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INTEGRAL SOLUTIONS OF AN INFINITE ELLIPTIC CONE $x^2 = 9y^2 + 11z^2$

MANJU SOMANATH[1], K. RAJA[2], J. KANNAN[3] and A. AKILA[4]

[1,2]PG and Research Department of Mathematics
National College, Trichy, India

[3,4]Department of Mathematics
Ayya Nadar Janaki Ammal College (Autonomous)
Sivakasi, India
E-mail: jayram.kannan@gmail.com
aradakila1999@gmail.com

Abstract

In this manuscript, we investigate the ternary Diophantine equation in place of inestimable elliptic cone specified by $x^2 = 9y^2 + 11z^2$ is analyzed for its non-zero distinctive integer points lying on it. Barely some dissimilar patterns of integer points gratifying the cone in deliberation are obtained.

I. Introduction

The theory of numbers has always occupied a unique position in the world of mathematics. This is due to the unquestioned historical importance of the subject: it is one of the few disciplines having demonstrable results that predate the very idea of a university or an academy. Nearly every century since classical antiquity has witnessed new and fascinating discoveries relating to the properties of numbers; and, at some point in their careers, most of the great masters of the mathematical sciences have contributed to this body of knowledge.

The Pythagoreans believed that the key to an explanation of the universe lay in number and form, their general thesis is that “Everything is number”. (By number, they meant, of course, a positive integer). For a rational

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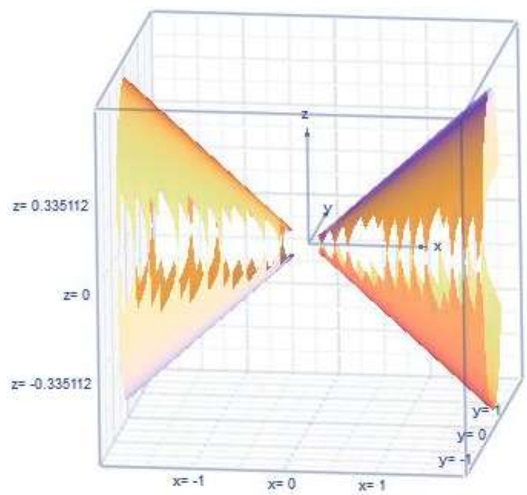
understanding of nature, they considered it sufficient to analyze the properties of certain numbers. Pythagoras himself, we are old "seems to have attached supreme importance to the study of arithmetic, which he advanced and took out of the realm of commercial utility."

Number theory is closely tied to the other areas of mathematics, most especially to abstract algebra, but also to linear algebra, combinatory, analysis, geometry, and even topology.

In this manuscript, the ternary quadratic Diophantine Equation $x^2 = 9y^2 + 11z^2$ is analyzed for a variety of patterns of integer solutions.

Pictorial representation of the equation:



II. Method of Analysis

The quadratic Diophantine equation with three unknowns to be solved is

$$x^2 = 9y^2 + 11z^2. \tag{1}$$

We present below different patterns of integral solutions of (1):

Pattern 1.

Think about (1) as

$$x^2 = 11z^2 + (3y)^2 \tag{2}$$

The solutions are originated to be

$$x = 11m^2 + n^2$$

$$3y = 11m^2 - n^2$$

$$z = 2mn.$$

Given that our attention centers on discovery integral solutions, substitute m by $3M$ and n by $3N$ in the on top of equations. Accordingly, the consequent solutions to (1) are specified by

$$x = 3^2(11M^2 + N^2)$$

$$y = 3(11M^2 - N^2)$$

$$z = 3^2(2MN).$$

Pattern 2.

Think about (1) as:

$$x^2 - 9y^2 = 11z^2 \tag{3}$$

which can be on paper in the appearance of the quotient as

$$\frac{x + 3y}{11z} = \frac{z}{x - 3y} = \frac{p}{q}\,(say),\ q \neq 0.$$

Expressing this as a structure of immediate equations

$$qx + 3qy - 11pz = 0 \tag{4}$$

$$px - 3py - qz = 0 \tag{5}$$

and solving (4) and (5) we search out

$$x = -(33p^2 + 3q^2)$$

$$y = q^2 - 11p^2$$

$$z = -6pq.$$

Pattern 3.

Equation (3) can also be on paper as:

$$\frac{x+3y}{z} = \frac{11z}{x-3y} = \frac{p}{q}\,(say),\ q \neq 0.$$

Going on as above, we get hold of

$$x = -(33p^2 + 3q^2)$$

$$y = -p^2 + 11q^2$$

$$z = -6pq.$$

Pattern 4. Regard as equation (1) as

$$x^2 - 11z^2 = 9y^2 \tag{6}$$

Take for granted y like

$$y = a^2 - 11b^2 \tag{7}$$

Inscribe 9 seeing that

$$9 = p^2 - 11q^2 \tag{8}$$

here the on top of equation

$$(x^2 - 11z^2) = 9(a^2 - 11b^2)^2. \tag{9}$$

In an examination of (7) and (8), (9) is a factorizable structure at the same time as

$$(x + \sqrt{11}z)(x - \sqrt{11}z) = 9\,(a + \sqrt{11}b)^2(a - \sqrt{11}b)^2$$

Characterize

$$x + \sqrt{11}\,z = (p + \sqrt{11}q)(a + \sqrt{11}b)^2.$$

Equating the rational and irrational parts, we acquire

$$x = p(a^2 + 11b^2) + 33abq,\ z = q(a^2 + 11b^2) + 3abp. \tag{10}$$

Thus, (7) and (10) stand for the common answer of (1).

In the direction of locating the value of p, q we carry on as follows:

Let (p_0, q_0) be the preliminary key of $p^2 - 11q^2 = 9.$

Employing the linear conversion $p = p_0 + 10h, q = q_0 + 3h$ frequently, the supplementary principles of p and q agreeable the equation (8) are specified by

$$p = -199p_0 + 660q_0, \; q = -60p_0 + 199q_0.$$

A hardly any arithmetical examples are obtainable below:

	Algebraic appearance used for	Solutions
Case I	$(30 + 9\sqrt{11})$ $(30 - 9\sqrt{11})$	$x = 30(a^2 + 11b^2) + 198ab$ $y = a^2 - 11b^2$ $z = 9(a^2 + 11b^2) + 60ab$
Case II	$(597 + 180\sqrt{11})$ $(597 - 180\sqrt{11})$	$x = 597(a^2 + 11b^2) + 3960ab$ $y = a^2 - 11b^2$ $z = 1194ab + 180(a^2 + 11b^2)$
Case III	$(11910 + 3591\sqrt{11})$ $(11910 - 3591\sqrt{11})$	$x = 11910(a^2 + 11b^2) + 79002ab$ $y = a^2 - 11b^2$ $z = 23820ab + 3591(a^2 + 11b^2)$
Case IV	$(237603 + 71640\sqrt{11})$ $(237603 - 71640\sqrt{11})$	$x = 237603(a^2 + 11b^2)$ $+ 1576080ab$ $y = a^2 - 11b^2$ $z = 475206ab + 71640(a^2 + 11b^2)$

III. Conclusion

The ternary quadratic Diophantine Equations are well-off within multiplicity, solitary possibly determination look for additional choices of Diophantine Equations to locate their parallel integer solutions.

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