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NEW CLASS OF CONNECTEDNESS IN SOFT TOPOLOGICAL SPACES

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Abstract

The main aim of this paper is provide some comparison theorem related to S_{soft} Jc-connectedness with some other mapping. Further we have introduced S_{soft} Jc-Component and some of its comparison theorem was defined. Moreover we have studied some properties of S_{soft} Jc-connectedness in detail.

1. Introduction

The methodology of connectedness via various generalized open sets is not a new idea in topological spaces. Njastad [6] introduced the α-open sets and investigated the topological structure on the class of these sets; the α-open sets form a topology. The notion of S_{soft} set theory was introduced in

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1999 by Molodtsov [4]. Maji et al. [5] have initiated some operations on S_{soft} sets. Further, many researchers paved way to the S_{soft} set theory and its applications.

'Shabir and Naz [7] was found the notion of S_{soft} topological spaces with a fixed set of parameters'. The authors of this paper have found a S_{soft} generalized closed set namely S_{soft} Jc closed set [1] and studied their properties. The main aim of this paper is provide some comparison theorem related to S_{soft} Jc connectedness with some other mapping. Further we have introduced S_{soft} Jc Component and some of its comparison theorem was defined. Moreover we have studied some properties of S_{soft} Jc connectedness in detail.

2. Preliminaries

In this work, '$\widetilde{X}$ refers to an initial universe', $P(\widetilde{X})$ is the power set of $\widetilde{X}$, E, K denote the set of parameters and $(\widetilde{X}, \tau_E)$, $(\widetilde{Y}, \sigma_K)$ denote S_{soft} topological spaces where no S_{soft} separation axioms are assumed unless it is explicitly stated.

Definition 2.1. A S_{soft} Jc-closed [1] if $S\alpha cl(A_E) \tilde{\subseteq} Int(A_E)$ whenever $(A_E) \tilde{\subseteq} (U_E)$ and (U_E) is S_{soft} $\hat{g}$-open in $(\widetilde{X}, \tau_E)$, the complement of S_{soft} Jc closed set is called a S_{soft} Jc-open set.

Definition 2.2. A function $f_c : (\widetilde{X}, \tau_E) \to (\widetilde{Y}, \sigma_K)$ is called S_{soft} Jc-continuous [3] if $f_c^{-1}(V_K)$ is S_{soft} Jc-closed in $(\widetilde{X}, \tau_E)$ for every S_{soft} closed set (V_K) of $(\widetilde{Y}, \sigma_K)$.

Definition 2.3. A bijection $f_c : (\widetilde{X}, \tau_E) \to (\widetilde{Y}, \sigma_K)$ is called S_{soft} Jc-homeomorphism [2] if f_c is both S_{soft} Jc-continuous and S_{soft} Jc-open.

Definition 2.4[8]. A S_{soft} topological space $(\widetilde{X}, \tau_E)$ is called S_{soft}-connected-space if $\widetilde{X}$ can't be written as a disjoint S_{soft} union of any two non-empty S_{soft} open sets.

Theorem 2.5. *Let* $\{B\gamma, \gamma \widetilde{\in} \Gamma\}$ *is a non-empty family of* S_{soft} Jc*-connected-subsets of a* S_{soft} *topological space* $\widetilde{X}$ *such that* $\widetilde{\bigcap} B\gamma \neq \widetilde{\emptyset}$, *then* $\widetilde{\bigcup} B\gamma$ *is* S_{soft} Jc*-connected.*

3. Soft Jc-Connectedness and Mappings

Theorem 3.1. *Let* f_c *be a* S_{soft} Jc*-irresolute function from a* S_{soft} *space* $(\widetilde{X}, \tau_E)$ *onto a* S_{soft} *space* $(\widetilde{Y}, \sigma_K)$. *If* $(\widetilde{X}, \tau_E)$ *is* S_{soft} Jc*-connected, then* $(\widetilde{Y}, \sigma_K)$ *is* S_{soft} Jc*-connected.*

Proof. Our assumption is that $(\widetilde{Y}, \sigma_K)$ is not S_{soft} Jc-connected.

Therefore there exist a S_{soft} non-empty proper subset A_K of $(\widetilde{Y}, \sigma_K)$ which is both S_{soft} Jc-open and S_{soft} Jc-closed.

