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DIOPHANTINE TRIPLES INVOLVING SQUARE PYRAMIDAL NUMBERS

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Abstract

In this paper, we scan for three particular polynomials with whole number coefficients to such an extent that the result of any two numbers expanded by a non-zero number (or polynomials with number coefficients) is an ideal square.

Introduction

In mathematics, a Diophantine condition is a polynomial condition, conventionally in at any rate two inquiries, so much that lone the entire number game plans are searched for or analyzed (an entire number course of action is a response so much that all the inquiries take entire number values). The word Diophantine suggests the Greek mathematician of the third century, Diophantus of Alexandria, who made an examination of such conditions and was one of the foremost mathematicians to bring symbolism into variable based math. The mathematical examination of Diophantine issues that Diophantus began is as of now called Diophantine analysis. While particular conditions present such a confound and have been considered from the start of time, the meaning of general speculations of Diophantine conditions (past the theory of quadratic constructions) was an achievement of

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the 20th century.

Notation. p_n^4 : square pyramidal number of rank n.

In [1-6], speculation of numbers was discussed. In [7-9], Diophantine altogether increments with the property for any abstract number and besides for any straight polynomials were discussed. Lately, in [10, 11] pentatope numbers were analyzed for its interesting Dio-essentially increments and evaluated using z-change. This paper targets creating Dio-Triples where the consequence of any two people from the triple with the development of a non-zero entire number or a polynomial with number coefficients satisfies the vital property. In like manner, we present three fragments where in all of which we find the Diophantine altogether increments from Square Pyramidal number of different situations with their relating properties.

Basic Definition

A set of three distinct polynomials with integer coefficients (a_1, a_2, a_3) is said to be Diophantine triple with property $D(n)$ if $a_i^* a_j + n$ is a perfect square for all $1 \leq i < j \leq 3$, where n may be non zero or polynomial with integer coefficients.

Method of Analysis

Section A.

Construction of the Diophantine triples involving square pyramidal number of rank n and $n-1$

Let $a = 6p_n^4$ and $b = 6p_{n-1}^4$ be square pyramidal numbers of rank n and $n-1$ respectively. Now, $a = 6p_n^4$ and $b = 6p_{n-1}^4$

$$ab + (-3n^4 + 3n^2) = 4n^6 - 5n^4 + n^2 - 3n^4 + 3n^2$$

$$= (2n^3 - 2n)^2$$

$$ab + (-3n^4 + 3n^2) = (2n^3 - 2n)^2 = \alpha^2 \tag{1}$$

Equation (1) is a perfect square.

$$ab + (-3n^4 + 3n^2) = \alpha^2 \text{ where } \alpha = 2n^3 - 2n.$$

Let c be non zero-integer such that

$$ac + (-3n^4 + 3n^2) = \beta^2 \quad (2)$$

$$bc + (-3n^4 + 3n^2) = \gamma^2 \quad (3)$$

setting $\beta = a + \alpha$ and $\gamma = b + \alpha$,

$$(2)\text{-}(3) \Rightarrow c(b - a) = \gamma^2 - \beta^2$$

$$\Rightarrow c(b - a) = (b + a + 2\alpha)(b - a)$$

$$\Rightarrow c = a + b + 2\alpha$$

$$c = 8n^3 - 2n$$

$$\Rightarrow c = (2(a + b - 3n)).$$

Therefore, the triples $\{a, b, (2(a + b - 3n))\} = \{6p_n^4, 6p_{n-1}^4, (2(6p_n^4 + 6p_{n-1}^4 - 3n))\}$ is a Diophantine triples with the property $D(-3n^4 + 3n^2)$.

Some numerical examples are given below in the following table.

n	Diophantine Triples	$D(-3n^4 + 3n^2)$.
2	(30, 6, 60)	-36
3	(84, 30, 210)	-216
4	(180, 84, 504)	-720

Section B.

Construction of the Diophantine triples involving square pyramidal number of rank n and $n - 2$

Let $a = 6p_n^4$ and $b = 6p_{n-2}^4$ be square pyramidal numbers of rank n and $n - 2$ respectively.

