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$g^{\#}\alpha$- LOCALLY CLOSED SETS AND $g^{\#}\alpha$- LOCALLY CONTINUOUS FUNCTIONS IN SUPRA TOPOLOGICAL SPACES

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Abstract

The aim of this paper is to introduce decompositions namely supra $g^{\#}\alpha$- locally closed sets and define supra $g^{\#}\alpha$- locally continuous functions. This paper also discussed some of their properties.

1. Introduction

In 1983 Mashhour et al. [8] introduced Supra topological spaces and studied S-continuous maps and S^{*}- continuous maps. In 1997, Arokiarani .I [2] introduced and investigated some properties of regular generalized locally closed sets and RGL-continuous functions. In 2011, Bharathi. S [3] introduced and investigated several properties of generalization of locally b-closed sets

In this paper, we define a new set called supra $g^{\#}\alpha$- locally closed and also define supra $g^{\#}\alpha$- locally continuous functions and investigated some of the basic properties for this class of functions.

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2. Preliminaries

Definition 2.1. A subfamily of μ of X is said to be a supra topology on X, if

(i) X, ϕ, $\phi \in \mu$

(ii) if $A_i \in \mu$ for all $i \in J$ then $\cup A_i \in \mu$.

The pair (X, μ) is called supra topological space. The elements of μ are called supra open sets in (X, μ) and complement of a supra open set is called a supra closed set.

Definition 2.2. (i) The supra closure of a set A is denoted by $cl^{\mu}(A)$ and is defined as $cl^{\mu}(A) = \cap\{B : B$ is a supra closed set and $A \subseteq B\}$.

(ii) The supra interior of a set A is denoted by $\text{int}^{\mu}(A)$ and defined as $\text{int}^{\mu}(A) = \cup\{B : B$ is a supra open set and $A \supseteq B\}$.

Definition 2.3. Let (X, τ) be a topological spaces and μ be a supra topology on X. We call μ a supra topology associated with τ if $\tau \subset \mu$.

Definition 2.4. Let (X, μ) be a supra topological space. A Subset A of X is called supra α-open set if $A \subseteq \text{int}^{\mu}(cl^{\mu}(\text{int}^{\mu}(A)))$. The complement of supra α-open set is supra α-closed set.

Definition: 2.5. Let (X, μ) be a supra topological space. A Subset A of X is called supra g^* closed set if $cl^{\mu}(A) \subseteq U$ whenever $A \subseteq U$ and U is supra g-open set of X. The complement of supra g-closed set is supra g-open set.

Definition 2.6. Let (X, μ) be a supra topological space. A Subset A of X is called supra $g^{\#}\alpha$-closed set if $\alpha cl^{\mu}(A) \subseteq U$, whenever $A \subseteq U$ and U is supra g-open set of X. The complement of supra $g^{\#}\alpha$-closed set is called supra $g^{\#}\alpha$-open set.

Definition 2.7. Let (X, τ) and (Y, σ) be two topological spaces and $\tau \subset \mu$. A function $f : (X, \tau) \to (Y, \sigma)$ is called supra continuous, if the inverse image of each supra open set of Y is a supra open set in X.

Definition 2.8. A map $f : (X, \tau) \to (Y, \sigma)$ is said to be

(i) supra $g^{\#}\alpha$- continuous if $f^{-1}(V)$ is supra $g^{\#}\alpha$- closed in (X, τ) for every supra closed set V of (Y, σ).

(ii) supra $g^{\#}\alpha$- irresolute if $f^{-1}(V)$ is supra $g^{\#}\alpha$- closed in (X, τ) for every supra $g^{\#}\alpha$- closed set V of (Y, σ).

Definition 2.9. Let A and B be subsets of X. Then A and B are said to be supra separated, if $A \cap cl^{\mu}(B) = B \cap cl^{\mu}(B) = \phi$.

3. Supra $g^{\#}\alpha$- Locally Closed Sets

Definition 3.1. Let (X, μ) be a supra topological space. A subset A of (X, μ) is called supra $g^{\#}\alpha$- locally closed set, if $= U \cap V$, where U is supra $g^{\#}\alpha$- open in (X, μ) and V is supra $g^{\#}\alpha$- closed in (X, μ). The collection of all supra $g^{\#}\alpha$- locally closed set S of X will be denoted by $S\text{-}g^{\#}\alpha LC(X)$.

