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A MODIFICATION FOR THE IMAGE EUCLIDEAN DISTANCE MEASURE

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Abstract

Measuring the distance between images is one of the major challenging, problems in various image processing and pattern recognition applications. Though image Euclidean distance measure (IMED) is a suitable measure for these problems. It is relevant to take up new means for measuring distance for solving the defects of this measure. This paper contributes two different IMED by modifying the metric coefficient which is easy to calculate and applicable for various image recognition programmes. Moreover, the paper treats some properties of these distance measures. Finally, we have compared all the Euclidean distance measures (ED) and it was an eye opening to the importance of the proposed IMED.

1. Introduction

Determining the distance between images is a specific problem in image recognition and computer vision. The distance measure of high standard is most likely to be intangible correspondence with the spectator's subjective evaluation. Consider $A = (\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_{mn}), B = (\beta_1, \beta_2, \beta_3, \ldots, \beta_{mn})$ be two m by n images, where $\alpha_n(k-1)+l, \beta_n(k-1)+l$ are the grey levels at location (k, l). The ED DED (A, B) is given by $D_{ED}^2(A, B) = \sum_{k=1}^{mn}(\alpha_k - \beta_k)^2$. Due to easiness this measure was commonly used among

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all image measures [1], [2], [3], [4]. But unfortunately, it does not give good subjective evaluation for an observer. This can be understood from the following example.

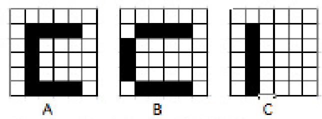


Figure 1.

The ED between A and B as well as A and C is 6. By observing the pattern this answer is not reasonable because the ED only considering the pixel intensity difference.

A digital image is a discrete representation of visual objects that have spatial and intensity information. So, if we neglect the spatial distance, the distance measure shows high sensitivity even to small deformation. In order to overcome this problem generalised Euclidean distance (GED) [5] and IMED [6] were designed to evaluate image distances which are also spatially dependent. Even though these few methods exist, it is required to find a suitable measure which produce better reasonable results.

This paper devises a new approach to find a distance by modifying the IMED and study some properties of the measures. Moreover, the new measures are compared with the existing measures. Finally, we illustrate the significance of the proposed distance measure with appropriate examples.

2. Preliminaries

In this section some basic definitions are discussed.

Definition 2.1 [8]**.** Consider σ be any nonempty set and $I = [0, 1]$. Then any map $E : \sigma \to I$ is called a fuzzy subset of σ. The collection of all fuzzy subset of σ is denoted by $FS(\sigma)$.

Definition 2.2 [8]**.** For E_1, E_2 be two $FS(\sigma)$ and for each $x \in \sigma$, the following operations are defined:

1. $E_1 \subseteq E_2$ if and only if $E_1(x) \leq E_2(x)$, $E_1^c(x) = 1 - E_1(x)$

2. $(E_1 + E_2)(x) = E_1(x) + E_2(x) - E_1(x)E_2(x)$.

Definition 2.3 [7]. A real valued function $D : FS(\sigma) \times FS(\sigma) \to [0, 1]$ is said to be a distance measure between $E_1 \in FS(\sigma)$ and $E_2 \in FS(\sigma)$ if D satisfies the following postulates:

1. $0 \leq D(E_1, E_2) \leq 1; D(E_1, E_2) = 0 \Leftrightarrow E_1 = E_2$

2. $D(E_1, E_2) = D(E_1, E_2)$;

3. If $E_1 \subseteq E_2 \subseteq E_3$, then $D(E_1, E_2) \leq D(E_1, E_3)$ and $D(E_2, E_3) \leq D(E_1, E_3)$ for all $E_3 \in FS(\sigma)$.

Definition 2.4 [5]. Consider $A = (\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_{mn})$, $B = (\beta_1, \beta_2, \beta_3, \ldots, \beta_{mn})$ be two m by n images. Then a generalised Euclidean distance, D^2_{GED} is be defined as $\sum_{i,j=1}^{mn} M_{ij}(\alpha_i - \beta_i)(\alpha_j - \beta_j)$ where $M(i, j) = r^d$ is used to encode the relative location of two pixels. Here $d = | K_i - K_j | + | l_i - l_j |$ is distance of two pixels and r is decay constant $(r = 0.6)$.

