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ON FUZZY PSEUDO PERFECTLY CONTINUOUS FUNCTIONS AND FUZZY d_δ FUNCTIONS

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Abstract

In this paper, the notions of fuzzy pseudo perfectly continuous functions and fuzzy d_δ functions between fuzzy topological spaces, are introduced and studied. The fuzzy Baire resolvability, fuzzy non-hyper connectedness and fuzzy disconnected nesss of fuzzy topological spaces under fuzzy pseudo perfectly continuous functions, are explored. By means of fuzzy d_δ functions, the mathematical means under which fuzzy topological spaces becoming fuzzy Baire spaces, fuzzy second category space sand fuzzy weakly Volterra spaces, are established.

1. Introduction

The introduction of fuzzy sets by L. A. Zadeh [25] in 1965, as an approach to a mathematical representation of vagueness in everyday language, was realized by many researchers and has successfully been applied in every branch of Mathematics. Fuzzy topological spaces introduced by C. L. Chang [7] in 1968, had a significant role in the subsequent tremendous growth of the numerous fuzzy topological notions.

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Continuity is one of the most important and fundamental properties that have been widely used in mathematical analysis. In the recent years, a considerable amount of research has been done on various types of fuzzy continuity between fuzzy topological spaces. In 2013, the notion of pseudo perfectly continuous functions in classical topology was introduced and studied by J. K. Kohli et al. [8] .In this paper by means of fuzzy regular F_σ-sets, the notions of fuzzy pseudo perfectly continuous functions and fuzzy d_δ functions between fuzzy topological spaces, are introduced and studied. The fuzzy Baire resolvability, fuzzy non-hyper connectedness and fuzzy disconnectedness of fuzzy topological spaces under fuzzy pseudo perfectly continuous functions, are explored. The conditions for fuzzy d_δ functions to become fuzzy pseudo perfectly continuous functions between fuzzy topological spaces, are obtained. It is shown that the inverse images of fuzzy regular F_σ-sets in fuzzy topological spaces under fuzzy d_δ functions are fuzzy Baire dense sets in fuzzy pseudo P-spaces and the inverse images of fuzzy regular G_δ-sets in fuzzy topological spaces under fuzzy d_δ functions are fuzzy open sets in weak fuzzy P-spaces. Also it is shown that the inverse images of fuzzy regular F_σ-sets in fuzzy topological spaces under fuzzy d_δ functions are not fuzzy β-open and fuzzy σ-nowhere dense in weak fuzzy P-spaces. By means of fuzzy d_δ functions, the conditions under which fuzzy topological spaces becoming fuzzy Baire spaces, fuzzy second category spaces, fuzzy almost irresolvable spaces and fuzzy weakly Volterra spaces, are explored in this paper.

2. Preliminaries

For the purpose of having the exposition self- contained, some basic concepts and results used in the sequel, are presented. In this paper, (X, T) or simply X, means a fuzzy topological space due to Chang (1968). A fuzzy set λ defined in X, is a mapping from the set X into the unit interval $I = [0, 1]$. The fuzzy set 0_X will be defined as $0_X(x) = 0$, for all $x \in X$ and the fuzzy set 1_X will be defined as $1_X(x) = 1$, for all $x \in X$.

Definition 2.1 [7]**.** Let λ be any fuzzy set in the fuzzy topological space

(X, T). The fuzzy interior, the fuzzy closure and the fuzzy complement of λ are defined respectively as follows.

(i) $\text{Int}(\lambda) = \vee\{\mu / \mu \leq \lambda, \mu \in T\}$;

(ii) $\text{cl}(\lambda) = \wedge\{\mu / \lambda \leq \mu, 1 - \mu \in T\}$.

(iii) $\lambda'(x) = 1 - \lambda(x)$, for all $x \in X$.

Lemma 2.1 [2]. *For a fuzzy set λ of a fuzzy space X,*

(i) $1 - int(\lambda) = cl(1 - \lambda)$ *and*

(ii) $1 - cl(\lambda) = int(1 - \lambda)$.

Definition 2.2. A fuzzy set λ in a fuzzy topological space (X, T) is called

(i) **fuzzy regular-open set** in (X, T) if $\lambda = \text{int cl}(\lambda)$; **fuzzy regular-closed set** in (X, T) if $\lambda = \text{cl int}(\lambda)$ [2].

(ii) **fuzzy G_δ-set** in (X, T) if $\lambda = \wedge_{i=1}^{\infty}(\lambda_i)$, where $\lambda_i \in T$ for $i \in I$; **fuzzy F_σ-set** in (X, T) if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where $1 - \lambda_i \in T$ for $i \in I$ [4].

(iii) **fuzzy regular G_δ-set** in (X, T) if $\lambda = \wedge_{i=1}^{\infty}(\text{int}(\lambda_i))$, where $1 - \lambda_i \in T$; **fuzzy regular F_σ-set** in (X, T) if $\lambda = \vee_{i=1}^{\infty}(\text{cl}(\mu_i))$, where $\mu_i \in T$ [12].

(iv) **fuzzy β-open set** in (X, T) if $\lambda \leq \text{cl int cl}(\lambda)$ **fuzzy β-closed set** in (X, T) if $\text{int cl int}(\lambda) \leq \lambda$ [5].

(v) **fuzzy pre-open set** if $\lambda \leq \text{int cl}(\lambda)$ **fuzzy pre-closed set** if $\text{cl int}(\lambda) \leq \lambda$ [6].

Definition 2.3. Let λ be the fuzzy set in the fuzzy space (X, T). Then λ is called a

(i) **fuzzy dense set** if there exists no fuzzy closed set μ in (X, T) such that $\lambda < \mu < 1$. That is, $\text{cl}(\lambda) = 1$, in the fuzzy space (X, T) [13].

(ii) **fuzzy nowhere dense set** if there exists no non-zero fuzzy open set μ in (X, T) such that $\mu < \text{cl}(\lambda)$. That is, $\text{int}[\text{cl}(\lambda)] = 0$, in (X, T) [13].

(iii) **fuzzy first category set** if $\lambda = \vee_{i=1}^{\infty} (\lambda_i)$, where (λ_i)'s are fuzzy now here dense sets in (X, T). Any other fuzzy set in (X, T) is said to be of fuzzy second category [13].

(iv) **fuzzy Baire dense set** if for a non-zero fuzzy open set μ in (X, T) $\lambda \wedge \mu$ is an fuzzy second category set in (X, T) [18].

(v) **Fuzzy somewhere dense set** if $\text{int}\,\text{cl}(\lambda) \neq 0$ in (X, T) [14].

(vi) **fuzzy residual set** if $1 - \lambda$ is an fuzzy first category set in (X, T) [15].

(vii) **fuzzy σ-nowhere dense set** if λ is a fuzzy F_σ-set with $\text{int}(\lambda) = 0$, in (X, T) [16].

(viii) **fuzzy σ-first category set** if $\lambda = \vee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy σ-nowhere dense sets in (X, T). Any other fuzzy set in (X, T) is said to be of **fuzzy σ-second category** [16].

Definition 2.4 [20]**.** If λ is the fuzzy somewhere dense set in the fuzzy topological space (X, T), then the fuzzy set $1 - \lambda$ is called the fuzzy cs dense set in (X, T).

Definition 2.5. Let (X, T) be the fuzzy topological space and (X, T) is called.

(i) **Fuzzy Baire space** if $\text{int}(\vee_{i=1}^{\infty} (\lambda_i)) = 0$, where (λ_i)'s are fuzzy nowhere dense sets in (X, T) [15].

(ii) **fuzzy hyper- connected space** if every non- null fuzzy open subset of (X, T) is fuzzy dense in (X, T) [9].

(iii) **weak fuzzy *P*-space** if $\wedge_{i=1}^{\infty} (\lambda_i)$ is an fuzzy regular open set in (X, T) where (λ_i)'s are fuzzy regular open sets in (X, T) [17].

(iv) **fuzzy connected space** if it has no proper fuzzy cl open set [1].

(v) **fuzzy almost resolvable space** if $\vee_{i=1}^{\infty}(\lambda_i) = 1_X$, where the fuzzy sets (λ_i)'s in (X, T) are such that $\text{int}(\lambda_i) = 0$. Otherwise (X, T) is called an **fuzzy almost irresolvable space** [22].

(vi) **fuzzy extremally disconnected space** if closure of every fuzzy open set is an fuzzy open set in (X, T) [3].

(vii) fuzzy first category space if $1_X = \vee_{i=1}^{\infty}(\lambda_i)$, where (λ_i)'s are fuzzy nowhere dense sets in (X, T). A fuzzy topological space which is not of fuzzy first category is said to be of fuzzy second category [15].

