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FIXED-POINT APPROACH ON GENERALISED $\breve{E}$- FUZZY-METRIC SPACE BY IMPLICATION

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Abstract

We exhibit the fixed-point approach on generalized $\breve{E}$- fuzzy-metric space aimed at $\breve{w}$eak compatible mappings ($\breve{w}c$ mapping) providing the Property-E.A by implication.

1. Introduction

Lotfi A. Zadeh [1] hosted the basis of fuzzy-set premise in 1965. Principle of Fuzzy-set premise has solicitations on neural networks, Engineering, control theory, communication, Mathematical programming etc. In linear programming the problems are frequently revealed that optimising some goal function furnished with certain constraints propounded by certain concrete empirical state. There exist many initialise problems that examine several ideas and regularly furthermore its complicated to get a appropriate solution that steers us and get an optimum sequel of many impartial functions. Thus, an appropriate method of rectifying such problems are the implementation of Fuzzy-sets. Mishra et al. [2] elongate the conception of consistent and unambiguous of the fixed-point hypotheses to illustrate the progression of a mapping and inclusiveness of mentioned field. J. Michalek and Kramosil [7] hosted the model of a fuzzy-metric, then owned generalised over the probabilistic metric to a fuzzy state. Veeramani and George [3] enhanced the fuzzy-metric hosted by J. Michalek and Kramosil [7]. Popa ([9]-[10]) specified

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the implication in the hypothesis of fixed-point in metric spaces. S. Jain and M. Singh [11] further prolonged the consequence of Popa ([9]-[10]) on fuzzy-metric, also they weakened the estimation of $\breve{w}c$ mapping. In non-compatible mappings, the appraisal of trivial fixed-points introduced by V. Pant [25]. The fixed-point hypotheses have on discussed with rigid contractive situation by M. Aamri and D. E. U. Moutawakil [19]. V. Pant [25] gave some ideas around the fixed-point hypotheses of contractive situation in fuzzy-metric. M. Aamri and D. E. U. Moutawakil [19] prolonged a property-E. A for self-mappings in the evidence of non-compatible mappings in metric in 2002. M. Imdad and Ali [17] reveal the property-E.A approves to replace the inclusiveness of the fuzzy-metric with certain situations of closedness of the range. In 2006, Z. Mustafa and B. Sims [5] gave certain consequences of fixed-point in G-Metric. Guangpeng Sun and Kali Jang [16] gave some consequences of generalized fuzzy-metric with properties. K. P. Suakanya and Magie Jose ([12]-[13]) offered a generalized fuzzy-metric and conferred various possessions of generalized fuzzy-metric and moreover gained the fixed-point proposal in generalized fuzzy-metric. In fixed-point theory, the implication covers many contractive situations that one may verify the discrete proposition in all contractive conditions. S. Jain and M. Singh [11] prolonged the conception of semi-compatible mappings in fuzzy-metric to gratify an implication. Asha Rani and Sanjay Kumar [14] discussed some fixed-point hypothesis with implication. The two absolute modules of implications be defined, to access of certain fixed-point consequences of those sets of $\breve{w}c$ mapping employing property-E.A by D. Gopal, M. Imdad, C. Vetro [31]. A. Allouche [21] proved fixed-point theorems employing implication, also gave various properties in 2007. S. Jain and B. Mundra S. Aake, discussed an implication and showed the fixed-point results. M. Imdad and J. Ali [30] prolonged Generalised fixed-point theorem amusing implication. We demonstrate certain consequences of fixed-point in sense of Generalised $\breve{E}$-fuzzy-metric aimed at $\breve{w}c$ mappings providing the property-E.A through implication. Specific outcomes derived with several illustrations.

2. Preliminaries

Definition 2.1 [1]**.** A Fuzzy-set $\widetilde{A}$ by $\widetilde{A} = \{(x, \mu A(x)) : x, \mu A(x) \in [0, 1]\}$. Now $X \in A, \mu A(x) \in [0, 1]$ is named as a membership function.

Definition 2.2 [5]**.** The mapping $* : [0, 1] \to [0, 1]$ is named as continual t-norm if the resulting conditions are fulfilled,

1. $*$ is associative and commutative.

2. $*$ is continuous.