Remark 3.2. Every supra $g^{\#}\alpha$- closed set (resp. supra $g^{\#}\alpha$- open set) is $S\text{-}g^{\#}\alpha LC$.

Definition 3.3. For a subset A of supra topological space (X, μ), $A \in S\text{-}g^{\#}\alpha LC^{*}(X, \mu)$, if there exist a supra $g^{\#}\alpha$- open set U and a supra closed set V of (X, μ), respectively such that $A = U \cap V$.

Remark 3.4. Every supra $g^{\#}\alpha$- closed set (resp. supra $g^{\#}\alpha$- open set) is $S\text{-}g^{\#}\alpha LC^{*}$.

Definition 3.5. For a subset A of supra topological space (X, μ), $A \in S\text{-}g^{\#}\alpha LC^{**}(X, \mu)$, if there exist a supra open set U and a supra $g^{\#}\alpha$- closed set V of (X, μ), respectively such that $A = U \cap V$.

Remark 3.6. Every supra $g^{\#}\alpha$-closed set (resp. supra $g^{\#}\alpha$-open set) is $S\text{-}g^{\#}\alpha LC^{**}$.

Theorem 3.7. *Let A be a subset of* (X, μ). *If* $A \in S\text{-}g^{\#}\alpha LC^{*}(X, \mu)$ (*or*) $S\text{-}g^{\#}\alpha LC^{**}(X, \mu)$ *then A is* $S\text{-}g^{\#}\alpha LC(X, \mu)$.

Proof. Given $A \in S\text{-}g^{\#}\alpha LC^{*}(X, \mu)$ (or) $S\text{-}g^{\#}\alpha LC^{**}(X, \mu)$, by definition $A = U \cap V$, where U is supra $g^{\#}\alpha$-open set and V is supra closed set (or) U is supra open set and V is supra $g^{\#}\alpha$-closed set. By Theorem 3.3 [5] every supra closed set is supra $g^{\#}\alpha$-closed set, therefore V is supra $g^{\#}\alpha$-closed set (or) every supra open set is supra $g^{\#}\alpha$-closed set, therefore U is supra $g^{\#}\alpha$-open set. Then A is $S\text{-}g^{\#}\alpha LC(X, \mu)$.

Example 3.8. Let $X = \{a, b, c\}$ and $\mu = \{X, \varphi, \{b\}, \{c\}, \{a, c\}\}$. In this (X, μ), $A \in S\text{-}g^{\#}\alpha LC^{**}(X, \mu)$, and $S\text{-}g^{\#}\alpha LC(X, \mu)$ are the proper subset of $S\text{-}g^{\#}\alpha LC(X, \mu)$, because $S\text{-}g^{\#}\alpha LC(X, \mu) = P(X)$.

Theorem 3.9. *Let A be a subset of* (X, μ). *If* $A \in S\text{-}g^{\#}\alpha LC(X, \mu)$ *then A is* $S\text{-}g^{\#}\alpha LC(X, \mu)$.

Proof. Given $A \in S\text{-}LC$, by definition $A = U \cap V$, where U is supra open set and V is supra closed set. Since every supra open set is supra $g^{\#}\alpha$-open set and every supra closed set is supra $g^{\#}\alpha$-closed set. Then $A \in S\text{-}g^{\#}\alpha LC(X, \mu)$.

The converse of the above theorem is not true from the following example.

Example 3.10. Let $X = \{a, b, c\}$ and $\mu = \{X, \varphi, \{a\}\}$. $S\text{-}LC(X, \mu) = \{X, \varphi, \{a\}, \{b, c\}\}$ and $S\text{-}g^{\#}\alpha LC(X, \mu) = P(X)$.