(viii) **fuzzy almost *P*-space** if for every non-zero fuzzy G_δ-set λ in (X, T), $\text{int}(\lambda) \neq 0$ in (X, T) [17].

(ix) **fuzzy pseudo *P*-space** if it is both an fuzzy almost P-space and weak fuzzy P-space [23].

(x) **fuzzy Volterra space** if $(\wedge_{i=1}^{N}(\lambda_i)) = 1$, where the fuzzy sets (λ_i)'s are fuzzy dense and fuzzy G_δ-sets in (X, T) [12].

(xi) **fuzzy regular weakly Volterra space** if $\text{cl}(\wedge_{i=1}^{N}(\lambda_i)) = 1$, where the fuzzy sets (λ_i)'s are fuzzy dense and fuzzy regular G_δ-sets in (X, T) [21].

(xii) **fuzzy Baire resolvable space** if there exists an fuzzy Baire dense set λ in (X, T) such that $1 - \lambda$ is also an fuzzy Baire dense set in (X, T) [18].

Definition 2.6. Let (X, T) and (Y, S) be any two fuzzy topological spaces. The function $f : (X, T) \to (Y, S)$ is called an

(i) **fuzzy continuous function** if for the fuzzy open set λ in (Y, S), $f^{-1}(\lambda)$ is fuzzy open in (X, T) [7].

(ii) **slightly fuzzy continuous function** if for the fuzzy cl open set λ in (Y, S), $f^{-1}(\lambda)$ is an fuzzy open in (X, T) [11].

Theorem 2.1 [2]. *In a fuzzy space X,*

(a) *The closure of a fuzzy open set is a fuzzy regular closed set.*

(b) *The interior of a fuzzy closed set is a fuzzy regular open set.*

Theorem 2.2 [24]. *A fuzzy topological space* (X, T) *is a weak fuzzy P-space if and only if each fuzzy regular* F_σ*-set is a fuzzy regular closed set in* (X, T).

Theorem 2.3 [12]. *If* λ *is a fuzzy regular* G_δ*-set in a fuzzy topological space* (X, T) *if and only if* $1 - \lambda$ *is a fuzzy regular* F_σ*-set in* (X, T).

Theorem 2.4 [18]. *If* λ *is an fuzzy open set in the fuzzy Baire space* (X, T), *then* λ *is an fuzzy Baire dense set in* (X, T).

Theorem 2.5 [12]. *If* $\operatorname{int}(\lambda) = 0$, *for an fuzzy regular* F_σ*-set* λ *in an fuzzy topological space* (X, T), *then* λ *is an fuzzy first category set in* (X, T).

Theorem 2.6 [24]. *If* μ *is an fuzzy regular* F_σ-set *in an weak fuzzy P-space* (X, T), *then* μ *is an fuzzy closed set in* (X, T).

Theorem 2.7 [24]. *If* μ *is a fuzzy regular* F_σ*-set in a weak fuzzy P- space* (X, T), *then* $\operatorname{int}(\mu) \neq 0$ *in* (X, T).

Theorem 2.8 [23]. *If* λ *is an fuzzy regular* F_σ*-set in an fuzzy extremally disconnected and fuzzy pseudo P-space* (X, T), *then* λ *is an fuzzy clopen set in* (X, T).

Theorem 2.9 [23]. *If* μ *is an fuzzy regular* F_σ*-set in an fuzzy pseudo P-space* (X, T), *then* μ *is not an fuzzy first category set in* (X, T).

Theorem 2.10 [23]. *If* μ *is an fuzzy regular* F_σ*-set in an fuzzy pseudo P-space* (X, T), *then* μ *is an fuzzy Baire dense set in* (X, T).

Theorem 2.11 [18]. *If* λ *is an fuzzy Baire dense set in an fuzzy topological space* (X, T), *then there exists an fuzzy second category set* δ such that $\delta \leq \lambda$ *in* (X, T).

Theorem 2.12 [19]. *If* λ *is an fuzzy Baire dense set in the fuzzy topological space* (X, T), *then there is no fuzzy* F_σ*-set* μ, *with* $\text{int}(\mu) = 0$ *in* (X, T) *such that* $\lambda < \mu$.

Theorem 2.13 [18]. *If* λ *is an fuzzy Baire dense set in the fuzzy topological space* (X, T), *then there exist fuzzy second category sets* δ_1 and δ_2 *in* (X, T) *such that* $\delta_1 < \lambda < \delta_2$.

Theorem 2.14 [19]. *If* λ *is an fuzzy Baire dense set in the fuzzy topological space* (X, T), *then there is no fuzzy residual set* δ *in* (X, T) *such that* $1 - \lambda > \delta$.

Theorem 2.15 [19]. *If* λ *is an fuzzy somewhere dense set in the fuzzy hyper connected space* (X, T), *then* λ *is an fuzzy* β*-open set in* (X, T).

Theorem 2.16 [24]. *If* (X, T) *is an weak fuzzy P-space*, *then* (X, T) *is not an fuzzy hyper connected space*.

Theorem 2.17 [24]. *If* μ *is an fuzzy regular* F_σ*-set in the weak fuzzy P-space* (X, T), *then* μ *is not an fuzzy* σ*-nowhere dense set in* (X, T).

Theorem 2.18 [19]. *If there exists a fuzzy Baire dense set* λ *in a fuzzy topological space* (X, T) *such that* $\lambda < \mu$, *for each fuzzy open set* μ *in* (X, T), *then* (X, T) *is a fuzzy Baire space*.

Theorem 2.19 [15]. *If* (X, T) *is a fuzzy Baire space, then* (X, T) *is an fuzzy second category space*.

Theorem 2.20 [22]. *If* (X, T) *is a fuzzy Baire space*, *then* (X, T) is *an fuzzy almost irresolvable space*.

Theorem 2.21 [21]. *If the fuzzy topological space* (X, T) *is an fuzzy almost irresolvable space*, *then* (X, T) *is an fuzzy weakly Volterra space*.

Theorem 2.22 [12]. *If the fuzzy topological space* (X, T) *is an fuzzy weakly Volterra space*, *then* (X, T) *is an fuzzy regular weakly Volterra space*.

Theorem 2.23 [1]. *Let* (X, T) *be an fuzzy topological space*. *Then the following conditions are equivalent*:

(i) *X is disconnected.*

(ii) *There exists a non-empty proper fuzzy sub set of X which is both open and closed.*

Theorem 2.24 [3]. *For any fuzzy topological space* (X, T), *the following are equivalent*:

(a) *X is fuzzy extremally disconnected*

(b) *For each fuzzy closed set* λ, int (λ) *is fuzzy closed.*

(c) *For each fuzzy open set* λ, *we have* $cl(\lambda) + cl(1 - cl(\lambda)) = 1$,

(d) *For every pair of fuzzy open sets* λ, μ *in X* with $cl(\lambda) + \mu = 1$,

we have $cl(\lambda) + cl(\mu) = 1$.

3. Fuzzy Pseudo Perfectly Continuous Functions

Definition 3.1. Let (X, T) and (Y, S) be any two fuzzy topological spaces. The function $f : (X, T) \rightarrow (Y, S)$ is called an fuzzy pseudo perfectly continuous function, if for each fuzzy regular F_{σ}-set μ in (Y, S), $f^{-1}(\mu)$ is an fuzzy clopen set in (X, T).

Example 3.1. Consider the set $X = \{a, b, c\}$. Let $I = [0, 1]$. The fuzzy sets $\alpha, \beta, \gamma, \mu$ and δ are defined on X as follows.