Then the inverse image of A_K under f_c is both S_{soft} Jc-open and S_{soft} Jc-closed in $(\widetilde{X}, \tau_E)$, which contradicts our hypothesis.

Corollary 3.2. *A* S_{soft} Jc*-irresolute function maps* S_{soft} Jc*-connected set onto* S_{soft}*-connected set.*

Theorem 3.3. *Consider a* S_{soft} Jc*-continuous function* f_c *from a* S_{soft} *space* $(\widetilde{X}, \tau_E)$ *onto a* S_{soft} *space* $(\widetilde{Y}, \sigma_K)$. *If* $(\widetilde{X}, \tau_E)$ *is* S_{soft} Jc*-connected, then* $(\widetilde{Y}, \sigma_K)$ *is* S_{soft}*-connected.*

Proof. Consider $(\widetilde{Y}, \sigma_K)$ is not S_{soft}-connected. Then there is a nonempty proper subset of $(\widetilde{X}, \tau_E)$ which is both S_{soft}-open and S_{soft}-closed. Then the inverse image of A_K under f_c is both S_{soft} Jc-open and S_{soft} Jc-closed in $(\widetilde{X}, \tau_E)$, a contradiction.

Though the concept of S_{soft} continuity, S_{soft} Jc-continuity and S_{soft} Jc-irresolute are independent of each other but they behave similarly in case of S_{soft} Jc-connectedness, that is, these functions map a S_{soft} Jc-connected set onto a S_{soft} connected set.

Remark 3.4. A S_{soft}-homeomorphism preserves S_{soft} Jc-connectedness.

Theorem 3.5. *If* $(\widetilde{X}, \tau_E)$ *is* S_{soft} Jc*-connected space, then* $(\widetilde{X}, \tau_E) \times \{a\}$ *is also* S_{soft} Jc*-connected.*

Proof. Obviously, $(\widetilde{X}, \tau_E)$ is S_{soft} homeomorphic to $(\widetilde{X}, \tau_E) \times \{a\}$.

Then by the previous remark, $(\widetilde{X}, \tau_E) \times \{a\}$ is S_{soft} Jc-connected.

Theorem 3.6. *If* $(\widetilde{X}, \tau_E)$ *and* $(\widetilde{Y}, \sigma_K)$ *are two* S_{soft} Jc*-connected spaces, then* $(\widetilde{X}, \tau_E) \times (\widetilde{Y}, \sigma_K)$ *is also* S_{soft} Jc*-connected.*

Proof. For any S_{soft} point (a_e, b_e) in the product $(\widetilde{X}, \tau_E) \times (\widetilde{Y}, \sigma_K)$, then each of the subspace $(\widetilde{X}, \tau_E) \times \{b_e\} \widetilde{\cup} \{x_e\} \times (\widetilde{Y}, \sigma_K)$ is S_{soft} Jc-connected since it is the union of two S_{soft} Jc-connected subspaces with a point in common. Then by Theorem 2.5, $(\widetilde{X}, \tau_E) \times (\widetilde{Y}, \sigma_K)$ is S_{soft} Jc-connected.

Theorem 3.7. *Let* $X_\beta, \beta \widetilde{\in} A_E$, *be a family of* S_{soft} *spaces. If* ΠX_β *is* S_{soft} Jc*-connected, then each* X_β *is* S_{soft}*-connected.*

Proof. Let ΠX_β is S_{soft} Jc-connected. Then ΠX_β is S_{soft}-connected.

"Since $p_\gamma : \Pi X_\beta \to X_\gamma$ a projection which is a S_{soft}-continuous map. So each X_β is S_{soft} connected".

Theorem 3.8. *Let* $X_\beta, \beta \widetilde{\in} A_E$, *be a family of* S_{soft} *spaces. If each* X_β *is* S_{soft} Jc*-connected, then* ΠX_β *is* S_{soft}*-connected.*

Proof. The proof follows from the fact that $S_{soft}\ Jc$-connectedness implies S_{soft}-connectedness and each X_{β} is S_{soft}-connected if and only if ΠX_{β} is Soft-connected.

Remark 3.9. If $f_c : (\widetilde{X}, \tau_E) \to (\widetilde{Y}, \sigma_K)$ is $S_{soft}\ Jc$-homeomorphism, then f_c preserves $S_{soft}\ Jc$-connectedness.

