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A NEW MODIFIED RIDGE REGRESSION ESTIMATOR

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Abstract

Ordinary least squares (OLS) estimator will yield unstable estimates when there is multicollinearity in the data. Thus in order to overcome the problem of multicollinearity, we propose a new modified ridge estimator to the ridge parameter. Performance of the proposed estimator is compared through mean squared error (MSE) with some of the existing ridge estimators which are defined in the literature. Simulation study is used to compare MSEs of the suggested and the other estimators. Results indicate that the suggested estimator performs better than the OLS estimator and the other existing estimators.

1. Introduction

Consider the general form of multiple linear regression model defined by

$$y = X\beta + u, \tag{1}$$

where X is an $(n \times p)$ data matrix of predictors which are non stochastic, y is a $(n \times 1)$ response vector, β is a $(p \times 1)$ vector of unknown regression coefficients, and u be $(n \times 1)$ vector of random disturbances, such that $E(u) = 0$, and $E(uu') = \sigma^2 I$. Here, the matrix X is standardized for computational purpose and y is expressed in terms of deviations from mean. It is well known that in presence of multicollinearity, the ordinary least squares (OLS) estimator for β, i.e., $\hat{\beta}_{OLS} = (X'X)^{-1}X'y$, does not yield stable

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estimates even though it is unbiased, and it will tend to produce large variance for the estimated regression coefficients. Therefore in order to overcome these deficiencies of OLS, a new regression method called 'Ridge regression', which was first introduced by Hoerl [7], and then improved by Hoerl and Kennard [8] and later by Hoerl et al. [9] Note that at present, it is one of the widely used biased regression methods, among all the biased regression methods such as, principal component regression (PCR), partial least squares regression (PLSR), etc., as it yields more stable estimates to the regression coefficients under severe multicollinearity when the number of predictors less than the number of observations. Hoerl and Kennard [8] suggests that the ordinary ridge estimator for β as $\hat{\beta}_R = (X'X + kI)^{-1}X'y$, where $k > 0$, being a small positive constant called biasing constant, or 'ridge parameter or shrinkage parameter'. It should be noted that later than Hoerl and Kennard [8], there are numerous studies have been taken place using various types of ridge parameters, namely, Lawless and Wang [13], Dorugade and Kashid [4], Kibria [12], Khalaf and Shukur [10], etc.

Motivation for writing this article is to evaluate the performance of various ridge estimators under various degree of multicollinearity i.e., at low, moderate and high degree of multicollinearity, and try to suggest more robust estimator for the ridge parameter k. The performance of the suggested estimator and other existing ridge estimators will be verified using Monte-Carlo simulation technique and compared them in terms of ratio of average MSE (AMSE) of OLS estimator over other existing ridge estimators.

This article is outlined in such a manner that Section 2, discusses the methodology used in this paper, and in Section 3, some of the well known ordinary ridge estimators were defined. In Section 4, we propose a new modified ridge estimator. Section 5, deal with verification of performance of estimators for a real data set, and we discuss and conduct simulation study to verify the performance of proposed estimator and the other existing ridge estimators in Section 6, and finally we make some of our observations in Section 7.

2. **Concepts and Methods**

Let us suppose that W be a $(p \times p)$ matrix such that its columns are normalized eigen vectors of $X'X$, and let $Z = XW$, such that $Z'Z = W'X'XW = D = diag(\lambda_1, \lambda_2, \ldots, \lambda_p)$, where λ_j 's are the j[th] eigen value of $X'X$, then we write the equation (1) as

$$y = Z\gamma + u, \tag{2}$$

where, $\gamma = W'\beta$. The above form of equation (2) is usually called as canonical form. The ordinary least square (OLS) estimator for γ is then given by

$$\hat{\gamma}_{OLS} = (Z'Z)^{-1} Z'y = D^{-1} Z'y. \tag{3}$$

Since $\gamma = W'\beta$, implies $\hat{\beta} = W\hat{\gamma}$. In presence of severe multicollinearity, OLS might yield unstable estimates for the regression coefficients because $(X'X)^{-1}$ may not exist. Therefore in-order to get inverse of $X'X$, Hoerl and Kennard [8] initially came up with an innovative idea that simply by adding a small positive constant *k*, to every diagonal element of the matrix $X'X$, one could, get the inverse of $X'X$, and they termed it as Ridge regression. Later they have suggested an estimator for the ridge parameter *k*, such that the optimum value of *k*, should lie between $(0, \sigma^2/\hat{\beta}^2_{\max})$. Ridge regression is now one of the most widely used biased regression methods. The ordinary ridge estimator (ORR) for γ is then given by

$$\hat{\gamma}_R = (D + kI)^{-1} Z'y = A^{-1} Z'y, \tag{4}$$

where $A = D + kI$.