Theorem 3.11. *For a subset A of* (X, μ), *the following are equivalent*:

(i) $A \in S\text{-}g^{\#}\alpha LC^{**}(X, \mu)$

(ii) $A = U \cap \alpha cl^{\mu}(A)$, *for some supra open set* U

(iii) $\alpha cl^{\mu}(A) - A$ *is supra* $g^{\#}\alpha$- *closed*

(iv) $A \in [X\text{-}cl^{\mu}(A)]$ *is supra* $g^{\#}\alpha$- *open*

Proof. (i) $\Rightarrow$ (ii): Given $A \in S\text{-}g^{\#}\alpha LC^{**}(X, \mu)$, then there exist a supra open subset U and a supra $g^{\#}\alpha$- closed subset V such that $A = U \cap V$. Since $A \subset$ U and $A \subset \alpha cl^{\mu}(A)$, then $A \subset \alpha cl^{\mu}(A)$.

Conversely, we have $\alpha cl^{\mu}(A), \subset V$ and hence $U \cap V \supset U \cap \alpha cl^{\mu}(A)$. Therefore $U \cap \alpha cl^{\mu}(A)$.

(ii) $\Rightarrow$ (i) Let $A = U \cap \alpha cl^{\mu}(A)$, for some supra open set U. Clearly, $\alpha cl^{\mu}(A)$ is supra $g^{\#}\alpha$- closed and hence $A = U \cap \alpha cl^{\mu}(A) \in S\text{-}g^{\#}LC^{**}(X, \mu)$.

(ii) $\Rightarrow$ (iii) Let $A = U \cap \alpha cl^{\mu}(A)$, for some supra open set U. Then $A \in S\text{-}g^{\#}LC^{**}(X, \mu)$. This implies U is supra open and $\alpha cl^{\mu}(A)$ is supra $g^{\#}\alpha$- closed. Therefore $\alpha cl^{\mu}(A) - A$ is supra $g^{\#}\alpha$- closed.

(iii) $\Rightarrow$ (ii) Let $U = X - [\alpha cl^{\mu}(A) - A]$. By (iii) U is supra $g^{\#}\alpha$- open in X. We know that every supra open is supra $g^{\#}\alpha$- open, therefore U is supra open in X. Then $A = U \cap \alpha cl^{\mu}(A)$ holds.

(iii) $\Rightarrow$ (iv) Let $Q = \alpha cl^{\mu}(A) - A$ be supra $g^{\#}\alpha$- closed. Then $X - Q = X \alpha cl^{\mu}(A) - A = A \cup [X - \alpha cl^{\mu}(A)]$.

Since $X - Q$ is supra $g^{\#}\alpha$- open, $A \cup [X - \alpha cl^{\mu}(A)]$ is supra $g^{\#}\alpha$- open.

(iv) $\Rightarrow$ (iii) Let $U = A \cup [X - \alpha cl^{\mu}(A)]$. Since $X - U$ is supra $g^{\#}\alpha$- closed and $X - U = \alpha cl^{\mu}(A) - A$ is supra $g^{\#}\alpha$- closed.

Definition 3.12. Let A be subset of (X, μ). Then

(i) The supra $g^{\#}\alpha$-closure of a set A is denoted by $g^{\#}\alpha cl^{\mu}(A)$, define as $g^{\#}\alpha\text{-}cl^{\mu}(A) = \bigcap\{B : B$ is supra $g^{\#}\alpha$-closed and $A \subseteq B\}$.

(ii) The supra $g^{\#}\alpha$-interior of a set A is denoted by $g^{\#}\alpha\text{-int}^{\mu}(A)$, define as $g^{\#}\alpha\text{-int}^{\mu}(A) = \bigcup\{B : B$ is supra $g^{\#}\alpha$-open and $B \subseteq A\}$.

Theorem 3.13. *For a subset A of* (X, μ), *the following are equivalent*:

(i) $A \in S\text{-}g^{\#}\alpha LC(X, \mu)$

(ii) $A = U \cap g^{\#}\alpha\text{-}cl^{\mu}(A)$, *for some supra* $g^{\#}\alpha$-*open set* U

(iii) $g^{\#}\alpha\text{-}cl^{\mu}(A) - A$ *is supra* $g^{\#}\alpha$-*closed*

(iv) $A \cup [X - g^{\#}\alpha\text{-}cl^{\mu}(A)]$ *is supra* $g^{\#}\alpha$-*open*

Proof. (i) $\Rightarrow$ (ii) Given $A \in S\text{-}g^{\#}\alpha LC(X, \mu)$, then there exist a supra $g^{\#}\alpha$-open subset U and a supra $g^{\#}\alpha$-closed subset V such that $A = U \cap V$. Since $A \subset U$ and $A \subset g^{\#}\alpha\text{-}cl^{\mu}(A)$, then $A \subset g^{\#}\alpha\text{-}cl^{\mu}(A)$. Conversely, we have $g^{\#}\alpha\text{-}cl^{\mu}(A) \subset V$ and hence $A = U \cap V \supset U \cap g^{\#}\alpha\text{-}cl^{\mu}(A)$. Therefore $A = U \cap g^{\#}\alpha\text{-}cl^{\mu}(A)$.