$a : X \rightarrow [0, 1]$ is defined as $\alpha(a) = 0.6;\ \alpha(b) = 0.4;\ \alpha(c) = 0.6$,

$\beta : X \rightarrow [0, 1]$ is defined as $\beta(a) = 0.5;\ \beta(b) = 0.5;\ \beta(c) = 0.5$,

$\gamma : X \rightarrow [0, 1]$ is defined as $\gamma(a) = 0.4;\ \gamma(b) = 0.6;\ \gamma(c) = 0.4$,

$\mu : X \rightarrow [0, 1]$ is defined as $\mu(a) = 0.4;\ \mu(b) = 0.5;\ \mu(c) = 0.6$,

$\delta : X \rightarrow [0, 1]$ is defined as $\delta(a) = 0.5;\ \delta(b) = 0.6;\ \delta(c) = 0.4$,

Then, $T = \{0, \mu, \delta, \beta, \mu \vee \delta, \mu \vee \beta, \delta \vee \beta, \mu \wedge \delta, \mu \wedge \beta, \delta \wedge \beta, 1\}$ and $S = \{0, \alpha, \beta, \gamma, \alpha \vee \beta, \alpha \vee \gamma, \beta \vee \gamma, \alpha \wedge \beta, \alpha \wedge \gamma, \beta \wedge \gamma, 1\}$ are fuzzy topologies on X. On computation one will find that $\mathrm{cl}(\alpha) = 1 - \gamma = \alpha$; $\mathrm{cl}(\beta) = 1 - \beta$

$= \beta$; $\mathrm{cl}(\gamma) = 1 - \alpha = \gamma$; $\mathrm{cl}(\alpha \vee \beta) = 1 - [\beta \wedge \gamma] = \alpha \vee \beta$; $\mathrm{cl}(\alpha \vee \gamma) = 1 - [\alpha \wedge \gamma] = \alpha \vee \gamma$; $\mathrm{cl}(\beta \vee \gamma) = 1 - [\alpha \wedge \beta] = \beta \vee \gamma$; $\mathrm{cl}(\alpha \wedge \beta) = 1 - [\beta \vee \gamma] = \alpha \wedge \beta$; $\mathrm{cl}(\alpha \wedge \gamma) = 1 - [\alpha \vee \gamma] = \alpha \wedge \gamma$; $\mathrm{cl}(\beta \wedge \gamma) = 1 - [\alpha \vee \beta] = \beta \wedge \gamma$. Since the closure of fuzzy open sets are fuzzy regular closed sets in an fuzzy topological space, $\mathrm{cl}(\alpha), \mathrm{cl}(\beta), \mathrm{cl}(\gamma), \mathrm{cl}(\alpha \vee \beta), \mathrm{cl}(\alpha \vee \gamma), \mathrm{cl}(\beta \vee \gamma), \mathrm{cl}(\alpha \wedge \beta), \mathrm{cl}(\alpha \wedge \gamma), \mathrm{cl}(\beta \wedge \gamma)$ are fuzzy regular closed sets in (X, T). Also on computation, $\mathrm{cl}(\alpha) \vee \mathrm{cl}(\beta) \vee \mathrm{cl}(\gamma) \vee \mathrm{cl}(\alpha \vee \beta) \vee \mathrm{cl}(\alpha \vee \gamma) \vee \mathrm{cl}(\beta \vee \gamma) = \alpha \vee \gamma$ and $\mathrm{cl}(\alpha \vee \gamma) \vee \mathrm{cl}(\beta \vee \gamma) \vee \mathrm{cl}(\alpha \vee \beta) = \beta$, in (X, S). Hence $\alpha \vee \gamma$ and β are fuzzy regular F_σ-sets in (X, S).

Now define a function $f : (X, T) \to (X, S)$ by $f(a) = b$; $f(b) = c$; $f(c) = a$. By computation, for the non-zero fuzzy regular F_σ-set $\alpha \vee \gamma$ in (X, S), $f^{-1}(\alpha \vee \gamma) = \mu \vee \delta$. Since $\mu \vee \delta = 1 - [\mu \wedge \delta]$, $\mu \vee \delta$ is fuzzy closed in (X, T) Thus $f^{-1}(\alpha \vee \gamma)$ is both an fuzzy open and fuzzy closed set in (X, T) and for the non-zero fuzzy regular F_σ-set β in (X, S), $f^{-1}(\beta) = \beta$, which is both fuzzy open and fuzzy closed set in (X, T). Hence $f : (X, T) \to (X, S)$ is an fuzzy pseudo perfectly continuous function from (X, T) into (X, S).

Example 3.2. Consider the set $X = \{a, b, c\}$. Let $I = [0, 1]$. The fuzzy sets $\alpha, \beta, \gamma, \mu$ and δ are defined on X as follows:

$\alpha : X \to [0, 1]$ is defined as $\alpha(a) = 0.6$; $\alpha(b) = 0.4$; $\alpha(c) = 0.6$,

$\beta : X \to [0, 1]$ is defined as $\beta(a) = 0.5$; $\beta(b) = 0.5$; $\beta(c) = 0.5$,

$\gamma : X \to [0, 1]$ is defined as $\gamma(a) = 0.4$; $\gamma(b) = 0.6$; $\gamma(c) = 0.4$,

$\mu : X \to [0, 1]$ is defined as $\mu(a) = 0.4$; $\mu(b) = 0.5$; $\mu(c) = 0.6$,

$\delta : X \to [0, 1]$ is defined as $\delta(a) = 0.5$; $\delta(b) = 0.6$; $\delta(c) = 0.4$,

Then, $T = \{0, \mu, \delta, \mu \vee \delta, \mu \wedge \delta, 1\}$ and $S = \{0, \alpha, \beta, \gamma, \alpha \vee \beta, \alpha \vee \gamma, \beta \vee \gamma, \alpha \wedge \beta, \alpha \wedge \gamma, \beta \wedge \gamma, 1\}$ are fuzzy topologies on X. On computation as in example 3.1, $\alpha \vee \gamma$ and β are fuzzy regular F_σ-sets in (X, S). Now define a function $f : (X, T) \to (X, S)$ by $f(a) = b$; $f(b) = c$; $f(c) = a$. By computation, for the non-zero fuzzy regular F_σ-set $\alpha \vee \gamma$ in (X, S), $f^{-1}(\alpha \vee \gamma) = \mu \vee \delta$, which is both fuzzy open and fuzzy closed set in (X, T) and for the non-zero fuzzy regular F_σ-set β in (X, S), $f^{-1}(\beta) = \beta$ and β is not both fuzzy open and fuzzy closed set in (X, T). Hence the function $f : (X, T) \to (X, S)$ is not an fuzzy pseudo perfectly continuous. From (X, T) into (Y, S).

Proposition 3.1. *If $f : (X, T) \to (X, S)$ is an fuzzy pseudo perfectly continuous function from the fuzzy topological space (X, T) into the fuzzy topological space (Y, S) and μ is an fuzzy regular F_σ-set in (Y, S), then $f^{-1}(\mu)$ is an fuzzy somewhere dense set in (X, T).*

Proof. Let $f : (X, T) \to (Y, S)$ be an fuzzy pseudo perfectly continuous function from (X, T) into (Y, S) and μ be an fuzzy regular F_σ-set in (Y, S). Then, $f^{-1}(\mu)$ is an fuzzy clopen set in (X, T). This implies that $\operatorname{cl}[f^{-1}(\mu)] = f^{-1}(\mu)$ and $\operatorname{int}[f^{-1}(\mu)] = f^{-1}(\mu)$ and hence $\operatorname{int}\operatorname{cl}[f^{-1}(\mu)] = \operatorname{int}[f^{-1}(\mu)] = f^{-1}(\mu) \neq 0$, in (X, T). This shows that $f^{-1}(\mu)$ is an fuzzy somewhere dense set in (X, T).

Proposition 3.2. *If $f : (X, T) \to (Y, S)$ is an fuzzy pseudo perfectly continuous function from the fuzzy topological space (X, T) into the fuzzy topological space (Y, S) and μ is an fuzzy regular F_σ-set in (Y, S), then there exists an fuzzy regular closed set η in (X, T) such that $\eta \leq \operatorname{cl}[f^{-1}(\mu)]$.*

Proof. Let $f : (X, T) \to (Y, S)$ be an fuzzy pseudo perfectly continuous function from (X, T) into (Y, S) and μ be an fuzzy regular F_σ-set in (Y, S). Then, by the proposition 3.1, $f^{-1}(\mu)$ is an fuzzy somewhere dense set in (X, T). Then, $\operatorname{int}\operatorname{cl}[f^{-1}(\mu)] \neq 0$, in (X, T) and this implies that there exist a non-zero fuzzy open set δ in (X, T) such that $\delta \leq \operatorname{cl}[f^{-1}(\mu)]$. Now $\operatorname{cl}(\delta) \leq \operatorname{cl}(\operatorname{cl}[f^{-1}(\mu)]) = \operatorname{cl}[f^{-1}(\mu)]$. That is, $\operatorname{cl}(\delta) \leq \operatorname{cl}[f^{-1}(\mu)]$. Since δ is an fuzzy open set in (X, T), by the theorem 2.1, the closure of δ is an fuzzy regular closed set in (X, T). Let $\operatorname{cl}(\delta) = \eta$. Hence, for the fuzzy regular F_σ-

set in (Y, S), there exists an fuzzy regular closed set η in (X, T) such that $\eta \leq \text{cl}\,[f^{-1}(\mu)]$.