Definition 2.5 [6]. Consider $A = (\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_{mn})$, $B = (\beta_1, \beta_2, \beta_3, \ldots, \beta_{mn})$ be two m by n images. Then an image Euclidean distance is given by $D^2_{IMED}(A, B) = \sum_{i,j=1}^{mn} (\alpha_i - \beta_i)(\alpha_j - \beta_j)$ where the metric coefficients are given by the Gaussian function, $M_{ij} = f(| S_i - S_j |)$ $= \frac{1}{2n} \exp\{-(| S_i - S_j |)^2/2\}$, where $| S_i - S_j |$ denotes the pixel distance.

3. An improved image Euclidean distance measure (IIMED)

In this section we illustrate some examples which have motivated the modification of the metric coefficient used in the IMED.

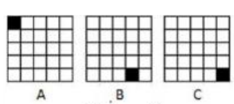


Figure 2.

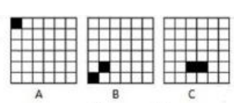


Figure 3.

Example 3.1. In figure 2 $D^2{}_{IMED}(A, B) = 0.318 = D^2{}_{IMED}(A, C)$ for digital image of size 5×5 and in figure 3, $D^2{}_{IMED}(A, C) = 0.669$, $D^2{}_{IMED}(B, C) = 0.681$ for digital image of size 6×6.

Remark 3.2. The IMED distance function become insensitive to small deformation at a larger distance. In example 3.1, figure 2, there is a small difference between the images B and C but IMED gives same distance between A, B and A, C respectively. When the pixel distance exceeds $4, M_{ij}$ is always almost the same. In figure 3 by human perception we analysed that the distance between the images A, C be larger than distance between the images B, C. But here we get an illogical result that is $D_{IMED}(A, C) < D_{IMED}(B, C)$. These defects are due to the metric coefficient function M_{ij} which is ineffective for the large pixel distance. In order to overcome these limits we propose an IIMED.

Definition 3.3. Let $A = (\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_{mn}), B = (\beta_1, \beta_2, \beta_3, \ldots, \beta_{mn})$, be two digital image. Then an improved image Euclidean distance measure (IIMED) in given by $D_{IIMED}(A, B) = \sqrt{\sum_{i,\, j=1}^{mn} M_{ij}(\alpha_i - \beta_i)(\alpha_j - \beta_j)}$ where $M_{i,j} = \frac{1}{1 + (| S_i - S_j |)^2}$. Here $| S_i - S_j |$ indicates the pixel distance.

Example 3.4. In figure 2, $D^2{}_{IIMED}(A, B) = 1.86$, $D^2{}_{IIMED}(A, C) = 1.89$ and in figure 3, $D^2{}_{IIMED}(A, C) = 3.82$, $D^2{}_{IIMED}(B, C) = 3.75$. Note that these results satisfy our human perception.

Theorem 3.5. *The IIMED measure is a distance measure.*

Proof of Theorem 3.5. Since M_{ij} is a symmetric matrix, we have $M_{i,j} = M_{j,i}$ and hence

$$D^2{}_{IIMED}(A, B) = \sum_{i=j}^{mn} M_{i,j}[\alpha_i - \beta_i]^2 + 2\sum_{i,j=1,i<j}^{mn} M_{i,j}[\alpha_i - \beta_i][\alpha_j - \beta_j].$$

D1: If $\alpha_i \leq \beta_i$ for all i or $\alpha_i \geq \beta_i$ for all i, $D_{IIMED}(A, B) > 0$. If $\alpha_i \leq \beta_i$, for some i and $\alpha_j \geq \beta_j$, for some j, then we let $u = \{i \in \{1, 2, 3, \ldots, mn\} / \alpha_i < \beta_i\}$, $v = \{i / \alpha_i > \beta_i\}$ and $w = \{i \in \{1, 2, 3, \ldots, K\} / \alpha_i = \beta_i\}$ and $\| u \| = s, \| v \| = t$ and $\| w \| = r$. Then

$$D^2_{IIMED}(AB) = \sum_{i \neq j \in u}^{mn} M_{i,j}[\alpha_i - \beta_i] + 2\sum_{i \neq j \in u}^{mn} M_{i,j}[\alpha_i - \beta_i]^2[\alpha_i - \beta_i]^2$$