(ii) $\Rightarrow$ (i) Let $A = U \cap g^{\#}\alpha\text{-}cl^{\mu}(A)$, for some supra $g^{\#}\alpha$-open set U. Clearly, $g^{\#}\alpha\text{-}cl^{\mu}(A)$ is supra $g^{\#}\alpha$-closed and hence $A = U \cap g^{\#}\alpha\text{-}cl^{\mu}(A) \in S\text{-}g^{\#}\alpha LC(X, \mu)$.

(ii) $\Rightarrow$ (iii) Let $A = U \cap g^{\#}\alpha\text{-}cl^{\mu}(A)$, for some supra $g^{\#}\alpha$-open set U. Then $A \in S\text{-}g^{\#}\alpha LC(X, \mu)$. This implies U is supra $g^{\#}\alpha$-open and $g^{\#}\alpha\text{-}cl^{\mu}(A)$ is supra $g^{\#}\alpha$-closed. Therefore $g^{\#}\alpha\text{-}cl^{\mu}(A) - A$ is supra $g^{\#}\alpha$-closed.

(iii) $\Rightarrow$ (ii) Let $U = X - [g^{\#}\alpha\text{-}cl^{\mu}(A) - A]$. By (iii) U is supra $g^{\#}\alpha$-open in X. Then $A = U \cap g^{\#}\alpha\text{-}cl^{\mu}(A)$ holds.

(iii) $\Rightarrow$ (iv) Let $Q = g^{\#}\alpha\text{-}cl^{\mu}(A) - A$ be supra $g^{\#}\alpha$- closed. Then $X - Q = X - [g^{\#}\alpha\text{-}cl^{\mu}(A) - A] = A \cup [X - g^{\#}\alpha\text{-}cl^{\mu}(A)]$. Since $X - Q$ is supra $g^{\#}\alpha$- open, $A \cup [X - g^{\#}\alpha\text{-}cl^{\mu}(A)]$ is supra $g^{\#}\alpha$- open.

(iv) $\Rightarrow$ (iii) Let $U = A \cup [X - g^{\#}\alpha\text{-}cl^{\mu}(A)]$. Since $X - U$ is supra $g^{\#}\alpha$- closed and $X - U = g^{\#}\alpha\text{-}cl^{\mu}(A) - A$ is supra $g^{\#}\alpha$- closed.

Theorem 3.14. *For a subset A of* (X, μ), *the following are equivalent*:

(i) $A \in S\text{-}g^{\#}\alpha LC^{*}(X, \mu)$

(ii) $A = U \cap cl^{\mu}(A)$, *for some supra* $g^{\#}\alpha$- *open set U*

(iii) $\alpha cl^{\mu}(A) - A$ *is supra* $g^{\#}\alpha$- *closed*

(iv) $A \cup [X - cl^{\mu}(A)]$ *is supra* $g^{\#}\alpha$- *open*

Proof. (i) $\Rightarrow$ (ii) Given $A \in S\text{-}g^{\#}\alpha LC^{*}(X, \mu)$, then there exist a supra $g^{\#}\alpha$- open subset U and a supra closed subset V such that $A = U \cap V$. Since $A \subset U$ and $A \subset \alpha cl^{\mu}(A)$, then $A \subset U \cap \alpha cl^{\mu}(A)$.

Conversely, we have $cl^{\mu}(A) \subset V$ and hence $A = U \cap V \supset cl^{\mu}(A)$. Therefore $A = U \cap \alpha cl^{\mu}(A)$.

