatter, introduced by Maronna [15]. Because of high dimensionality and heavy-tailedness of asset return data, estimating the covariance matrix of asset returns is challenging task. The reason for Heavy-tailed asset returns is due to no. of assets under management larger than the sample size and extreme events. To overcome heavy tailed data, robust estimators of covariance matrix are recommended. Classical robust covariance matrix estimators include: M-estimators, Minimum volume ellipsoid (MVE), Minimum covariance determinant (MCD) estimators, S-estimators and Estimators based on data outlyingness and depth. These estimators are specifically designed for data with very low dimensions and large sample sizes. A general class of Robust estimators that can be derived from robust scaling, namely the minimum volume ellipsoid estimator, the minimum covariance determinant (MCD) estimator and the S-Estimator. M-Estimation is an extension of maximum likelihood method and is a robust estimation. While *S* Estimation is the development of *M*-Estimation method. In order to eliminate some of the data these methods were used. *S* Estimators for Multivariate and scatter was introduced by Davies, who extended the idea of the *S* Estimators for Regression (Rouseeuw and Yohai [21]). A Special form of robust *M* Estimator was introduced by Yohai [24] and called it MM Estimator. He proposed the computation of the MM Estimator in three steps. Nevertheless, the subject has become very active in the last decade, as seen in the works of Lauprête [12], Lauprete et al. 13], Mendes and Leal [16], Perret-Gentil and Victoria-Feser [18], Welsch and Zhou [23], and DeMiguel and Nogales [6].

C. Robust high break down point estimators

Robust estimation is sometimes criticized for being overly conservative. Likewise, a robust optimization approach only preserve against the given uncertainty model, while potentially becoming very vulnerable to realizations outside of the realm of the model. According to Rousseeuw and Driessen [19] there are various methods for estimating multivariate location and scatter break down in the presence of outliers, where is the number of observations and p is the number of variables. Moreover, several positive-breakdown estimators of multivariate location and scatter have been proposed. One of these is the minimum volume ellipsoid (MVE) method of Rousseeuw. There are several reasons for replacing the MVE by the minimum covariance determinant (MCD) estimator $n = (p + 1)$.

D. Robust optimization

Practitioners often misrepresents robust portfolio optimization with robust mean variance model. Robust optimization has traditionally modelled the stock returns as uncertain parameters belonging to known uncertainty sets. It has also been extended to active portfolio management problems. Robust optimization approach in the operations research introduced by Ben-tal and Nemirovski [3]. They gave a robust optimization approach for portfolio optimization using factor models. Ben-tal and Nemirovski [3] has applied the robust optimization approach not on uncertain parameters but on random variables. Ben-tal et al. [4] developed a new model by a new methodology for optimization under uncertainty, that is the robust Counterpart approach.

4. Application of Robust Statistics in Portfolio Models

A. Practical considerations

Depending on the size of the portfolio, the type of assets and their distributional characteristics, robust models considers as the best models for modelling financial portfolios. Fabozzi et al. [7] highlighted that, "The Robust optimization framework offers great flexibility and many new interesting possibility in management".

B. Robust portfolio models

Since the use of quantitative techniques universal in the investment industry, consideration of estimation risk and model risk has grown in importance. Robust estimation of model parameters are now common in financial applications. Recent contributions made from Operations research and finance to the theory of robust portfolio selection. In order to bring down the sensitivity of Markowitz's models several methodologies have been proposed, one among them is Robust Statistics. Methods have been proposed which generate robust statistical estimates of the uncertain parameters of the models. A different form to attain robustness is robust optimization. There are a number of places in finance where robust estimation has been used.

C. Computation of robust statistics

In order to present the result with robust statistical viewpoint, we ought to understand what to do the data and how to interpret it. Software that is designed for performing robust statistical analysis can make this process as smooth as easy as possible and provide the necessary results. Software packages such as S-PLUS, SYSTAT, STATA, Scout, JMP and many Add-ons in R-Package were used to analyse robust portfolio models.

5. Conclusion

In this paper the authors discussed the framework of robust statistics, estimators and the practical application of the robust statistics in modern portfolio theory and pointed out several approaches and models introduced by the practitioners. Then, reviewed the robust statistics approach in the Portfolio selection. Then, discussed about estimation and optimization distinctly and provided several robust estimators. It is very hard to estimate the underlying probability distributions of asset prices accurately, but traditionally portfolio management assumes so. But, Application of robust statistics in portfolio theory has been widely accepted, because it reduce the sensitivity of Markowitz model. Since, robust portfolio selection model extends to robust active portfolio management, the main objective is to combine the robust estimation with robust optimization and to introduce new testing methodology.

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