Theorem 3.10. *If a S_{soft} map $f_c : (\widetilde{X}, \tau_E) \to (\widetilde{Y}, \sigma_K)$ is a weakly $S_{soft}\ Jc$-irresolute, surjection and $\widetilde{X}$ is $S_{soft}\ Jc$-connected, then $\widetilde{Y}$ is $S_{soft}\ Jc$-connected.*

Proof. Consider that $\widetilde{Y}$ is not $S_{soft}\ Jc$-connected. There exist non-empty $S_{soft}\ Jc$-open sets V_K and W_K of $\widetilde{Y}$ so that $V_K \widetilde{\cup} W_K = \widetilde{Y}$ and $V_K \widetilde{\cap} W_K = \widetilde{\varphi}$.

Since f_c is weakly $S_{soft}\ Jc$-irresolute, $f_c^{-1}(V_K), f_c^{-1}(W_K) \widetilde{\in} JcO(\widetilde{X})$.

Moreover, we have $f_c^{-1}(V_K) \widetilde{\cup} f_c^{-1}(W_K) = \widetilde{X}$, $f_c^{-1}(V_K) \widetilde{\cap} f_c^{-1}(W_K) = \widetilde{\varphi}$ and $f_c^{-1}(V_K)$ and $f_c^{-1}(W_K)$ are non-empty. Therefore, $\widetilde{X}$ is not $S_{soft}\ Jc$-connected. This contradicts. Therefore $\widetilde{Y}$ is $S_{soft}\ Jc$-connected.

4. Soft Jc-components

Definition 4.1. Consider x_e any element of a S_{soft} space $(\widetilde{X}, \tau_E)$. The $S_{soft}\ Jc$-component containing x, $\widetilde{C}Jc(x_e)$, is the union of all $S_{soft}\ Jc$-connected subsets of $(\widetilde{X}, \tau_E)$ which contain x.

The component $\widetilde{C}Jc(x_e)$ is $S_{soft}\ Jc$-connected and hence S_{soft}-connected. It follows from its definition that $\widetilde{C}Jc(x_e)$ is not properly contained in any $S_{soft}\ Jc$-connected subset of $(\widetilde{X}, \tau_E)$. Thus, $\widetilde{C}Jc(x_e)$ is a S_{soft} maximal $S_{soft}\ Jc$-connected subset of $(\widetilde{X}, \tau_E)$.

Lemma 4.2. *If* A_E *is* S_{soft} Jc*-component of* $(\widetilde{X}, \tau_E)$ *containing* x_e, *then it is contained in* S_{soft} *component of* $(\widetilde{X}, \tau_E)$ *containing* x_e.

Theorem 4.3. *Consider* $(\widetilde{X}, \tau_E)$ *be a* S_{soft} *space. Then, each* S_{soft} Jc*-component of* $(\widetilde{X}, \tau_E)$ *is* S_{soft} Jc*-closed.*

Proof. If $\widetilde{C}Jc(x_e)$ is a S_{soft} Jc-component containing x_e in $(\widetilde{X}, \tau_E)$, then $\widetilde{C}Jc(x_e)$ is S_{soft} Jc-connected. So $JcCl(\widetilde{C}Jc(x_e))$ is also S_{soft} Jc-connected.

By the maximality of $\widetilde{C}Jc(x_e)$, we have $\widetilde{C}Jc(x_e) = JcCl(\widetilde{C}Jc(x_e))$.

Thus, $\widetilde{C}Jc(x_e)$ is S_{soft} Jc-closed in $(\widetilde{X}, \tau_E)$.

Theorem 4.4. *Consider* $(\widetilde{X}, \tau_E)$ *be a* S_{soft} *space. Then, each* S_{soft} Jc*-connected subset of* $(\widetilde{X}, \tau_E)$ *is contained in a* S_{soft} Jc*-component of* $(\widetilde{X}, \tau_E)$.

Proof. If A_E is a non-empty S_{soft} Jc-connected subset of $(\widetilde{X}, \tau_E)$.

Thus $A_E \mathrel{\widetilde{\subseteq}} \widetilde{C}Jc(a)$ for each a in A_E.

Theorem 4.5. *Let* $(\widetilde{X}, \tau_E)$ *be a* S_{soft} *space. Then, the set of all* S_{soft} Jc*-components of* $(\widetilde{X}, \tau_E)$ *forms a partition of* $(\widetilde{X}, \tau_E)$.