(ii) $\Rightarrow$ (i) Let $A = U \cap cl^{\mu}(A)$, for some supra $g^{\#}\alpha$- open set U. Clearly, $cl^{\mu}(A)$ is supra closed and hence $A = U \cap cl^{\mu}(A) \in S\text{-}g^{\#}\alpha LC^{*}(X, \mu)$.

(ii) $\Rightarrow$ (iii): Let $A = U \cap g^{\#}\alpha cl^{\mu}(A)$, for some supra $g^{\#}\alpha$- open set U. Then $A \in S\text{-}g^{\#}\alpha LC^{*}(X, \mu)$. This implies U is supra $g^{\#}\alpha$- open and $cl^{\mu}(A)$ is supra closed. Therefore $cl^{\mu}(A) - A$ is supra closed. We know that every supra closed is supra $g^{\#}\alpha$- closed, therefore $cl^{\mu}(A) - A$ is supra $g^{\#}\alpha$- closed.

(iii) $\Rightarrow$ (ii) Let $U = X - [cl^{\mu}(A) - A]$. By (iii) U is supra $g^{\#}\alpha$- open in X. Then $A = U \cap cl^{\mu}(A)$ holds.

(iii) $\Rightarrow$ (iv) Let $Q = \alpha cl^{\mu}(A) - A$ be supra $g^{\#}\alpha$- closed. Then $X - Q = X - [cl^{\mu}(A) - A] = A \bigcup [X - \alpha cl^{\mu}(A)]$.

Since $X - Q$ is supra $g^{\#}\alpha$- open, $A \cup [X - \alpha cl^{\mu}(A)]$ is supra $g^{\#}\alpha$- open.

(iv) $\Rightarrow$ (iii) Let $U = A \cup [X - \alpha cl^{\mu}(A)]$. Since $X - U$ is supra $g^{\#}\alpha$- closed and $X - U = \alpha cl^{\mu}(A) - A$ is supra $g^{\#}\alpha$- closed.

Theorem 3.15. *For a subset A of* (X, μ), *if* $A \in S\text{-}g^{\#}\alpha LC^{**}(X, \mu)$, *then there exist an supra open set G such that* $A = G \bigcap g^{\#}\alpha\text{-}cl(A)$.

Proof. Let $A \in S\text{-}g^{\#}\alpha LC^{**}(X, \mu)$. Then $A = G \bigcap V \Rightarrow A \subset G$. Then $A \subset g^{\#}\alpha\text{-}cl^{\mu}(A)$. Therefore, $A = G \bigcap g^{\#}\alpha\text{-}cl^{\mu}(A)$.

Also, we have $g^{\#}\alpha\text{-}cl^{\mu}(A) \subset V$. This implies $A = G \bigcap V \supset G \bigcap g^{\#}\alpha\text{-}cl^{\mu}(A)$ $\Rightarrow A \supset G \bigcap g^{\#}\alpha\text{-}cl^{\mu}(A)$. Thus $A = G \bigcap g^{\#}\alpha\text{-}cl^{\mu}(A)$.

Theorem 3.16. *For a subset A of* (X, μ) *if* $A \in S\text{-}g^{\#}\alpha LC^{**}(X, \mu)$, *then there exist an supra open set G such that* $A = G \bigcap \alpha\text{-}cl^{\mu}(A)$.

Proof Let $A \in S\text{-}g^{\#}\alpha LC^{**}(X, \mu)$. Then $A = G \bigcap V$, where G is supra open set and V is supra $g^{\#}\alpha$- closed set. Then

$A = G \bigcap V \Rightarrow A \subset G$. Obviously, $A \subset \alpha\text{-}cl^{\mu}(A)$.

Therefore,

$$A \subset G \bigcap \alpha\text{-}cl^{\mu}(A). \tag{1}$$

Also, we have $\alpha cl^{\mu}(A) \subset V$. This implies

$$A = G \bigcap V \supset G \bigcap \alpha cl^{\mu}(A) \Rightarrow A \supset G \bigcap \alpha cl^{\mu}(A). \tag{2}$$

From (1) and (2), we get $A = G \bigcap \alpha cl^{\mu}(A)$.