3. $a * 1 = a$ for all $a \in [0, 1]$

4. $a * b \leq c * d$, for all $a \leq c$, $b \leq d$.

Definition 2.3 [2]**.** A well-ordered tuple $(X, \breve{M}, *)$ is named as fuzzy-metric if X is erratic, $*$ is continual triangular-norm and $\breve{M}$ is a fuzzy-set on $X^2 \times [0, \infty)$ has the subsequent appropriate conditions $\forall x, y \in X$ and $t, s > 0$.

(a) $\breve{M}(x, y, t) > 0, t > 0$.

(b) $\breve{M}(x, y, t) = 1$ iff $x = y, t > 0$.

(c) $\breve{M}(x, y, t) = \breve{M}(y, x, t)$.

(d) $\breve{M}(x, y, t) * \breve{M}(y, z, s) \leq \breve{M}(x, z, t + s)$

(e) $\breve{M}(x, y, \cdot) : (0, \infty) \to [0, 1]$ is continuous, $\forall x, y \in X$ and $t, s > 0$.

Definition 2.4 [4]**.** Consider X is an erratic set and $G : X^2 \to [0, \infty)$, it has the subsequent conditions,

(a) $G(x, y, t) = 0$ if $x = y = z$.

(b) $G(x, x, y) > 0$ for all $x, y \in X$ with $x \neq y$.

(c) $G(x, x, y) \leq G(x, y, z)$, for all $x, y, z \in X$ with $z \neq y$.

(d) $G(x, y, z) = G(p\{x, y, z\})$ (Symmetry) p, a permutation function.

(e) $G(x, y, z) \leq G(x, a, a) + G(a, y, z)$, $\forall x, y, z, a \in X$.

Therefore (X, G) is designate as generalised metric on X.

Illustration 2.5. If $X = R$ and the function $G : X^3 \times [0, \infty)$ by $G(x, y, z) = |x - y| + |y - z| + |z - x|$, then (X, G) is G-metric.

Definition 2.6 [11]. An ordered tuple $(X, \breve{E}, *)$ named as $\breve{E}$-fuzzy-metric, $*$ is continual t-norm, X is erratic set, $\breve{E}$ is Fuzzy-set on $X^3 \times (0, \infty)$ which fulfills the following, $\forall x, y, z \in X$, and $t, s > 0$.

(a) $\breve{E}(x, x, y, t) > 0$; $\breve{E}(x, x, y, t) \geq \breve{E}(x, y, z, t)$, for all $x, y, z \in X$ with $z \neq y$.

(b) $\breve{E}(x, y, z, t) = 1$ iff $x = y = z$.

(c) $\breve{E}(x, y, z, t) = \breve{E}(p\{x, y, z\}, t)$ (Symmetry), p, a permutation function.

(d) $\breve{E}(x, a, z, t) * \breve{E}(a, y, z, s) \leq \breve{E}(x, y, z, t + s)$.

(e) $\breve{E}(x, y, z, \cdot) : (0, \infty) \to [0, 1]$ is continuous.

Then $\breve{E}$ is designated as generalisation of fuzzy-metric.

Illustration 2.7. Consider X as any unpredictable set and G-metric on X. The continual t-norm $a * b = \min\{a, b\}$ for all $a, b \in [0, 1], t > 0$, Ensures $\breve{E}(x, y, z, t) = \dfrac{t}{t + G(x, y, z)}$. Formerly $(X, E, *)$ designate as $\breve{E}$ fuzzy-metric.

Lemma 2.8 [11]. *Let* $(X, \breve{E}, *)$ *be* $\breve{E}$-*fuzzy-metric. Then* $\breve{E}(x, y, z, t)$ *is increasing related as* t, $\forall x, y, z \in X$.

Proof. Set $a = y, z = y$ in $\breve{E}(x, a, z, t) * \breve{E}(a, y, z, s) \leq \breve{E}(x, y, z, t + s)$.

We acquire, $\breve{E}(x, y, y, t) \leq \breve{E}(x, y, y, t + s)$. If possible let $\breve{E}(x, y, z, t) > \breve{E}(x, y, y, t + s)$.

Again uncertainty we acquire $z = y$, in the above condition gives contravention.

Therefore $\breve{E}(x, y, z, t)$ is a non decreasing function related as t.