Proposition 3.3. *If* $f : (X, T) \to (Y, S)$ *is an fuzzy pseudo perfectly continuous function from the fuzzy topological space* (X, T) *into the fuzzy topological space* (Y, S) *and* λ *is an fuzzy regular* G_δ*-set in* (Y, S), *then* $f^{-1}(\mu)$ *is an fuzzy cs dense set in* (X, T).

Proof. Let λ be an fuzzy regular G_δ-set in (Y, S). Then, by the theorem 2.3, $1 - \lambda$ is an fuzzy regular F_σ-set in (Y, S). Since $f : (X, T) \to (Y, S)$ is an fuzzy pseudo perfectly continuous function from (X, T) into (Y, S), by the proposition 3.1, $f^{-1}(1 - \lambda.)$ is an fuzzy somewhere dense set in (X, T). This implies that $1 - f^{-1}(\lambda)$ is an fuzzy somewhere dense set in (X, T) band thus $1 - [1 - f^{-1}(\lambda)]$ is an fuzzy cs dense set in (X, T). This gives that $f^{-1}(\lambda)$ is an fuzzy cs dense set in (X, T).

Proposition 3.4. *If* $f : (X, T) \to (Y, S)$ *is an fuzzy pseudo perfectly continuous function from the fuzzy topological space* (X, T) *into the fuzzy topological space* (Y, S) *and* λ *is an fuzzy regular* G_δ*-set in* (Y, S), *then* $\text{cl int}[f^{-1}(\lambda)] \neq 1$ *in* (X, T).

Proof. Let $f : (X, T) \to (Y, S)$ be an fuzzy pseudo perfectly continuous function from (X, T) into (Y, S) and λ be an fuzzy regular G_δ-set in (Y, S). By the proposition 3.3, $f^{-1}(\lambda)$ is an fuzzy cs dense set in (X, T) and then, $1 - f^{-1}(\lambda)$ is an fuzzy somewhere dense set in (X, T). This implies that $\text{int cl}\,[1 - f^{-1}(\lambda)] \neq 0$ and hence $1 - \text{cl int}\,[f^{-1}(\lambda)] \neq 0$ in (X, T). Thus, $\text{cl int}\,[f^{-1}(\lambda)] \neq 1$ in (X, T).

Proposition 3.5. *If* $f : (X, T) \to (Y, S)$ *is an fuzzy pseudo perfectly continuous function from the fuzzy topological space* (X, T) *into the fuzzy topological space* (Y, S) *and* λ *is an fuzzy regular* G_δ*-set in* (Y, S), *then there exists a fuzzy closed set* μ *in* (X, T) *such that* $\text{int}\,[f^{-1}(\lambda)] \leq \mu$.

Proof. Let λ be an fuzzy regular G_δ-set in (Y, S). Then, by the proposition 3.4, $\mathrm{cl}\,\mathrm{int}[f^{-1}(\lambda)] \neq 1$ in (X, T). Then, $\mathrm{int}[f^{-1}(\lambda)]$ is not an fuzzy dense set in (X, T) and hence there exists a fuzzy closed set μ in (X, T) such that $\mathrm{int}\,[f^{-1}(\lambda)] \leq \mu$.

Proposition 3.6. *If* $f : (X, T) \to (Y, S)$ *is an fuzzy pseudo perfectly continuous function from the fuzzy topological space* (X, T) *in to the fuzzy topological space* (Y, S), *then* (X, T) *is not an fuzzy hyper connected space.*

Proof. Let $f : (X, T) \to (Y, S)$ be an fuzzy pseudo perfectly continuous function from (X, T) into (Y, S) and λ be an fuzzy regular G_δ-set in (Y, S). Then, by the proposition 3.4, $\mathrm{cl}\,\mathrm{int}[f^{-1}(\lambda)] \neq 1$ in (X, T). Now $\mathrm{int}\,[f^{-1}(\lambda)]$ is an fuzzy open set in (X, T). Thus, $\mathrm{cl}\,\mathrm{int}[f^{-1}(\lambda)] \neq 1$ shows that the fuzzy open set $\mathrm{int}\,[f^{-1}(\lambda)]$, is not an fuzzy dense set in (X, T) and hence (X, T) is not an fuzzy hyper connected space.

Proposition 3.7. *If* $f : (X, T) \to (Y, S)$ *is an fuzzy pseudo perfectly continuous function from the fuzzy topological space* (X, T) *into the fuzzy topological space* (Y, S) *and* μ *is an fuzzy regular* F_σ*-set in* (Y, S), *then* *int* $[f^{-1}(\mu)] \neq 0$, *in* (X, T).

Proof. Let $f : (X, T) \to (Y, S)$ be an fuzzy pseudo perfectly continuous function from (X, T) into (Y, S) and μ be an fuzzy regular F_σ-set in (Y, S). Then, $f^{-1}(\mu)$ is an fuzzy clopen set in (X, T). This implies that $\mathrm{cl}\,[f^{-1}(\mu)] = f^{-1}(\mu)$ and $\mathrm{int}\,[f^{-1}(\mu)] = f^{-1}(\mu)$. Now $f^{-1}(\mu) \neq 0$, implies that $\mathrm{int}\,[f^{-1}(\mu)] \neq 0$, in (X, T).

Proposition 3.8. *If* $f : (X, T) \to (Y, S)$ *is the fuzzy continuous function from the fuzzy topological space* (X, T) *into the fuzzy topological space* (Y, S) *and* $g : (Y, S) \to (Z, W)$ *is the pseudo perfectly continuous function from* (Y, S) *into the fuzzy topological space* (Z, W), *then* $g \circ f : (X, T) \to (Z, W)$ *is an fuzzy pseudo perfectly continuous function from* (X, T) *into* (Z, W).

Proof. Let μ be the non-zero fuzzy regular F_σ-set in (Z, W). Since g is the fuzzy pseudo perfectly continuous function from (Y, S) into (Z, W), $g^{-1}(\mu)$ is an fuzzy clopen set in (Y, S). Since f is the fuzzy continuous function from (X, T) into (Y, S), $f^{-1}(g^{-1}(\mu))$ is an fuzzy clopen set in (X, T). Now $(g \circ f)^{-1}(\mu) = f^{-1}(g^{-1}(\mu))$, implies that $(g \circ f)^{-1}(\mu)$ is an fuzzy clopen set in (X, T), for the fuzzy regular F_σ-set μ in (Z, W). Therefore $g \circ f$ is an fuzzy pseudo perfectly continuous function from (X, T) into (Z, W).

Proposition 3.9. *If* $f : (X, T) \to (Y, S)$ *is an fuzzy pseudo perfectly continuous function from the fuzzy topological space* (X, T) *into the weak fuzzy P-space* (Y, S) *then* $f : (X, T) \to (Y, S)$ *is an fuzzy continuous function from* (X, T) *into* (Y, S).

Proof. Let $f : (X, T) \to (Y, S)$ be an fuzzy pseudo perfectly continuous function from (X, T) into (Y, S) and μ be an fuzzy regular F_σ-set in (Y, S). Then, $f^{-1}(\mu)$ is an fuzzy clopen set in (X, T). Since (Y, S) is an fuzzy weak fuzzy P-space, by the theorem 2.6, the fuzzy regular F_σ-set μ is an fuzzy closed set in (Y, S). Thus, $f^{-1}(\mu)$ is an fuzzy closed set in (X, T), for the fuzzy closed set μ in (Y, S) and hence $f : (X, T) \to (Y, S)$ is an fuzzy continuous function from (X, T) into (Y, S).

Proposition 3.10. *If* $f : (X, T) \to (Y, S)$ *is an fuzzy function from the fuzzy connected space* (X, T) *into the fuzzy topological space* (Y, S), *then the function* $f : (X, T) \to (Y, S)$ *is not an fuzzy pseudo perfectly continuous function from* (X, T) *into* (Y, S).

Proof. Let $f : (X, T) \to (Y, S)$ be an fuzzy function from the fuzzy connected space (X, T) into the fuzzy topological space (Y, S) and μ be an fuzzy regular F_σ-set in (Y, S). Suppose that $f : (X, T) \to (Y, S)$ is an fuzzy pseudo perfectly continuous function from (X, T) into (Y, S). Then, for the fuzzy regular F_σ-set μ in (Y, S), $f^{-1}(\mu)$ is an fuzzy clopen set in (X, T). But this is a contradiction to (X, T) being an fuzzy connected space which has no proper fuzzy clopen set. Hence the function $f : (X, T) \to (Y, S)$ is not an fuzzy pseudo perfectly continuous function from (X, T) into (Y, S).