Theorem 3.17. *Let A be a subset of* (X, μ). *If* $A \in S\text{-}g^{\#}\alpha LC^{**}(X, \mu)$, *then* $g^{\#}\alpha\text{-}cl^{\mu}(A) - A$ *is supra* $g^{\#}\alpha$- *closed and* $A \cup [X - g^{\#}\alpha\text{-}cl^{\mu}(A)]$ *is supra* $g^{\#}\alpha$- *open*.

Proof. Given $A \in S\text{-}g^{\#}\alpha LC^{**}(X, \mu)$. Then there exist a supra open subset U and a supra $g^{\#}\alpha$- closed subset V such that $A = U \cap V$. This implies $g^{\#}\alpha\text{-}cl^{\mu}(A)$ is supra $g^{\#}\alpha$- closed. Therefore, $g^{\#}\alpha\text{-}cl^{\mu}(A) - A$ is supra $g^{\#}\alpha$- closed.

Also, $[X - [g^{\#}\alpha\text{-}cl^{\mu}(A) - A]] = A \cup [X - g^{\#}\alpha\text{-}cl^{\mu}(A)]$. Therefore $A \cup [X - g^{\#}\alpha\text{-}cl^{\mu}(A)]$ is supra $g^{\#}\alpha$- open.

Remark 3.18. The converse of the above theorem need not be true as seen from the following example.

Example 3.19. Let $X = \{a, b, c, d\}$ and $\mu = \{X, \phi, \{a\}, \{b\}, \{a, b\}\}$. Then $\{X, \phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, c, d\}, \{b, c, d\}\}$ is the set of all supra $g^{\#}\alpha$- closed set in X and $S\text{-}g^{\#}\alpha LC^{**}(X, \mu) = P(X) - \{\{a, b, c\}, \{a, b, d\}\}$, If $A = \{a, b, c\}$, then $g^{\#}\alpha\text{-}cl^{\mu}(A) - A = \{d\}$ is supra $g^{\#}\alpha$- closed and $A \cup [X - g^{\#}\alpha\text{-}cl^{\mu}(A)] = A$ is supra $g^{\#}\alpha$- open but $A \notin S\text{-}g^{\#}\alpha LC^{**}(X, \mu)$.

Theorem 3.20. *If* $A \in S\text{-}g^{\#}\alpha LC^{**}(X, \mu)$ *and* B *is supra open*, *then* $A \cap B \in S\text{-}g^{\#}\alpha LC^{**}(X, \mu)$.

Proof. Suppose $A \in S\text{-}g^{\#}\alpha LC^{**}(X, \mu)$, then there exist a supra $g^{\#}\alpha$- open set U and supra closed set V such that $A = U \cap V$. So $A \cap B = U \cap V \cap B = (U \cap B) \cap V$, where $U \cap B$ is supra $g^{\#}\alpha$- open. Therefore, $A \in S\text{-}g^{\#}\alpha LC^{**}(X, \mu)$.

Theorem 3.21. *If* $A \in S\text{-}g^{\#}\alpha LC^{**}(X, \mu)$ *and* B *is supra open*, *then* $A \cap B \in S\text{-}g^{\#}\alpha LC^{**}(X, \mu)$.

Proof. Suppose $A \in S\text{-}g^{\#}\alpha LC^{**}(X, \mu)$, then there exist a supra $g^{\#}\alpha$- open set U and supra $g^{\#}\alpha$- closed set V such that $A = U \cap V$. So $A \cap B = U \cap V \cap B = (U \cap B) \cap V$, where $U \cap B$ is supra $g^{\#}\alpha$- open. Therefore, $A \cap B \in S\text{-}g^{\#}\alpha LC(X, \mu)$.

Theorem 3.22. *If* $A \in S\text{-}g^{\#}\alpha LC^{**}(X, \mu)$, *and B is supra closed*, *then* $A \cap B \in S\text{-}g^{\#}\alpha LC^{**}(X, \mu)$.

Proof. Suppose $A \in S\text{-}g^{\#}\alpha LC^{**}(X, \mu)$, then there exist a supra open set U and supra $g^{\#}\alpha$-closed set V such that $A = U \cap V$. So $A \cap B = U \cap V \cap B = U \cap (B \cap V)$, where $B \cap V$ is supra $g^{\#}\alpha$-closed. Hence, $A \cap B \in S\text{-}g^{\#}\alpha LC^{**}(X, \mu)$.