Definition 2.9. Define A, T are the functions from $(X, \breve{E}, *)$ into $(X, \breve{E}, *)$. The couple $\{A, T\}$ is designate as compatible, if $\lim_{n\to\infty} \breve{E}(ATx_n, TAx_n, TAx_n, t) = 1$, whenever $\{x_n\} \in X$ such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Tx_n = w$ for some $w \in X$ and for all $t > 0$.

Definition 2.10. Consider A, T are depicts from $(X, \breve{E}, *)$ into $(X, \breve{E}, *)$ are said as w̆eak compatible mapping, if $Ax = Tx$, implies that $ATx = TAx$, for some $x \in X$.

Definition 2.11. Consider A, T are depicts from $(X, \breve{E}, *)$ into $(X, \breve{E}, *)$. A couple $\{A, T\}$ is designate as non compatible, if $\lim_{n\to\infty} \breve{E}(ATx_n, TAx_n, TAx_n, t) \neq 1$, whenever $\{x_n\} \in X$ ensures $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Tx_n = w$ for some $w \in X$ and $t > 0$.

Definition 2.12. The couple $\{A, T\}$ of metric (X, G) reveals that to persuade a property-E.A if $\exists \{x_n\}$ in X ensures $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Tx_n = w$ for some $w \in X$.

Definition 2.13. The couple $\{A, T\}$ of $\breve{E}$-fuzzy-metric $(X, \breve{E}, *)$ reveals that to induce a property-E.A if $\exists \{x_n\}$ in X ensures $\lim_{n\to\infty} \breve{E}(Ax_n, Tx_n, w, t) = 1$, for some $w \in X$.

Definition 2.14. Self-mappings (A, T), (B, S) and (C, U) on $(X, \breve{E}, *)$ fulfills the common property-E.A if $\exists$ sequences $\{x_n\}, \{y_n\}$ and $\{z_n\}$ in X ensures, $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Tx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Sy_n = \lim_{n\to\infty} Cz_n = \lim_{n\to\infty} Uz_n = w$ for some $w \in X$.

Illustration 2.15. Let $(X, \breve{E}, *)$ be $\breve{E}$-fuzzy-metric, where $X = [0, 2]$ with min t-norm and $\breve{E}(x, y, z, t) = \frac{t}{t + G(x, y, z)}$, $\forall t > 0$ and $x, y, z \in X$.

Express T and A as $Tx = \begin{cases} 0, \text{ if } x \in [0, 1) \\ x, \text{ if } x \in [1, 2] \end{cases}$, $Ax = \begin{cases} x, \text{ if } x \in [0, 1) \\ 1, \text{ if } x \in [1, 2] \end{cases}$.

Let $\{x_n\} = \left\{2 - \frac{1}{n}\right\} \in X$ ensures $\lim_{n \to \infty} Ax_n = \lim_{n \to \infty} Tx_n = w$, where $w = \{1\} \in X$.

So $\lim_{n \to \infty} \breve{E}(Ax_n, Tx_n, w, t) = 1$, for some $w \in X$.

Thus $\{A, T\}$ fulfills the common property-E.A.

3. Implicit Relations

Denote (φ) be the set of all real continuous $\varphi : (R^+)^5 \to R^+$, takes the sufficient conditions,

(i) For $u, v \geq 0$, $\varphi(u, v, u, v, v) \geq 0$ (or) $\varphi(u, u, v, v, v) \geq 0$ implies $u \geq v$.

(ii) $\varphi(u, 1, 1, u, 1) \geq 0$ (or) $\varphi(u, 1, 1, 1, u) \geq 0$ implies $u \geq 1$.

Illustration 3.1.