$$+ 2\sum_{i \neq j \in v}^{mn} M_{i,j}[\alpha_i - \beta_i][\alpha_j - \beta_j] + 2\sum_{i \neq j \in w}^{mn} M_{i,j}[\alpha_i - \beta_i][\alpha_j - \beta_j]$$

$$+ 2\sum_{i \in v \, and \, j \in w}^{mn} M_{i,j}[\alpha_i - \beta_i][\alpha_j - \beta_j] + 2\sum_{i \in u \, and \, j \in ww}^{mn} M_{i,j}[\alpha_i - \beta_i][\alpha_j - \beta_j]$$

$$+ 2\sum_{i \in u \, and \, j \in v}^{mn} M_{i,j}[\alpha_i - \beta_i][\alpha_j - \beta_j].$$

In this expression among the $mn = k$ number of diagonal terms (M_{ii} terms), we have the relation $s + t + r = k$. Among non diagonal terms (M_{ij} terms), there are 2 $(sC_1 + tC_2)$ number of positive terms and 2st number of negative terms, $2 * rC_2 + 2sr + 2tr$ number of term with zero value. The number of negative terms is less than number of positive terms. Clearly $| 2M_{ij}(\alpha_i - \beta_i)(\alpha_j - \beta_j) | < M_{ii}(\alpha_i - \beta_i)^2$ or $| 2M_{ij}(\alpha_i - \beta_i)(\alpha_j - \beta_j) | < M_{jj}(\alpha_j - \beta_j)^2$ where $i \in u$, $j \in v$ or $i \in v$, $j \in u$.

Hence, summing up the results that number of negative terms is less than number of positive terms and the modulus value of each negative term is less than the value of positive term, we conclude that $D_{IIMED}(A, B) > 0$. D2 and D3 are straight forward from the definition.

D4. Assume that $A = (\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_{mn})$, $B = (\beta_1, \beta_2, \beta_3, \ldots, \beta_{mn})$, $C = (\gamma_1, \gamma_2, \gamma_3, \ldots, \gamma_{mn})$, be three digital images. Since $A \subseteq B \subseteq C$, we have $\alpha_i \leq \beta_i \leq \gamma_i$. $\Rightarrow (\gamma_i - \alpha_i)(\gamma_j - \alpha_j) \geq (\beta_i - \alpha_i)(\beta_j - \alpha_j) \Rightarrow D_{IIMED}(A, C)$ $D_{IIMED}(A, B)$. Similarly, $D_{IIMED}(A, C) \geq D_{IIMED}(B, C)$. Hence the proof.

Proposition 3.8. *Let* A, B, C *be digital images. If* $A \subseteq B \subseteq C$, *then* $D_{IIMED}(A, C) \geq \frac{D_{IIMED}(A, B) + D_{IIMeD}(B, C)}{2}$.

Proposition 3.9. *If A be digital image. Then* $D_{IIMED}(0, A) = D_{IIMED}(A^c, 1); D_{IIMED}(A, 1) = D_{IIMED}(A^c, 0); D_{IIMED}(0, 1) = D_{IIMED}(1, 0)$ $= \sqrt{\sum_{i,j=1}^{mn} M_{i,j}}.$

Proposition 3.10. *Let A, B, C be digital images and $B \subseteq A$, then* $D_{IIMED}(A + C, B + C) \leq D_{IIMED}(A, B)$.

Proof of Proposition 3.10. By definition, $D_{IIMED}(A + C, B + C)$

$$= \sqrt{\sum_{i,j=1}^{mn} M_{i,j}[(\alpha_i - \beta_i)(1 - \gamma,)][(\alpha_j - \beta_j)(1 - \gamma,)]}$$

$$\leq \sqrt{\sum_{i,j=1}^{mn} M_{i,j}[\alpha_1 - \beta][\alpha_j - \beta_j]}$$

4. A New Image Euclidean Distance Measure (NIMED)

While handling the IIMED, we found some drawbacks of this measure. For large pictures when pixel distance exceeds a certain limit the value of the $M_{i,j}$ function remains the same and will not show significant effect. Hence it is not robust to small deformation. In this situation it is desirable to seek a distance measure which is significant for larger pixel distances so that it is sensitive to small deformation.