Now,

$$a = 6p_n^4 \text{ and } b = 6p_{n-1}^4$$

$$ab + (-2n^3 + 3n^2 + 2n + 1) = 4n^6 - 12n^5 + n^4 + 16n^3 - 2n^2 - 4n + 1$$

$$= (2n^3 - 3n^2 - 2n + 1)^2$$

$$ab + (-2n^3 + 3n^2 + 2n + 1) = (2n^3 - 3n^2 - 2n + 1)^2 = \alpha^2. \tag{4}$$

Equation (4) is a perfect square.

$ab + (-2n^3 + 3n^2 + 2n + 1) = \alpha^2$ where $\alpha = 2n^3 - 3n^2 - 2n + 1$.

Let c be non zero-integer such that,

$$ac + (-2n^3 + 3n^2 + 2n + 1) = \beta^2 \tag{5}$$

$$bc + (-2n^3 + 3n^2 + 2n + 1) = \gamma^2 \tag{6}$$

setting $\beta = a + \alpha$ and $\gamma = b + \alpha$,

$$(5)\text{-}(6) \Rightarrow c(b - a) = \gamma^2 - \beta^2$$

$$\Rightarrow c(b - a) = (b + a + 2\alpha)(b - a)$$

$$\Rightarrow c = a + b + 2\alpha$$

$$c = 8n^3 - 12n^2 + 10n - 4$$

$$\Rightarrow c = (2(a + b - 9n + 4))$$

Therefore, the triples $\{a, b, (2(a + b - 9n + 4))\} = \{6p_n^4, 6p_{n-2}^4, (2(6p_n^4 + 6p_{n-2}^4 - 9n + 4))\}$ is a Diophantine triples with the property $D(-2n^3 + 3n^2 + 2n + 1)$.

Some numerical examples are given below in the following table.

Table 2.

n	Diophantine	$D(-2n^3 + 3n^2 + 2n + 1)$
1	(6, 0, 2)	4
2	(30, 0, 32)	1

3	(84, 6,134)	-20

Section C.

Construction of the Diophantine triples involving square pyramidal number of rank n and $n-3$

Let $a = 6p_n^4$ and $b = 6p_{n-3}^4$ be square pyramidal numbers of rank n and $n-3$ respectively.

Now,

$$a = 6p_n^4 \text{ and } b = 6p_{n-3}^4$$

$$ab + (n^4 - 18n^2 + 18n + 36) = 4n^6 - 24n^5 + 32n^4 + 36n^3 - 71n^2 - 12n + 36$$

$$= (2n^3 - 6n^2 - n + 6)^2$$

$$ab + (n^4 - 18n^2 + 18n + 36) = (2n^3 - 6n^2 - n + 6)^2 = \alpha^2 \quad (7)$$

Equation (7) is a perfect square.

$ab + (n^4 - 18n^2 + 18n + 36) = \alpha^2$ where $\alpha = 2n^3 - 6n^2 - n + 6$

Let c be non zero-integer such that

$$ac + n^4 - 18n^2 + 18n + 36 = \beta^2 \quad (8)$$

$$bc + n^4 - 18n^2 + 18n + 36 = \gamma^2 \quad (9)$$

setting $\beta = a + \alpha$ and $\gamma = b + \alpha$,

(8)-(9) $\Rightarrow c(b-a) = \gamma^2 - \beta^2$

$$\Rightarrow c(b-a) = (b + a + 2\alpha)(b-a)$$

$$\Rightarrow c = a + b + 2\alpha$$

$$c = 8n^3 - 24n^2 + 36n - 18$$

$$\Rightarrow c = (2(a + b - 20n + 21))$$

Therefore, the triples, $\{a, b, (2(a + b - 20n + 21))\} = \{6p_n^4, 6p_{n-3}^4, (2(6p_n^4 + 6p_{n-3}^4 - 20n + 21))\}$ is a Diophantine triples with the property

$D(n^4 - 18n^2 + 18n + 36)$.

Some numerical examples are given below in the following table.

Table 3.

n	Diophantine	$D(-n^4 + 18n^2 + 18n + 36)$
1	(6, -6, 2)	37
2	(30, 0, 22)	16
3	(84, 0, 90)	9

Conclusion

We have presented the Diophantine triples and the special dio 3-triples involving square pyramidal numbers. To conclude one may look for triples or quadruples for different numbers with their relating properties.

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