From equations (3) and (4), we write

$$\hat{\gamma}_R = (I - A^{-1}k)\hat{\gamma}_{OLS}. \tag{5}$$

Then, the bias of $\hat{\gamma}_R$ is given by

$$\text{bias}(\hat{\gamma}_R) = -A^{-1}k\gamma. \tag{6}$$

Consequently, the bias of $\hat{\beta}$ is given by

$$\text{bias}(\hat{\beta}) = \text{Wbias}(\hat{\gamma}_R) = -kWA^{-1}W'\beta. \tag{7}$$

The mean square error (MSE) of $\hat{\gamma}_R$ is given by

$$\text{MSE}\ (\hat{\gamma}_R) = \text{trace}\ [\text{var}\ (\hat{\gamma}_R)] + [\text{bias}\ (\hat{\gamma}_R)]'\, bias\ (\hat{\gamma}_R)$$

$$= \hat{\sigma}^2 \sum_{i=1}^{\rho} \frac{\lambda_i}{(\lambda_i + k)^2} + k^2 \sum_{i=1}^{p} \frac{\gamma_i^2}{(\lambda_i + k)^2}. \qquad (8)$$

3. Some Ordinary Ridge Estimators

Several authors have suggested different types of ordinary ridge estimators in the literature, namely, McDonald and Galarneau [16], Nomura [19], Kibria [12], Khalaf and Shukur [10], Mardkyan and Cetin [15], Muniz and Kibria [18], Dorugade and Kashid [4], El-Dereny and Rashwan [6], Khalaf [11], Yazid [25], Alkhamisi and Shukur [1], Dorugade [5], Satish and Vidya [20], [21], [22].

Now in the following section we consider some of the well known existing ridge estimators which are defined in the literature for the ridge parameter *k*, and then these estimators will be compared with the proposed estimator and they are defined as under:

(i) $k_1 = \frac{p\hat{\sigma}^2}{\hat{\gamma}'\hat{\gamma}}$, (Hoerl, Kennard and Baldwin-[9]) (9)

(ii) $k_2 = \max\left(0, \frac{p\hat{\sigma}^2}{\hat{\gamma}'\hat{\gamma}} - \frac{1}{n(VIF_j)_{\max}}\right)$, (Dorugade and Khashid [4]), (10)

where, $VIF_j = \frac{1}{1 - R_j^2}$, $j = 1, 2, \ldots, p$ is the variance inflation factor of the j^{th} regressor.

(iii) $k_3 = \frac{p\hat{\sigma}^2}{\sum_{i=1}^{p} \lambda_i \hat{\gamma}_i^2}$, (Lawless and Wang-[13]) (11)

(iv) $k_4 = \frac{\lambda_{\max}\hat{\sigma}^2}{(n - p - 1)\hat{\sigma}^2 + \lambda_{\max}\hat{\gamma}_{\max}^2}$, (Khalaf and Shukur-[10]) (12)

where, $\lambda_{\max}$ is the largest eigen value of $X'X$.

(v) $k_5 = \frac{p\hat{\sigma}^2}{\sum_{i=1}^{p}\{\hat{\gamma}_j^2/[1+(1+\lambda_i[\hat{\gamma}_i^2/\hat{\sigma}^2]^{1/2})]\}}$, (Nomura-[19]) (13)

(vi) $k_6(\text{Harmonic Mean}) = \frac{2p}{\lambda_{max}}\sum_{i=1}^{p}\frac{\hat{\sigma}^2}{\hat{\gamma}_i^2}$, (Dorugade [5]) (14)

(vii) $k_7 = \frac{p\hat{\sigma}^2}{\hat{\gamma}'\gamma} + \frac{1}{\lambda_{max}\hat{\gamma}'\hat{\gamma}} = k_1 + \frac{1}{\lambda_{max}\hat{\gamma}'\hat{\gamma}}$ (Satish and Vidya-[20]) (15)

(viii) $k_8 = \frac{p\hat{\sigma}^2}{\hat{\gamma}'\hat{\gamma}} + \frac{1}{2(\sqrt{\lambda_{max}/\lambda_{min}})^2} = k_1 + \frac{1}{2m^2}$, (Satish and Vidya-[20]) (16)

Where $m = \sqrt{\lambda_{max}/\lambda_{min}}$ denote condition number, which measures a severity of problem of multicollinearity [Weisberg [24], Chatterjee and Hadi [3], Montgomery et al. [17]. Condition number indicates the severity of multicollinearity, i.e., higher value of m indicates high degree of multicollinearity. If m lies in between 30-100, indicate a moderate to strong correlation, and if $m > 100$ indicate severe multicollinearity (Liu [14]).