1. Define $\varphi : (R^+)^5 \to R^+$ by $\varphi(t_1, t_2, t_3, t_4, t_5) = t_1 - \min\{t_2, t_3, t_4, t_5\}$

2. Define $\varphi : (R^+)^5 \to R^+$ by $\varphi(t_1, t_2, t_3, t_4, t_5) = 20t_1 - 18t_2 + 10t_3 - 10t_4 - 2t_5$.

4. Main Results

Theorem 4.1. *Let* $(X, E, *)$ *be* $\breve{E}$*-fuzzy-metric through* $*$ *is a continual t-norm. Self-mappings* (A, T), (B, S) *and* (C, U) *of X which gratifying,*

(i) $A(X) \subset U(X)$, $B(X) \subset T(X)$, $C(X) \subset S(X)$,

(ii) (A, T), (B, S), *and* (C, U) *are fulfills the common property* $(E.A)$,

(iii) *For some* $\varphi \in (\varphi)$ *and* $\forall x, y, z \in X$, $t > 0$,

$$\varphi(\breve{E}(Ax, By, Cz, t), \breve{E}(Ax, Tx, Tx, t), \breve{E}(By, Sy, Sy, t), \breve{E}(Cz, Uz, Uz, t), \breve{E}(Tx, Sy, Uz, t)) \geq 0 \tag{4.1}$$

(iv) *The couples* $(A, T), (B, S)$, *and* (C, U) *be there* $\breve{w}c$ *mapping*,

(v) $A(X)$ *or* $B(X)$ *or* $C(X)$ *or* $S(X)$ *or* $T(X)$ *or* $U(X)$ *is a closed subset of* X,

Then occurs a unique fixed-point of S, A, T, B, U *and* C.

Proof. Suppose (B, S) and (C, U) consumes the common property-E.A, then $\exists\{x_n\}$ and $\{y_n\}$ in X ensures, $\lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Sx_n = w$, and $\lim_{n\to\infty} Cy_n = \lim_{n\to\infty} Uy_n = w$ for some $w \in X$.

Since $B(X) \subset T(X)$, then $\exists$a sequence $\{z_n\}$ in X ensures $\lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Tz_n = w$, for some $w \in X$.

On putting $x = z_n, y = x_n, z = y_n$ in (4.1), we have, $\varphi(\breve{E}(Az_n, Bx_n, Cy_n, t)$, $\breve{E}(Az_n, Tz_n, Tz_n, t)$, $\breve{E}(Bx_n, Sx_n, Sx_n, t)$, $\breve{E}(Cy_n, Uy_n, Uy_n, t)$, $\breve{E}(Tz_n, Sx_n, Uy_n, t)) \geq 0$.

Limiting as $n \to \infty$,

$$\varphi(\breve{E}(Az_n, w, w, t), \breve{E}(Az_n, w, w, t), \breve{E}(w, w, w, t), \breve{E}(w, w, w, t), \breve{E}(w, w, w, t)) \geq 0.$$

From (φ), $\lim_{n\to\infty} Az_n = w$, for some $w \in X$.

Supposing $T(x)$ is closed, then $\exists\, p \in X$ ensures $Tp = w$, for some $u \in X$.

On putting $x = p$, $y = x_n$, $z = y_n$ in (4.1), we have,

$$\varphi(\breve{E}(Ap, Bx_n, Cy_n, t), \breve{E}(Ap, Tp, Tp, t), \breve{E}(Bx_n, Sx_n, Sx_n, t),$$
$$\breve{E}(Cy_n, Uy_n, Uy_n, t), \breve{E}(Tp, Sx_n, Uy_n, t) \geq 0.$$

Limiting as $n \to \infty$,

$$\varphi(\breve{E}(Ap, w, w, t), \breve{E}(Ap, w, w, t), \breve{E}(w, w, w, t), \breve{E}(w, w, w, t), \breve{E}(w, w, w, t)) \geq 0.$$

From (φ), $Ap = w$, for some $w \in X$. Hence $Tp = Ap = w$, for some $w \in X$.

Whereas the $\breve{w}c$ mapping of A, T indicates that $ATp = TAp$, then it $Aw = ATp = TAp = Tw$

$\therefore Aw = Tw.$

Since $A(X) \subset U(X)$, subsequently $\exists\, q \in X$ ensures $Ap = Uq = w$, for some $w \in X$.

On putting $x = p,\ y = x_n,\ z = q$ in (4.1),

$$\varphi(\breve{E}(Ap, Bx_n, Cq, t), \breve{E}(Ap, Tp, Tp, t), \breve{E}(Bx_n, Sx_n, Sx_n, t),$$
$$\breve{E}(Cq, Uq, Uq, t), \breve{E}(Tp, Sx_n, Uq, t)) \geq 0.$$

Limiting as $n \to \infty$,

$$\varphi(\breve{E}(w, w, Cq, t), \breve{E}(w, w, w, t), \breve{E}(w, w, w, t), \breve{E}(Cq, w, w, t), \breve{E}(w, w, w, t)) \geq 0.$$

From (φ), $Cq = w$, for some $w \in X$.