Proposition 3.11. *If* $f : (X, T) \to (Y, S)$ *is an fuzzy pseudo perfectly continuous function from the fuzzy Baire space* (X, T) *into the fuzzy topological space* (Y, S), *then* $f^{-1}(\mu)$ *is an fuzzy Baire dense and fuzzy closed set in* (X, T), *for the fuzzy regular* F_σ*-set* μ *in* (Y, S).

Proof. Let $f : (X, T) \to (Y, S)$ be an fuzzy pseudo perfectly continuous function from (X, T) into (Y, S) and μ be an fuzzy regular F_σ-set in (Y, S). Then, $f^{-1}(\mu)$ is an fuzzy clopen set in (X, T). This implies that $f^{-1}(\mu)$ is an fuzzy open and fuzzy closed set in (X, T). Since (X, T) is an fuzzy Baire space, by the theorem 2.4, the fuzzy open set $f^{-1}(\mu)$ is an fuzzy Baire dense set in (X, T). Hence $f^{-1}(\mu)$ is an fuzzy Baire dense and fuzzy closed set in (X, T), for the fuzzy regular F_σ-set μ in (Y, S).

Proposition 3.12. *If* $f : (X, T) \to (Y, S)$ *is an fuzzy pseudo perfectly continuous function from the fuzzy Baire space* (X, T) *into the fuzzy topological space* (Y, S), *then there exists a fuzzy second category set* δ *in* (X, T). *such that* $\delta \leq f^{-1}(\mu)$ *such that* $\mathrm{cl}[\delta] \neq 1$, *for the fuzzy regular* F_σ*-set* μ *in* (Y, S).

Proof. Let $f : (X, T) \to (Y, S)$ be an fuzzy pseudo perfectly continuous function from the fuzzy Baire space (X, T) into (Y, S) and μ be an fuzzy regular F_σ-set in (Y, S). Then, by the proposition 3.11, $f^{-1}(\mu)$ is an fuzzy Baire dense and fuzzy closed set in (X, T). Since $f^{-1}(\mu)$ is an fuzzy Baire dense set in (X, T), by the theorem 2.11, there exists an fuzzy second category set δ in (X, T) such that $\delta \leq f^{-1}(\mu)$ in (X, T). Now $\delta \leq f^{-1}(\mu)$, implies that $\mathrm{cl}[\delta] \leq \mathrm{cl}[f^{-1}(\mu)] = f^{-1}(\mu)$ and hence $\mathrm{cl}[\delta] \neq 1$, in (X, T).

Proposition 3.13. *If* $f : (X, T) \to (Y, S)$ *is an fuzzy pseudo perfectly continuous function from the fuzzy Baire space* (X, T) *into the fuzzy topological space* (Y, S), *then there exist fuzzy second category sets* δ_1 *and* δ_2 *in* (X, T) *such that* $\delta_1 \leq f^{-1}(\mu) < \delta_2$, *for the fuzzy regular* F_σ*-set* μ *in* (Y, S).

Proof. Let $f : (X, T) \to (Y, S)$ be an fuzzy pseudo perfectly continuous function from the fuzzy Baire space (X, T) into (Y, S) and μ be an fuzzy regular F_σ-set in (Y, S). Then, by the proposition 3.11, $f^{-1}(\mu)$ is an fuzzy Baire dense and fuzzy closed set in (X, T). Since $f^{-1}(\mu)$ is an fuzzy Baire dense set in (X, T), by the theorem 2.13, there exist fuzzy second category sets δ_1 and δ_2 in (X, T) such that $\delta_1 \leq f^{-1}(\mu) < \delta_2$, for the fuzzy regular F_σ-set μ in (Y, S).

Proposition 3.14. *If* $f : (X, T) \to (Y, S)$ *is an fuzzy pseudo perfectly continuous function from the fuzzy Baire space* (X, T) *into the fuzzy topological space* (Y, S), *then there is no fuzzy residual set* δ *in* (X, T) *such that* $1 - f^{-1}(\mu) > \delta$, *for the fuzzy regular* F_σ*-set* μ *in* (Y, S).

Proof. Let $f : (X, T) \to (Y, S)$ be an fuzzy pseudo perfectly continuous function from the fuzzy Baire space (X, T) into (Y, S) and μ be an fuzzy regular F_σ-set in (Y, S). Then, by the proposition 3.11, $f^{-1}(\mu)$ is an fuzzy Baire dense and fuzzy closed set in (X, T). Since $f^{-1}(\mu)$ is an fuzzy Baire dense set in (X, T), by the theorem 2.14, there is no fuzzy residual set δ in (X, T) such that $1 - f^{-1}(\mu) > \delta$.

Proposition3.15. *If* $f : (X, T) \to (Y, S)$ *is an fuzzy pseudo perfectly continuous function from the fuzzy Baire space* (X, T) *into the fuzzy topological space* (Y, S), *then* (X, T) *is an fuzzy Baire resolvable space.*

Proof. Let $f : (X, T) \to (Y, S)$ be an fuzzy pseudo perfectly continuous function from the fuzzy Baire space (X, T) into (Y, S) and μ be an fuzzy regular F_σ-set in (Y, S). Then, $f^{-1}(\mu)$ is an fuzzy clopen set in (X, T). This implies that $f^{-1}(\mu)$ is an fuzzy open set and fuzzy closed set in (X, T). Since (X, T) is an fuzzy Baire space, by the theorem 2.4, the fuzzy open set $f^{-1}(\mu)$ is an fuzzy Baire dense set in (X, T). Since $f^{-1}(\mu)$ is an fuzzy closed set, $1-f^{-1}(\mu)$ is an fuzzy open set and by the theorem 2.4, the fuzzy open set $1-f^{-1}(\mu)$ is an fuzzy Baire dense set in (X, T). This implies that there exists an fuzzy Baire dense set $f^{-1}(\mu)$ in (X, T) such that $1-f^{-1}(\mu)$ is also an fuzzy Baire dense set in (X, T) and hence (X, T) is an fuzzy Baire resolvable space.

Proposition 3.16. *If $f:(X, T) \to (Y, S)$ is an fuzzy pseudo perfectly continuous function from the fuzzy Baire space (X, T) into the fuzzy topological space (Y, S), then there exist fuzzy second category sets δ_1 and δ_2 in (X, T) such that $\delta_1 \leq 1-\delta_2$.*

Proof. Let $f:(X, T) \to (Y, S)$ be an fuzzy pseudo perfectly continuous function from the fuzzy Baire space (X, T) into the fuzzy topological space (Y, S). Then, by the proposition 3.15, there exists an fuzzy Baire dense set $f^{-1}(\mu)$ in (X, T) such that $1-f^{-1}(\mu)$ is also an fuzzy Baire dense set in (X, T). Since $f^{-1}(\mu)$ is an fuzzy Baire dense set in (X, T), by the theorem 2.11, there exists an fuzzy second category set δ_1 in (X, T) such that $\delta_1 \leq f^{-1}(\mu)$ in (X, T). Also since $1-f^{-1}(\mu)$ is an fuzzy Baire dense set in (X, T), by the theorem 2.11, there exist san fuzzy second category set δ_2 in (X, T) such that $\delta_2 \leq 1-f^{-1}(\mu)$ in (X, T). Then, $f^{-1}(\mu) \leq 1-\delta_2$. This implies that $\delta_1 \leq 1-\delta_2$. in (X, T).

Proposition 3.17. *If $f:(X, T) \to (Y, S)$ is an fuzzy pseudo perfectly continuous function from the fuzzy topological space (X, T) into the fuzzy topological space (Y, S), then (X, T) is not an connected space.*

Proof. Let $f:(X, T) \to (Y, S)$ be an fuzzy pseudo perfectly continuous function from (X, T) into (Y, S) and μ be an fuzzy regular F_σ-set in (Y, S).

Then, $f^{-1}(\mu)$ is an fuzzy clopen set in (X, T). Thus, there exists a non-empty proper fuzzy subset $f^{-1}(\mu)$ of X which is both open and closed in (X, T). This implies by the theorem 2.23, that (X, T) is disconnected and hence (X, T) is not an connected space.

Proposition 3.18. *If* $f : (X, T) \to (Y, S)$ *is an fuzzy pseudo perfectly continuous function from the fuzzy extremally disconnected space* (X, T) *into the fuzzy topological space* (Y, S), *then for the fuzzy regular* F_σ*-set* μ *in* (Y, S), $f^{-1}(\mu)$ *is an fuzzy pre closed and fuzzy pre open set in* (X, T).