Proof. For $x_e \mathrel{\widetilde{\in}} (\widetilde{X}, \tau_E)$, $\{x_e\}$ is S_{soft} Jc-connected.

Then there is a S_{soft} Jc-component $U_E \mathrel{\widetilde{\subseteq}} (\widetilde{X}, \tau_E)$ containing x_e.

So $(\widetilde{X}, \tau_E)$ will be contained in the union of S_{soft} Jc-components.

Let $\widetilde{C}_1$ and $\widetilde{C}_2$ be two distinct S_{soft} Jc-components such that $\widetilde{C}_1 \mathrel{\widetilde{\cap}} \widetilde{C}_2 \neq \widetilde{\emptyset}$.

Then $\widetilde{C}_1 \mathbin{\widetilde{\cup}} \widetilde{C}_2$ is S_{soft} Jc-connected, which contradicts the fact that $\widetilde{C}_1$ and $\widetilde{C}_2$ are S_{soft} Jc-components.

Therefore $\widetilde{C}_1$ and $\widetilde{C}_2$ are S_{soft} disjoint.

Thus, the S_{soft} Jc-components constitute a partition of $(\widetilde{X}, \tau_E)$.

Theorem 4.6. *If* $(\widetilde{X}, \tau_E)$ *has finite number of* S_{soft}*-components, then each* S_{soft}*-component is both* S_{soft} Jc*-open and* S_{soft} Jc*-closed.*

Proof. If a S_{soft} space $(\widetilde{X}, \tau_E)$ has finite number of S_{soft} components, then each S_{soft} component is both S_{soft} open and S_{soft} closed and since "A clopen subset of a soft topological space $\widetilde{X}$ is both S_{soft} Jc-open and S_{soft} Jc-closed, Hence the proof follows.

Remark 4.7. If $(\widetilde{X}, \tau_E)$ has finite number of S_{soft} Jc-components, then each S_{soft} Jc-component is both S_{soft} Jc-open and S_{soft} Jc-closed.

Theorem 4.8. *If* $f_c : (\widetilde{X}, \tau_E) \to (\widetilde{Y}, \sigma_K)$ *is* S_{soft} Jc*-continuous or* S_{soft} Jc*-irresolute and* $\widetilde{C}(x_e)$ *is the component containing* x_e *in* $(\widetilde{X}, \tau_E)$*, then* $f_c(\widetilde{C}Jc(x_e)) \mathrel{\widetilde{\subseteq}} \widetilde{C}(f_c(x_e))$.

Proof. Follows from Corollary 3.2 and Theorem 3.3.

Corollary 4.9. *If* f_c *is* S_{soft} Jc*-homeomorphism, then* $f_c(\widetilde{C}Jc(x_e)) = \widetilde{C}(f_c(x_e))$.

Corollary 4.10. *If* f_c *is* S_{soft} *homeomorphism, then* $f_c(\widetilde{C}Jc(x_e)) = \widetilde{C}Jc(f_c(x_e))$.

Remark 4.11. Let $(\widetilde{X}, \tau_E)$ be a space with finite S_{soft} α-topology on it and f_c be S_{soft} Jc-homeomorphism. Then $f_c(\widetilde{C}(x)) = \widetilde{C}(f_c(x))$.

Theorem 4.12. *A S_{soft} Jc-connected, S_{soft} Jc-open and S_{soft} Jc-closed subset A_E of a S_{soft} space $(\widetilde{X}, \tau_E)$ is a S_{soft} Jc-component of $(\widetilde{X}, \tau_E)$.*

Proof. If possible, suppose that A_E is not a S_{soft} Jc-component of $(\widetilde{X}, \tau_E)$.

Then $A_E \widetilde{\subset} B_E$ and B_E is S_{soft} Jc-component of $(\widetilde{X}, \tau_E)$.

Obviously BE is S_{soft} Jc-closed. Therefore, $B_E \setminus A_E$ is S_{soft} Jc-closed.

Then A_E and $B_E \setminus A_E$ constitute a S_{soft} Jc-separation of BE, a contradiction.

Remark 4.13. A S_{soft} Jc-connected, S_{soft} Jc-open and S_{soft} Jc-closed subset A_E of a S_{soft} space $(\widetilde{X}, \tau_E)$ with finite S_{soft} α-topology, is a S_{soft}-component of $(\widetilde{X}, \tau_E)$.