$\therefore Cq = Uq = w$, for some $w \in X$.

Whereas $\breve{w}c$ mapping of C and U indicates that $CUq = UCq$.

$\therefore Cw = CUq = UCq = Uw$, for some $w \in X$.

Since $C(X) \subset S(X)$, then $\exists\, r \in X$ ensures $Cq = Sr = w$, for some $w \in X$.

On putting $x = p,\ y = r,\ z = q$ in (4.1),

$$\varphi(\breve{E}(Ap, Br, Cq, t), \breve{E}(Ap, Tp, Tp, t), \breve{E}(Br, Sr, Sr, t),$$
$$\breve{E}(Cq, Uq, Uq, t), \breve{E}(Tp, Sr, Uq, t)) \geq 0.$$

Limiting as $n \to \infty$,

$$\varphi(\breve{E}(w, Br, w, t), \breve{E}(w, w, w, t), \breve{E}(Br, w, w, t), \breve{E}(w, w, w, t), \breve{E}(w, w, w, t)) \geq 0.$$

From (φ), $Br = w$, for some $w \in X$.

$\therefore Br = Sr = w$, for some $w \in X$.

Whereas $\breve{w}c$ mapping of B and S indicates that $BSr = SBr$.

$\therefore Bw = BSr = SBr = Sw$, for some $w \in X$.

Replace x by w, y by r, z by q in (4.1), we have,

$$\varphi(\breve{E}(Aw, Br, Cq, t), \breve{E}(Aw, Tw, Tw, t), \breve{E}(Br, Sr, Sr, t),$$

$$\breve{E}(Cq, Uq, Uq, t), \breve{E}(Tw, Sr, Uq, t)) \geq 0.$$

Proceeding limit as $n \to \infty$, $\varphi(\breve{E}(Aw, w, w, t), \breve{E}(Aw, Aw, Aw, t), \breve{E}(w, w, w, t), \breve{E}(w, w, w, t), \breve{E}(Aw, w, w, t)) \geq 0$.

In view of (φ), $Aw = w$.

$\therefore Aw = Sw = w$.

Replace x by p, y by w, z by q in (4.1), we have,

$$\varphi(\breve{E}(Ap, Bw, Cq, t), \breve{E}(Ap, Tp, Tp, t), \breve{E}(Bw, Sw, Sw, t),$$

$$\breve{E}(Cq, Uq, Uq, t), \breve{E}(Tp, Sw, Uq, t)) \geq 0.$$

Limiting as $n \to \infty$,

$$\varphi(\breve{E}(w, Bw, w, t), \breve{E}(w, w, w, t), \breve{E}(Bw, Bw, Bw, t), \breve{E}(w, w, w, t), \breve{E}(w, Bw, w, t)) \geq 0.$$

From (φ), $Bw = w$.

$\therefore Bw = Sw = w$.

Replace x by p, y by r, z by w in (4.1), we have,

$$\varphi(\breve{E}(Ap, Br, Cw, t), \breve{E}(Ap, Tp, Tp, t), \breve{E}(Br, Sr, Sr, t),$$

$$\breve{E}(Cw, Uw, Uw, t), \breve{E}(Tp, Sr, Uw, t)) \geq 0.$$

Limiting as $n \to \infty$,

$$\varphi(\breve{E}(w, w, Cw, t), \breve{E}(w, w, w, t), \breve{E}(w, w, w, t), \breve{E}(Cw, Cw, Cw, t),$$

$$\breve{E}(w, w, Cw, t)) \geq 0.$$

From (φ), $Cw = w$.

$\therefore Cw = Uw = w$.

The proof is similar when $S(X)$ or $U(X)$ is a closed subset of X.

Whether $A(X)$ or $B(X)$ or $C(X)$ is a closed subset of X, which are related to the cases such as $U(X)$ or $T(X)$ or $S(X)$ respectively is closed.

To confirm the uniqueness.