Definition 4.1. Let $A = (\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_m)$, $B = (\beta_1, \beta_2, \beta_3, \ldots, \beta_{mn})$, be two digital image. Then NIMED is given by

$$D_{IIMED}(A, B) = \sqrt{\sum_{i,j=1}^{mn} M_{i,j}[\alpha_i - \beta_i][\alpha_j - \beta_j]} \text{ where}$$

$$Mij = \begin{cases} \dfrac{(| s_{mn} - s_1 |) - (| s_i - s_j |)}{((s_{mn} - s_1 |))} & \text{if } m, n \geq 2, \\ 1 & \text{if } m, n = 1 \end{cases}$$

To illustrate the effectiveness of the new IMED measure, the following examples can be considered.

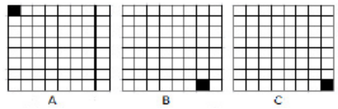


Figure 4.

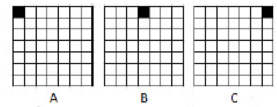


Figure 5.

Example 4.2. In figure 4, $D^2{}_{IIMED}(A, B) = 1.98 = D^2{}_{IIMED}(A, C)$, $D^2{}_{NIMED}(A, B) = 1.86 D^2{}_{NIMED}(A, C) = 2$. In figure 5, $D^2{}_{IIMED}(A, B) = 1.8 = D^2{}_{IIMED}(B, C)$, $D^2{}_{IIMED}(A, C) = 1.9$ $D^2{}_{NIMED}(A, B) = 0.707 = D^2{}_{NIMED}(B, C)$, $D^2{}_{NIMED}(A, C) = 1.414$.

Remarks 4.3. In the example 4.2, figure 4, shows that while increasing the size of image representation our IIMED also inefficient. In figure 5 distance between the image A, C and C are almost same for IIMED whereas the intuition, provided by visualization of figure-5 is not supported. However, the NIMED gives good result.

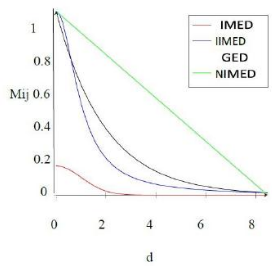
5. Comparative study of the ED measures

In this section a comparison between the proposed NIMED measures with the existing measures are done and analysed. The advantage of the proposed measure over existing measure is illustrated in the following table1. It reveals that the four ED measures exhibit illogical results when the number of pixels increased. But the NIMED satisfies our human perception.

6. Significance of the new IMED

Computation of the proposed NIMED is effortless and easier than the existing ones. The main drawback of the existing measures is that sometimes even for smaller number of pixel size we could not acquire a suitable and intuitively satisfactory result. This is because of the inefficiently of $M_{i,j}$ function used in the image ED formula. From the graph we can see that $M_{i,j}$ function IMED suddenly falls to zero, when pixel distance is grater that 2.5. While improved $M_{i,j}$ function slowly tends to zero as pixel distance is increased. Since the new $M_{i,j}$ function is linear it gives a measure which is robust to ignorable deformations however large the pixel distance be.

Therefore, the new image Euclidean distance measure shows better result for any picture and are good in subjective perception.



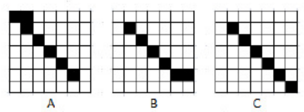


Figure 6.

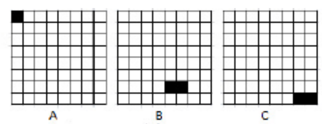


Figure 7.

Table 1.

Figure	D^2	ED	GED	IMED	IIMED	NIMED	NOTE
6	(A, B)	3	4.18	0.67	3.9	4.6	Though the images B, C are different, existing measures exhibit the same distance. But NIMED gives logic result.
	(A, C)	3	4.18	0.67	3.9	4.6	
7	(A, C)	3	4.2	0.67	3.96	4.66	According to subjective perception, $D(B, C) < D(A, C)$. But the existing measures shows illogical $(D(A, C) < D(B, C))$. But NIMED gives precise result.
	(B, C)	3	4.5	0.85	4.48	1.42	

7. Future scope and Conclusion

In the existing image recognition methods, IMED plays a vital role. In this paper we have met with some defects of the existing measures. Hence we introduced new IMED measures which covered the limitations of the existing measures. The major contributions is that, however larger the pixel size the proposed NIMED shows great outcome. This paper describes the mathematical properties of IIMED measure. One possible future direction is to search this measure embedding in image recognition algorithms. The other one is to extend these concepts into intuitionistic fuzzy set up.

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