Proof. Let $f : (X, T) \to (Y, S)$ be an fuzzy pseudo perfectly continuous function from (X, T) into (Y, S) and μ be an fuzzy regular F_σ-set in (Y, S). Then, $f^{-1}(\mu)$ is an fuzzy clopen set in (X, T). Thus, there exists a non-empty proper fuzzy subset $f^{-1}(\mu)$ of (X, T) which is both open and closed in (X, T).

Now $f^{-1}(\mu)$ is the fuzzy closed set in the fuzzy extremally disconnected space (X, T). Then ,by the theorem 2.24 (ii), $\operatorname{int}[f^{-1}(\mu)]$ is fuzzy closed and hence $\operatorname{cl}(\operatorname{int}[f^{-1}(\mu)]) = \operatorname{int}[f^{-1}(\mu)]$, in (X, T). This implies that $\operatorname{cl}(\operatorname{int}[f^{-1}(\mu)]) = \operatorname{int}[f^{-1}(\mu)] \leq f^{-1}(\mu)$. Thus, $\operatorname{cl}\operatorname{int}[f^{-1}(\mu)] \leq f^{-1}(\mu)$ and hence $f^{-1}(\mu)$ is an fuzzy pre closed set in (X, T).

(ii) Now $f^{-1}(\mu)$ is the fuzzy open set in the fuzzy extremally disconnected space (X, T). Then, by the theorem 2.24 (iii), $\operatorname{cl}(f^{-1}(\mu)) + \operatorname{cl}((1 - \operatorname{cl}(f^{-1}(\mu))) = 1$, in (X, T). This implies that $\operatorname{cl}(f^{-1}(\mu)) + 1 - \operatorname{int}\operatorname{cl}(f^{-1}(\mu) = 1$ and hence $\operatorname{int}\operatorname{cl}(f^{-1}(\mu)) = \operatorname{cl}(f^{-1}(\mu) \geq (f^{-1}(\mu))$. Then, $\operatorname{int}\operatorname{cl}(f^{-1}(\mu) \geq f^{-1}(\mu)$. Thus $f^{-1}(\mu)$ is an fuzzy pre open set in (X, T). Thus, for the fuzzy regular F_σ-set μ in (Y, S), $f^{-1}(\mu)$ is an fuzzy pre closed and fuzzy pre open set in (X, T).

Proposition 3.19. *If* $f : (X, T) \to (Y, S)$ *is the slightly fuzzy continuous function from the fuzzy topological space* (X, T) *into the fuzzy topological* *space* (Y, S) *and* $g : (Y, S) \to (Z, W)$ *is the pseudo perfectly continuous function from* (Y, S) *into the fuzzy topological space* (Z, W), *then* $(g \circ f)^{-1}(\mu)$ *is an fuzzy open set in* (X, T), *for the fuzzy regular* F_σ*-set* μ *in* (Z, W).

Proof. Let μ be the non-zero fuzzy regular F_σ-set in (Z, W). Since g is the fuzzy pseudo perfectly continuous function from (Y, S) into (Z, W), $g^{-1}(\mu)$ is an fuzzy clopen set in (Y, S). Since f is the slightly fuzzy continuous function from (X, T) into (Y, S), $f^{-1}(g^{-1}(\mu))$ is an fuzzy open set in (X, T). Now $(g \circ f)^{-1}(\mu) = f^{-1}(g^{-1}(\mu))$, implies that $(g \circ f)^{-1}(\mu)$ is an fuzzy open set in (X, T), for the fuzzy regular F_σ-set μ in (Z, W).

4. Fuzzy d_δ Functions

Definition 4.1. Let (X, T) and (Y, S) be any two fuzzy topological spaces. The function $f : (X, T) \to (Y, S)$ is called an fuzzy d_δ function, if for each fuzzy regular F_σ-set μ in (Y, S), $f^{-1}(\mu)$ is an fuzzy regular F_σ-set in (X, T).

Example 4.1. Consider the set $X = \{a, b, c\}$. Let $I = [0, 1]$. The fuzzy sets α, β, λ and μ are defined on X as follows:

$a : X \to [0, 1]$ is defined as $\alpha(a) = 0.5$; $\alpha(b) = 0.5$; $\alpha(c) = 0.4$,

$\beta : X \to [0, 1]$ is defined as $\beta(a) = 0.5$; $\beta(b) = 0.5$; $\beta(c) = 0.6$,

$\lambda : X \to [0, 1]$ is defined as $\lambda(a) = 0.5$; $\lambda(b) = 0.4$; $\lambda(c) = 0.5$,

$\mu : X \to [0, 1]$ is defined as $\mu(a) = 0.5$; $\mu(b) = 0.6$; $\mu(c) = 0.5$.

Then, $T = \{0, \alpha, \beta, 1\}$ and $S = \{0, \lambda, \mu, 1\}$ are fuzzy topologies on X. On computation one will find that $\mathrm{cl}(\alpha) = 1 - \beta = \alpha$; $\mathrm{cl}(\beta) = 1 - \alpha = \beta$. Since the closure of fuzzy open sets are fuzzy regular closed sets in an fuzzy topological space, $\mathrm{cl}(\alpha), \mathrm{cl}(\beta)$, are fuzzy regular closed sets in (X, T) and $\beta = \mathrm{cl}(\alpha) \vee \mathrm{cl}(\beta) = \alpha \vee \beta$ and β is an fuzzy regular F_σ-set in (X, T). Also on computation $\text{cl}(\lambda) = 1 - \mu = \lambda$; $\text{cl}(\mu) = 1 - \lambda = \mu$ and $\text{cl}(\lambda), \text{cl}(\mu)$ are fuzzy regular closed sets in (X, S) and $\mu = \text{cl}(\lambda) \vee \text{cl}(\mu)$ and μ is an fuzzy regular F_σ-set in (X, S).

Now define the function $f : (X, T) \to (X, S)$ by $f(a) = a$; $f(b) = c$; $f(c) = b$. By computation, for the non-zero fuzzy regular F_σ-set μ in (X, S), $f^{-1}(\mu) = \beta$, which is an fuzzy regular F_σ-set in (X, T). Hence the function $f : (X, T) \to (X, S)$ is an fuzzy d_δ function from (X, T) into (X, S).

Proposition 4.1. *If $f : (X, T) \to (Y, S)$ is the fuzzy pseudo perfectly continuous function from the fuzzy topological space (X, T) into the fuzzy topological space (Y, S) and $g : (Y, S) \to (Z, W)$ is an fuzzy d_δ function from (Y, S) into the fuzzy topological space (Z, W), then $g \circ f : (X, T) \to (Z, W)$ is an fuzzy pseudo perfectly continuous function from (X, T) into (Z, W).*

Proof. Let μ be the non-zero fuzzy regular F_σ-set in (Z, W). Since g is the fuzzy d_δ function from (Y, S) into (Z, W), $g^{-1}(\mu)$ is an fuzzy regular F_σ-set in (Y, S). Since f is the fuzzy pseudo perfectly continuous function from (X, T) into (Y, S), $f^{-1}(g^{-1}(\mu))$ is an fuzzy clopen set in (X, T). Now $(g \circ f)^{-1}(\mu) = f^{-1}(g^{-1}(\mu))$ implies that $(g \circ f)^{-1}(\mu)$ is an fuzzy clopen set in (X, T), for the fuzzy regular F_σ-set μ in (Z, W). Therefore $g \circ f$ is an fuzzy pseudo perfectly continuous function from (X, T) into (Z, W).

Definition 4.2. Let (X, T) be the fuzzy topological space and (X, T) is called fuzzy pseudo hyper connected space if there exists no non-zero fuzzy regular G_δ-set in (X, T) or equivalently there exists no non-zero fuzzy regular F_σ-set in (X, T).

Proposition 4.2. *If $f : (X, T) \to (Y, S)$ is an fuzzy d_δ function from the fuzzy topological space (X, T) into the fuzzy topological space (Y, S), then (X, T) is not an fuzzy pseudo hyper connected space.*

Proof. Let $f : (X, T) \to (Y, S)$ be an fuzzy d_δ function from (X, T) into (Y, S) and μ be an fuzzy regular F_σ-set in (Y, S). Then $f^{-1}(\mu)$ is an fuzzy regular F_σ-set in (X, T) and hence (X, T) is not an fuzzy pseudo hyper connected space.