Suppose $w' \neq w$ be a fixed-point different from w, then from (4.1)

$$\varphi(\breve{E}(Aw', Bw, Cw, t), \breve{E}(Aw', Tw', Tw', t), \breve{E}(Bw, Sw, Sw, t),$$
$$\breve{E}(Cw, Uw, Uw, t), \breve{E}(Tw', Sw, Uw, t)) \geq 0.$$

Limiting as $n \to \infty$,

$$\varphi(\breve{E}(w', w, w, t), \breve{E}(w', w', w', t), \breve{E}(w, w, w, t), \breve{E}(w, w, w, t), \breve{E}(w, w, w, t)) \geq 0.$$

From (φ), $w' = w$.

Henceforth the theorem.

Theorem 4.2. *Let $(X, \breve{E}, *)$ be $\breve{E}$-fuzzy-metric space with $*$ is a continual t-norm. The Self-mappings (A, T), (B, S) and (C, U) of X amusing, for some $\varphi \in (\varphi)$, for all $x, y, z \in X$, $t > 0$,*

$$\varphi(\breve{E}(Ax, By, Cz, t), \breve{E}(Ax, Tx, Tx, t), \breve{E}(By, Sy, Sy, t),$$
$$\breve{E}(Cz, Uz, Uz, t), \breve{E}(Tx, Sy, Uz, t)) \geq 0. \tag{4.2}$$

Then A, T, B, S and C, U have eccentric fixed-point which is in X, provided (A, T), (B, S) and (C, U) are assures the common property $(E.A)$, $S(x)$, $T(x)$ and $U(x)$ are closed subsets of X and (A, T), (B, S) and (C, U) remain $\breve{w}c$ mapping.

Proof. Suppose (A, T), (B, S) and (C, U) of self-mappings of a $\breve{E}$-fuzzy-metric space $(X, E, *)$ are fulfill the common property (E.A) if $\exists$ sequences

$\{x_n\}, \{y_n\}$ and $\{z_n\}$ in X ensures, $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Tx_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Sy_n = \lim_{n\to\infty} Cz_n = \lim_{n\to\infty} Uz_n = w$ for some $w \in X$.

Suppose $T(x), U(x)$ and $S(X)$ are closed subsets of X, then $\exists\, p, q, r$ in X ensures, $Tp = Uq = Sr = w$, for some $w \in X$.

Replace x by p, y by y_n, z by z_n in (4.2), we have,

$$\varphi(\breve{E}(Ap, By_n, Cz_n, t), \breve{E}(Ap, Tp, Tp, t), \breve{E}(By_n, Sy_n, Sy_n, t),$$
$$\breve{E}(Cz_n, Uz_n, Uz_n, t), \breve{E}(Tp, Sy_n, Uz_n, t)) \geq 0.$$

Limiting as $n \to \infty$,

$$\varphi(\breve{E}(Ap, w, w, t), \breve{E}(Ap, w, w, t), \breve{E}(w, w, w, t), \breve{E}(w, w, w, t), \breve{E}(w, w, w, t)) \geq 0.$$

From (φ), $Ap = w$.

Hence $Ap = Tp = w$.

Whereas $\breve{w}c$ mapping of A, $TATp = TAp$

$\therefore\ Aw = ATp = TAp = Tw$.

Therefore, $Aw = Tw$.

Replace x by p, y by r, z by y_n in (4.2), We have,

$$\varphi(\breve{E}(Ap, Br, Cz_n, t), \breve{E}(Ap, Tp, Tp, t), \breve{E}(Br, Sr, Sr, t),$$
$$\breve{E}(Cz_n, Uz_n, Uz_n, t), \breve{E}(Tp, Sy_n, Uz_n, t)) > 0.$$

Limiting as $n \to \infty$,

$$\varphi(\breve{E}(w, Br, w, t), \breve{E}(w, w, w, t), \breve{E}(Br, w, w, t), \breve{E}(w, w, w, t), \breve{E}(w, w, w, t)) \geq 0.$$

From (φ), $Br = w$.

$\therefore\ Br = Sr = w$.

Whereas $\breve{w}c$ mapping of B, S $BSr = SBr$.

$Bw = BSr = SBr = Sw$, for some $w \in X$.