4. Supra $g^{\#}\alpha$-Locally Continuous Functions

In this section, we define a new type of functions called supra $g^{\#}\alpha$-locally continuous functions ($S\text{-}g^{\#}\alpha L$-continuous functions), supra $g^{\#}\alpha$-locally irresolute functions ($S\text{-}g^{\#}\alpha L$-irresolute) and study some of their properties.

Definition 4.1. Let (X, τ) and (Y, σ) be two topological spaces and $\tau \subseteq \mu$. A function $f : (X, \tau) \to (Y, \sigma)$ is called $S\text{-}g^{\#}\alpha L$-continuous (resp., $S\text{-}g^{\#}\alpha L^{*}$-continuous, and $S\text{-}g^{\#}\alpha L^{**}$-continuous), if $f^{-1}(A) \in S\text{-g}^{\#}\alpha LC(X, \mu)$, (resp., $f^{-1}(A) \in S\text{-g}^{\#}\alpha LC^{*}(X, \mu)$, and $f^{-1}(A) \in S\text{-g}^{\#}\alpha LC^{**}(X, \mu)$ for each $A \in \sigma$.

Definition 4.2. Let (X, τ) and (Y, σ) be two topological spaces and μ and λ be a supra topologies associated with τ and respectively. A function $f : (X, \tau) \to (Y, \sigma)$ is said to be $S\text{-}g^{\#}\alpha L$-irresolute (resp., $S\text{-}g^{\#}\alpha L^{*}$-irresolute, resp., $S\text{-}g^{\#}\alpha L^{**}$-irresolute) if $f^{-1}(A) \in S\text{-g}^{\#}\alpha LC(X, \mu)$, (resp., $f^{-1}(A) \in S\text{-g}^{\#}\alpha LC^{*}(X, \mu)$, resp., $f^{-1}(A) \in S\text{-g}^{\#}\alpha LC^{**}(X, \mu)$ for each $A \in S\text{-g}^{\#}\alpha LC(Y, \lambda)$ (resp., $A \in S\text{-g}^{\#}\alpha LC^{*}(Y, \lambda)$, resp., $A \in S\text{-g}^{\#}\alpha LC^{**}(Y, \lambda)$.

Theorem 4.3. *Let* (X, τ) *and* (Y, σ) *be two topological spaces and* μ *be a supra topology associated with* τ. *Let* $f : (X, \tau) \to (Y, \sigma)$ *be a function. If f is* $S\text{-}g^{\#}\alpha L^{*}$-*continuous* (*or*) $S\text{-}g^{\#}\alpha L^{**}$-*continuous*, *then it is* $S\text{-}g^{\#}\alpha L$-*continuous*.

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be a function. If f is $S\text{-}g^{\#}\alpha L^{*}$-continuous (or) $S\text{-}g^{\#}\alpha L^{**}$-continuous, by definition $f^{-1}(A) \in S\text{-g}^{\#}\alpha LC^{*}(X, \mu)$, and $f^{-1}(A) \in S\text{-g}^{\#}\alpha LC^{*}(X, \mu)$, for each $A \in \sigma$. By theorem $f^{-1}(A) \in S\text{-g}^{\#}\alpha LC(X, \mu)$. Then it is $S\text{-}g^{\#}\alpha L$-continuous.

Theorem 4.4. *Let* (X, τ) *and* (Y, σ) *be two topological spaces and* μ *and* λ *be a supra topologies associated with* τ *and* σ *respectively. Let* $f : (X, \tau) \to (Y, \sigma)$ *be a function. If f is* $S\text{-}g^{\#}\alpha L$-*irresolute* (*resp*., $S\text{-}g^{\#}\alpha L^{*}$-*irresolute*, *and* $S\text{-}g^{\#}\alpha L^{**}$-*irresolute*), *then it is* $S\text{-}g^{\#}\alpha L$-*continuous* (*resp*., $S\text{-}g^{\#}\alpha L^{*}$-*continuous*, *and* $S\text{-}g^{\#}\alpha L^{**}$-*continuous*).