Proposition 4.3. *If* $f : (X, T) \to (Y, S)$ *is an fuzzy* d_δ *function from the weak fuzzy P-space* (X, T) *into the fuzzy topological space* (Y, S) *and* μ *is an fuzzy regular* F_σ*-set in* (Y, S), *then* $f^{-1}(\mu)$ *is an fuzzy closed and fuzzy regular* F_σ*-set in* (X, T).

Proof. Let $f : (X, T) \to (Y, S)$ be an fuzzy d_δ function from (X, T) into (Y, S) and μ be an fuzzy regular F_σ-set in (Y, S). Then, $f^{-1}(\mu)$ is an fuzzy regular F_σ-set in (X, T). Since (X, T) is an weak fuzzy P-space, by the theorem 2.6, the fuzzy regular F_σ-set $f^{-1}(\mu)$ is an fuzzy closed set in (X, T). Thus, $f^{-1}(\mu)$ is an fuzzy closed and fuzzy regular F_σ-set in (X, T).

The following proposition shows that the inverse images of fuzzy regular F_σ-sets under fuzzy d_δ functions are fuzzy somewhere dense sets in weak fuzzy P-spaces.

Proposition 4.4. *If* $f : (X, T) \to (Y, S)$ *is an fuzzy* d_δ *function from the weak fuzzy P-space* (X, T) *into the fuzzy topological space* (Y, S) *and* μ *is an fuzzy regular* F_σ*-set in* (Y, S), *then* $f^{-1}(\mu)$ *is an fuzzy somewhere dense set in* (X, T).

Proof. Let $f : (X, T) \to (Y, S)$ be an fuzzy d_δ function from (X, T) into (Y, S) and μ be an fuzzy regular F_σ-set in (Y, S). Then, by the proposition 4.3, $f^{-1}(\mu)$ is an fuzzy closed and fuzzy regular F_σ-set in (X, T). Since (X, T) is an weak fuzzy P-space , by the theorem 2.7, for the fuzzy regular F_σ-set $f^{-1}(\mu)$ in (Y, S), int $(f^{-1}(\mu)) \neq 0$. Since $\text{int}\,(f^{-1}(\mu)) \leq \text{int cl}\,(f^{-1}(\mu))$, $\text{int cl}\,(f^{-1}(\mu)) \neq 0$, in (X, T). Hence $f^{-1}(\mu)$ is an fuzzy somewhere dense set in (X, T).

The following proposition shows that the inverse images of fuzzy regular G_δ-sets under fuzzy d_δ functions are fuzzy open sets in weak fuzzy P-spaces.

Proposition 4.5. If $f : (X, T) \to (Y, S)$ is an fuzzy d_δ function from the weak fuzzy P-space (X, T) into the fuzzy topological space (Y, S) and γ is an fuzzy regular G_δ-set in (Y, S), then $f^{-1}(\gamma)$ is an fuzzy regular G_δ-set and fuzzy open set in (X, T).

Proof. Let $f : (X, T) \to (Y, S)$ be an fuzzy d_δ function from (X, T) into (Y, S) and γ be an fuzzy regular G_δ-set in (Y, S). Then $1 - \gamma$ is an fuzzy regular F_σ-set in (Y, S) and hence $f^{-1}(1 - \gamma)$ is an fuzzy regular F_σ-set in (X, T). Since (X, T) is an weak fuzzy P-space, by the theorem 2.6, the fuzzy regular F_σ-set $f^{-1}(1 - \gamma)$ is an fuzzy closed set in (X, T). Since $f^{-1}(1 - \gamma) = 1 - f^{-1}(\gamma)$, $1 - f^{-1}(\gamma)$ is an fuzzy closed set in (X, T) and hence $f^{-1}(\gamma)$ is an fuzzy open set in (X, T). Also $1 - f^{-1}(\gamma)$ is an fuzzy regular F_σ-set, implies that $f^{-1}(\gamma)$ is an fuzzy regular G_δ-set in (X, T). Thus $f^{-1}(\gamma)$ is an fuzzy regular G_δ-set and fuzzy open set in (X, T).

The following proposition gives a condition for fuzzy d_δ functions to become fuzzy pseudo perfectly continuous functions between fuzzy topological spaces.

Proposition 4.6. *If $f : (X, T) \to (Y, S)$ is an fuzzy d_δ function from the fuzzy extremally disconnected and fuzzy pseudo P-space (X, T) into the fuzzy topological space (Y, S), then $f : (X, T) \to (Y, S)$ is an fuzzy pseudo perfectly continuous function from (X, T) into (Y, S).*

Proof. Let $f : (X, T) \to (Y, S)$ be an fuzzy d_δ function from (X, T) into (Y, S) and μ be an fuzzy regular F_σ-set in (Y, S). Then, $f^{-1}(\mu)$ is an fuzzy regular F_σ-set in (X, T). Since (X, T) is an fuzzy extremally disconnected and fuzzy pseudo P-space, by the theorem 2.8, the fuzzy regular F_σ-set $f^{-1}(\mu)$ is an fuzzy clopen set in (X, T). Thus, $f^{-1}(\mu)$ is an fuzzy clopen set in

(X, T), for the regular F_σ-set μ in (Y, S), implies that $f : (X, T) \to (Y, S)$ is an fuzzy pseudo perfectly continuous function from (X, T) into (Y, S).

Proposition 4.7. *If* $f : (X, T) \to (Y, S)$ *is an fuzzy* d_δ *function from the fuzzy pseudo P-space* (X, T) *into the fuzzy topological space* (Y, S) *and* μ *is an fuzzy regular* F_σ*-set in* (Y, S), *then* $f^{-1}(\mu)$ *is an fuzzy second category set in* (X, T).

Proof. Let $f : (X, T) \to (Y, S)$ be an fuzzy d_δ function from (X, T) into (Y, S) and μ be an fuzzy regular F_σ-set in (Y, S). Then, $f^{-1}(\mu)$ is an fuzzy regular F_σ-set in (X, T). Since (X, T) is an fuzzy pseudo P-space, by the theorem 2.9, the fuzzy regular F_σ-set $f^{-1}(\mu)$ is not an fuzzy first category set in (X, T). Hence $f^{-1}(\mu)$ is an fuzzy second category set in (X, T).

The following proposition shows that the inverse images of fuzzy regular F_σ-sets under fuzzy d_δ functions are fuzzy Baire dense sets in fuzzy pseudo P-spaces.

Proposition 4.8. *If* $f : (X, T) \to (Y, S)$ *is an fuzzy* d_δ *function from the fuzzy pseudo P-space* (X, T) *into the fuzzy topological space* (Y, S) *and* μ *is an fuzzy regular* F_σ*-set in* (Y, S), *then* $f^{-1}(\mu)$ *is an fuzzy Baire dense set in* (X, T).

Proof. Let $f : (X, T) \to (Y, S)$ be an fuzzy d_δ function from (X, T) into (Y, S) and μ be an fuzzy regular F_σ-set in (Y, S). Then, $f^{-1}(\mu)$ is an fuzzy regular F_σ-set in (X, T). Since (X, T) is an fuzzy pseudo P-space, by the theorem 2.10, the fuzzy regular F_σ-set $f^{-1}(\mu)$ is an fuzzy Baire dense set in (X, T). Hence $f^{-1}(\mu)$ is an fuzzy Baire dense set in (X, T), for the fuzzy regular F_σ-set μ in (Y, S).

Proposition 4.9. *If* $f : (X, T) \to (Y, S)$ *is an fuzzy* d_δ *function from the fuzzy pseudo P-space* (X, T) *into the fuzzy topological space* (Y, S) *and* μ *is an fuzzy regular* F_σ*-set in* (Y, S), *then there exists an fuzzy second category set* δ *such that* $\delta \leq f^{-1}(\mu)$ *in* (X, T).

Proof. Let $f : (X, T) \to (Y, S)$ be an fuzzy d_δ function from (X, T) into (Y, S) and μ be an fuzzy regular F_σ-set in (Y, S). Then, by the proposition 4.8, $f^{-1}(\mu)$ is an fuzzy Baire dense set in (X, T). By the theorem 2.11, there exists an fuzzy second category set δ such that $\delta \leq f^{-1}(\mu)$ in (X, T), for the fuzzy regular F_σ-set μ in (Y, S).

Proposition 4.10. *If* $f : (X, T) \to (Y, S)$ *is an fuzzy* d_δ *function from the fuzzy pseudo P-space* (X, T) *into the fuzzy topological space* (Y, S) *and* μ *is an fuzzy regular* F_σ*-set in* (Y, S), *then there is no fuzzy* F_σ*-set* γ *with* $int\ (\gamma) = 0$ *in* (X, T) *such that* $f^{-1}(\mu) < \gamma$.