Definition 4.14. A S_{soft} space $(\widetilde{X}, \tau_E)$ is known as locally S_{soft} Jc-connected at $x_e \widetilde{\in} (\widetilde{X}, \tau_E)$ if for each S_{soft} Jc-open set U_E containing x_e, there is a S_{soft} Jc-connected, S_{soft} Jc-open set V_E such that $x_e \widetilde{\in} V_E \widetilde{\subseteq} U_E$. The S_{soft} space $(\widetilde{X}, \tau_E)$ is locally S_{soft} Jc-connected if it is locally S_{soft} Jc-connected at each of its S_{soft} points.

Theorem 4.15. *A S_{soft} space $(\widetilde{X}, \tau_E)$ is locally S_{soft} Jc-connected if and only if the S_{soft} Jc-components of each S_{soft} Jc-open subset of $(\widetilde{X}, \tau_E)$ are S_{soft} Jc-open.*

Proof. Suppose that $(\widetilde{X}, \tau_E)$ is locally S_{soft} Jc-connected. Let U_E be a S_{soft} Jc-open subset of $(\widetilde{X}, \tau_E)$ and $\widetilde{C}$ be a S_{soft} Jc-component of U_E.

If $x_e \widetilde{\in} \widetilde{C}$, then there is a S_{soft} Jc-connected S_{soft} Jc-open set $V_E \widetilde{\subseteq} (\widetilde{X}, \tau_E)$ such that $x_e \widetilde{\in} V_E \widetilde{\subseteq} U_E$.

Since $\widetilde{C}$ is a S_{soft} Jc-component of U_E and V_E is a S_{soft} Jc-connected subset of U_E containing $x_e \widetilde{\in} V_E \widetilde{\subseteq} \widetilde{C}$. Thus, $\widetilde{C}$ is a S_{soft} Jc-open set.

Conversely, let $U_E \widetilde{\subseteq} (\widetilde{X}, \tau_E)$ be a S_{soft} Jc-open set, and $x_e \widetilde{\in} U_E$.

By our hypothesis, the S_{soft} Jc-component V_E of U_E containing x_e is S_{soft} Jc-open, so $(\widetilde{X}, \tau_E)$ is locally S_{soft} Jc-connected at x.

Theorem 4.16. *Let* $f_c : (\widetilde{X}, \tau_E) \to (\widetilde{Y}, \sigma_K)$ *be a* S_{soft} Jc*-irresolute,* S_{soft} Jc*-closed surjection. If* $(\widetilde{X}, \tau_E)$ *is locally* S_{soft} Jc*-connected, then* $(\widetilde{Y}, \sigma_K)$ *is locally* S_{soft}*-*Jc*-connected.*

Proof. Suppose that $(\widetilde{X}, \tau_E)$ is locally S_{soft} Jc-connected.

Let $\widetilde{C}$ be a S_{soft} Jc-component of $U_E \widetilde{\in} JcO(\widetilde{Y}, \sigma_K)$, and let $x_e \widetilde{\in} f_c^{-1}(\widetilde{C})$. Then there exists a S_{soft} Jc-connected S_{soft} Jc-open set V_E in $(\widetilde{X}, \tau_E)$ such that $x_e \widetilde{\in} V_E \widetilde{\subseteq} f_c^{-1}(U_E)$, since $(\widetilde{X}, \tau_E)$ is locally S_{soft} Jc-connected and $f_c^{-1}(U_E)$ is S_{soft} Jc-open in $(\widetilde{X}, \tau_E)$. It follows that $f_c(x) \widetilde{\in} f_c(V_E) \widetilde{\subseteq} \widetilde{C}$ for $f(V_E)$ is S_{soft} Jc-connected.

So, by Theorem 4.15, $x \widetilde{\in} V_E \widetilde{\subseteq} f_c^{-1}(\widetilde{C})$, and $f_c^{-1}(\widetilde{C}) \widetilde{\in} JcO(\widetilde{X})$.

Since f_c is S_{soft} Jc-closed surjection, $\widetilde{Y} \setminus \widetilde{C} = f_c(\widetilde{X} \setminus f_c^{-1}(\widetilde{C}))$ is S_{soft} Jc-closed.

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