We proclaim that $Cq = w$

Replace x by p, y by r, z by q in (4.2). We have,

$$\varphi(\breve{E}(Ap, Br, Cq, t), \breve{E}(Ap, Tp, Tp, t), \breve{E}(Br, Sr, Sr, t),$$

$$\breve{E}(Cq, Uq, Uq, t), \breve{E}(Tp, Sr, Uq, t)) \geq 0.$$

Limiting as $n \to \infty$,

$$\varphi(\breve{E}(w, w, Cq, t), \breve{E}(w, w, w, t), \breve{E}(w, w, w, t), \breve{E}(Cq, w, w, t), \breve{E}(w, w, w, t)) \geq 0.$$

From (φ) indicates that, $Cq = w$.

$\therefore Cq = Uq = w$. Whereas $\breve{w}c$ mapping of C, $UCUq = UCq$.

$Cw = CUq = UCq = Uw$, for some $w \in X$.

Replace x by w, y by r, z by q in (4.2), we have,

$$\varphi(\breve{E}(Aw, Br, Cq, t), \breve{E}(Aw, Tw, Tw, t), \breve{E}(Br, Sr, Sr, t),$$

$$\breve{E}(Cq, Uq, Uq, t), \breve{E}(Tw, Sr, Uq, t)) \geq 0.$$

Proceeding limit as $n \to \infty$,

$$\varphi(\breve{E}(Aw, w, w, t), \breve{E}(Aw, Aw, Aw, t), \breve{E}(w, w, w, t), \breve{E}(w, w, w, t),$$

$$\breve{E}(Aw, w, w, t)) \geq 0.$$

In view of (φ), $Aw = w$.

$\therefore Aw = Sw = w$.

Replace x by p, y by w, z by q in (4.2),

$$\varphi(\breve{E}(Ap, Bw, Cq, t), \breve{E}(Ap, Tp, Tp, t), \breve{E}(Bw, Sw, Sw, t),$$

$$\breve{E}(Cq, Uq, Uq, t), \breve{E}(Tp, Sw, Uq, t)) \geq 0.$$

Taking limit as $n \to \infty$,

$$\varphi(\breve{E}(w, Bw, w, t), \breve{E}(w, w, w, t), \breve{E}(Bw, Bw, Bw, t), \breve{E}(w, w, w, t), \breve{E}(w, Bw, w, t)) \geq 0.$$

From (φ) indicates that, $Bw = w$.

$\therefore Bw = Sw = w$.

Replace x by p, y by r, z by w in (4.2),

$$\varphi(\breve{E}(Ap, Br, Cw, t), \breve{E}(Ap, Tp, Tp, t), \breve{E}(Br, Sr, Sr, t),$$

$$\breve{E}(Cw, Uw, Uw, t), \breve{E}(Tp, Sr, Uw, t)) \geq 0.$$

Limiting as $n \to \infty$,

$$\varphi(\breve{E}(w, w, Cw, t), \breve{E}(w, w, w, t), \breve{E}(w, w, w, t), \breve{E}(Cw, Cw, Cw, t), \breve{E}(w, w, Cw, t)) \geq 0.$$

Form (φ) indicates that, $Cw = w$.

$\therefore Cw = Uw = w$.

$\therefore$ The common fixed-point of A, T, B, S, C and U is w.

To confirm the uniqueness.

Suppose $w' \neq w$, be a fixed-point different from w, then from (4.2)

$$\varphi(\breve{E}(Aw', Bw, Cw, t), \breve{E}(Aw', Tw', Tw', t), \breve{E}(Bw, Sw, Sw, t),$$

$$\breve{E}(Cw, Uw, Uw, t), \breve{E}(Tw, Sw, Uw, t)) \geq 0.$$

Limiting as $n \to \infty$,

$$\varphi(\breve{E}(w', w, w, t), \breve{E}(w', w', w', t), \breve{E}(w, w, w, t), \breve{E}(w, w, w, t), \breve{E}(w, w, w, t)) \geq 0.$$

From (φ) indicates that, $w' = w$.

Henceforth the theorem.

5. Conclusion

Fixed-point system has several applications and numerous branches of sciences in the ranges of stability theory, nonlinear programming, economics, etc. In this paper, we confined the fixed-point approach on generalized $\breve{E}$-

fuzzy-metric space and this hypothesis aimed at $\breve{w}c$ mapping providing the property-E.A by implication.

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