Proof. Let $f : (X, T) \to (Y, S)$ be an fuzzy d_δ function from (X, T) into (Y, S) and μ be an fuzzy regular F_σ-set in (Y, S). Then, by the proposition 4.8 $f^{-1}(\mu)$ is an fuzzy Baire dense set in (X, T). By the theorem 2.12, there is no fuzzy F_σ-set γ with $\text{int}\ (\gamma) = 0$ in (X, T) such that $f^{-1}(\mu) < \gamma$.

Proposition 4.11. *If* $f : (X, T) \to (Y, S)$ *is an fuzzy* d_δ *function from the fuzzy pseudo P-space* (X, T) *into the fuzzy topological space* (Y, S) *and* μ *is an fuzzy regular* F_σ*-set in* (Y, S), *then there is no fuzzy residual set* δ *in* (X, T) *such that* $f^{-1}(1 - \mu) > \delta$.

Proof. Let $f : (X, T) \to (Y, S)$ be an fuzzy d_δ function from (X, T) into (Y, S) and μ be an fuzzy regular F_σ-set in (Y, S). Then, by the proposition 4.8, $f^{-1}(\mu)$ is an fuzzy Baire dense set in (X, T). By the theorem 2.14, there is no fuzzy residual set δ in (X, T) such that $1 - f^{-1}(\mu) > \delta$. Hence, there is no fuzzy residual set δ in (X, T) such that $f^{-1}(1 - \mu) > \delta$, for the fuzzy regular F_σ-set μ in (Y, S).

The following propositions show that the inverse images of fuzzy regular F_σ-sets under fuzzy d_δ functions are not fuzzy β-open and fuzzy σ-nowhere dense in weak fuzzy *P*-spaces.

Proposition 4.12. *If* $f : (X, T) \to (Y, S)$ *is an fuzzy* d_δ *function from the* *weak fuzzy P-space* (X, T) *into the fuzzy topological space* (Y, S) *and* μ *is an fuzzy regular* F_σ*-set in* (Y, S), *then* $f^{-1}(\mu)$ *is not an fuzzy* β*-open set in* (X, T).

Proof. Let $f : (X, T) \to (Y, S)$ be an fuzzy d_δ function from (X, T) into (Y, S) and μ be an fuzzy regular F_σ-set in (Y, S). Then, by the proposition 4.4, $f^{-1}(\mu)$ is an fuzzy somewhere dense set in (X, T). Since (X, T) is an weak fuzzy P-space, by the theorem 2.16, (X, T) is not an fuzzy hyper connected space. Hence, by the theorem 2.15, the fuzzy somewhere dense set $f^{-1}(\mu)$ is not an fuzzy β-open set in (X, T).

Proposition 4.13. *If* $f : (X, T) \to (Y, S)$ *is an fuzzy* d_δ *function from the weak fuzzy P-space* (X, T) *into the fuzzy topological space* (Y, S) *and* μ *is an fuzzy regular* F_σ*-set in* (Y, S). *then* $f^{-1}(\mu)$ *is not an fuzzy* σ*-nowhere dense set in* (X, T).

Proof. Let $f : (X, T) \to (Y, S)$ be an fuzzy d_δ function from (X, T) into (Y, S) and μ be an fuzzy regular F_σ-set in (Y, S). Then, $f^{-1}(\mu)$ is an fuzzy regular F_σ-set in (X, T). Since (X, T) is an weak fuzzy P-space, by the theorem 2.17, the fuzzy regular F_σ-set $f^{-1}(\mu)$ is not an fuzzy σ-nowhere dense set in (X, T).

Proposition 4.14. *If* $f : (X, T) \to (Y, S)$ *is an fuzzy function from the fuzzy pseudo hyper connected space* (X, T) *into the fuzzy topological space* (Y, S) *and then the function* $f : (X, T) \to (Y, S)$ *is not an fuzzy* d_δ *function from* (X, T) *into* (Y, S).

Proof. Let $f : (X, T) \to (Y, S)$ be an fuzzy function from the fuzzy pseudo hyper connected space (X, T) into the fuzzy topological space (Y, S) and μ be an fuzzy regular F_σ-set in (Y, S). Suppose that $f : (X, T) \to (Y, S)$ is an fuzzy d_δ function from (X, T) into (Y, S). Then, for the fuzzy regular F_σ-set μ in (Y, S), $f^{-1}(\mu)$ is an fuzzy regular F_σ-set in

(X, T). But this is a contradiction to (X, T) being an fuzzy pseudo hyper connected space which has no proper fuzzy regular F_σ-set. Hence $f : (X, T) \to (Y, S)$ is not an fuzzy d_δ function from (X, T) into (Y, S).

Proposition 4.15. *If $f : (X, T) \to (Y, S)$ is an fuzzy d_δ function from the fuzzy pseudo P-space (X, T) into the fuzzy topological space (Y, S) and μ is an fuzzy regular F_σ-set in (Y, S), then there exist fuzzy second category sets δ_1 and δ_2 such that $\delta_1 \leq f^{-1}(\mu) \leq \delta_2$ in (X, T).*

Proof. Let $f : (X, T) \to (Y, S)$ be an fuzzy d_δ function from (X, T) into (Y, S) and μ be an fuzzy regular F_σ-set in (Y, S). Then, by the proposition 4.8, $f^{-1}(\mu)$ is an fuzzy Baire dense set in (X, T). By the theorem 2.13, there exist fuzzy second category sets δ_1 and δ_2 in (X, T) such that $\delta_1 < f^{-1}(\mu) < \delta_2$.

The following proposition gives the condition for the inverse images of fuzzy regular F_σ-sets to become fuzzy first category sets under fuzzy d_δ functions.

Proposition 4.16. *If $f : (X, T) \to (Y, S)$ bean fuzzy d_δ function from the fuzzy topological space (X, T) into the fuzzy topological space (Y, S) and if* $\operatorname{int}[f^{-1}(\mu)] = 0$, *for an fuzzy regular F_σ-set μ in (Y, S), then $f^{-1}(\mu)$ is an fuzzy first category set in (X, T).*

Proof. Let $f : (X, T) \to (Y, S)$ be an fuzzy d_δ function from (X, T) into (Y, S) and μ be an fuzzy regular F_σ-set in (Y, S). Then $f^{-1}(\mu)$ is an fuzzy regular F_σ-set in (X, T). By the hypothesis, $\operatorname{int}[f^{-1}(\mu)] = 0$, in (X, T) and hence from the theorem 2.5, $f^{-1}(\mu)$ is an fuzzy first category set in (X, T).

The following proposition gives conditions for fuzzy pseudo P-spaces to become fuzzy Baire spaces, fuzzy second category spaces, fuzzy almost irresolvable spaces and fuzzy weakly Volterra spaces under fuzzy d_δ functions.

Proposition 4.17. *If $f : (X, T) \to (Y, S)$ is an fuzzy d_δ function from the fuzzy pseudo P-space (X, T) into the fuzzy topological space (Y, S) and $f^{-1}(\mu) < \lambda$, where $\lambda \in T$, for an fuzzy regular F_σ-set μ in (Y, S), then (X, T) is an fuzzy Baire space.*

Proof. Let $f : (X, T) \to (Y, S)$ be an fuzzy d_δ function from (X, T) into (Y, S) and μ be an fuzzy regular F_σ-set in (Y, S). Then, by the proposition 4.8, $f^{-1}(\mu)$ is an fuzzy Baire dense set in (X, T). By the hypothesis, $f^{-1}(\mu) < \lambda$, where $\lambda \in T$, Then, by the theorem 2.18, (X, T) is an fuzzy Baire space.

Proposition 4.18. *If $f : (X, T) \to (Y, S)$ is an fuzzy d_δ function from the fuzzy pseudo P-space (X, T) into the fuzzy topological space (Y, S) and $f^{-1}(\mu) < \lambda$, where $\lambda \in T$, for an fuzzy regular F_σ-set in (Y, S), then*

(i) (X, T) *is an fuzzy second category space*.

(ii) (X, T) *is an fuzzy almost irresolvable space*.

(iii) (X, T) *is an fuzzy weakly Volterra space*.

(iv) (X, T) *is an fuzzy regular weakly Volterra space*.

Proof. (i) The proof follows from the proposition 4.17 and the theorem 2.19.

(ii) The proof follows from the proposition 4.17 and the theorem 2.20.

(iii) The proof follows from (ii) and the theorem 2.21.

(iv) The proof follows from (iii) and the theorem 2.22.

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