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be a function. Let A is supra closed of Y. By theorem 3.3 [6] every supra closed set is supra $g^{\#}\alpha$-closed set, therefore A is supra $g^{\#}\alpha$-closed set. Since f is $S\text{-}g^{\#}\alpha L$-irresolute (resp., $S\text{-}g^{\#}\alpha L$-irresolute, and $S\text{-}g^{\#}\alpha L^{**}$-irresolute), $f^{-1}(A)$ is $S\text{-}g^{\#}\alpha L$-closed. Therefore f is $S\text{-}g^{\#}\alpha L$-continuous (resp., $S\text{-}g^{\#}\alpha L^{*}$-continuous, and $S\text{-}g^{\#}\alpha L^{**}$-continuous).

Theorem 4.5. *If* $g : X \to Y$ *is* $S\text{-}g^{\#}\alpha L$-*continuous and* $h : Y \to Z$ *is supra continuous*, *then* $hog : X \to Z$ *is* $S\text{-}g^{\#}\alpha L$-*continuous*.

Proof. Let $g : X \to Y$ is $S\text{-}g^{\#}\alpha L$-continuous and $h : Y \to Z$ is supra continuous. By definition, $g^{-1}(V) \in S\text{-}g^{\#}\alpha LC(X), V \in Y$ and $h^{-1}(W) \in Y, W \in Z$. Let $W \in Z$, then $(hog)-1(W) = (g^{-1}h^{-1})(W)$ $= g^{-1}(h^{-1}(W)) = g^{-1}(V)$, for $V \in Y$. This implies, $(h \circ g)^{-1}(W)$ $= g^{-1}(V) \in S\text{-}g^{\#}\alpha LC(X), W \in Z$. Hence hog is $S\text{-}g^{\#}\alpha L$-continuous.

Theorem 4.6. *If* $g : X \to Y$ *is* $S\text{-}g^{\#}\alpha L$-*irresolute and* $h : Y \to Z$ *is* $S\text{-}g^{\#}\alpha L$-*continuous*, then $hog : X \to Z$ is $S\text{-}g^{\#}\alpha L$-*continuous*.

Proof. Let $g : X \to Y$ is $S\text{-}g^{\#}\alpha L$-irresolute and $h : Y \to Z$ is

$S\text{-}g^{\#}\alpha L$-continuous. By definition, $g^{-1}(V) \in S\text{-}g^{\#}\alpha C(X)$, for $V \in S\text{-}g^{\#}\alpha LC(Y)$ and $h^{-1}(W) \in S\text{-}g^{\#}\alpha LC(Y)$, for $W \in Z$, Let $W \in Z$, then $(h \circ g)^{-1}(W) = (g^{-1}h^{-1})(W) = g^{-1}(h^{-1}(W)) = g^{-1}(V)$, for $V \in S\text{-}g^{\#}\alpha LC(Y)$. This implies, $(h \circ g)^{-1}(W) = g^{-1}(V) \in S\text{-}g^{\#}\alpha LC(X), W \in Z$. Hence hog is $S\text{-}g^{\#}\alpha L$-continuous.

Theorem 4.7. *If* $g : X \to Y$ *and* $h : Y \to Z$ *are* $S\text{-}g^{\#}\alpha L$-*irresolute*, *then hog* $: X \to Z$ *is* $S\text{-}g^{\#}\alpha L$-*irresolute*.

Proof. By the hypothesis and the definition, we have $g^{-1}(V) \in S\text{-}g^{\#}\alpha LC(X)$, for $V \in S\text{-}g^{\#}\alpha LC(Y)$ and $h^{-1}(W) \in S\text{-}g^{\#}\alpha LC(Y)$, for $W \in S\text{-}g^{\#}\alpha LC(Z)$. Let $W \in S\text{-}g^{\#}\alpha LC(Z)$, then $(h \circ g)^{-1}(W) = (g^{-1}h^{-1})(W)$ $= g^{-1}(h^{-1}(W)) = g^{-1}(V)$, for $V \in S\text{-}g^{\#}\alpha LC(Y)$. Therefore, $(h \circ g)^{-1}(W)$ $= g^{-1}(V) \in S\text{-g}^{*}\alpha LC(X), W \in S\text{-g}^{*}\alpha LC(Z)$. Thus, hog is $S\text{-}g^{\#}\alpha L$-irresolute.

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