(ix) $k_9 = GM(k_1, k_2) = \sqrt{k_1 \times k_2}$, (Satish and Vidya-[21]) (17)

(x) $k_{10} = HM(k_1, k_2) = 2k_1 \times k_2/(k_1 + k_2)$, (Satish and Vidya-[22]) (18)

where HM, indicates the Harmonic mean.

4. Proposed Estimator (A New Modified Ridge Estimator)

Most of the above existing estimators are verified under a high degree $(\rho \geq 0.9)$ of multicollinearity and compared through MSE. It is observed that the estimator due to Nomura [19] yields somewhat more unstable estimates and then followed by Lawless and Wang [13] and Kashid and Shukur [10] when the error variance is either very small or very large. Estimator due to Dorugade and Kashid [4] seems to be more stable when there exists a high degree of multicollinearity, and the estimators due to Satish and Vidya [20], [21], [22] seem to be more stable when high or moderate or low degree of multicollinearity is present in the data, but they even seem to be little overestimated in few instances, especially when the error variance is too large. Estimators due to Hoerl and Kennard [8], shows that the ridge estimator is biased and its squared bias is monotonically increasing and continuous function of the ridge parameter *k* when *k* is lies in the interval $(0, \sigma^2 / \hat{\gamma}^2_{\max})$, such that MSE of the ridge estimator become minimum, where $\hat{\gamma}^2_{\max}$ is the biggest element of $\hat{\gamma}^2_R$, and σ^2 replaced by its estimator $\hat{\sigma}^2 = \frac{y'y - \hat{\gamma}_{OLS} Z'y}{n - p - 1}$. Thirsted [23], pointed out that Hoerl et al. [9] seems to be unstable when number of observations is less than the number of predictors and appear to be little over estimated. Thus in-order to overcome the drawbacks of these existing estimators, here we propose a new modified ridge estimator for the ridge parameter *k*, as

$$\text{(xi)}\ k_{11} = \alpha \frac{\hat{\sigma}^2}{\hat{\gamma}'\hat{\gamma}} + (1 - \alpha) \max \left(0, \frac{(p - 1)\hat{\sigma}^2}{\hat{\gamma}'\hat{\gamma}} - \frac{1}{n (VIF_j)_{\max}} \right), \tag{19}$$

where $a = \sqrt{(\lambda_{\min} + 1) / \lambda_{\max}}$ and $0 < \alpha < 1$. The proposed estimator is the convex combination of Hoerl et al. [9], i.e., k_1, and Dorugade and Kashid [4] i.e., k_2, but with little modifications. This modification made the proposed estimator more robust as compared to that of k_1, and k_2. The Monte-Carlo simulation was carried out and it was indicated that the suggested estimator defined as in (equation 19), performed better in terms of MSE as compared to that of the existing estimators when the degree of multicollinearity is low, moderate or high. That is, the estimates which are obtained from the suggested estimator is little more closer to the true parameter value and thereby it is stable too, as compared to that of Hoerl et al. [9], Dorugade and Kashid [4] and other existing estimators which are considered under this study.

5. Example (Child Mortality and Fertility Data Set)

To illustrate the performance of suggested and the other existing estimators, a real data set on child mortality and fertility (ref. Bhuyan [2], p-123) is considered. The data matrix *X* contain $n = 60$, the sample size, and $p = 8$ the number of predictors. To verify the multicollinearity among the predictors, variance inflation factor (VIF) for each variable of *X* are: 1.1946, 1.6629, 1.6664, 2.4845, 2.1081, 1.1734, 1.8128, and 2.4931. These VIF's indicate that there is a moderate degree of multicollinearity between any two predictors. Then the MSE's of suggested and the existing estimators are obtained as follows:

Estimators: k_1 k_2 k_3 k_4 k_5 k_6 k_7 k_8 k_9 k_{10} k_{11}

MSE : 1.2422 1.2421 0.7979 0.8159 1.1615 0.9127 1.2422 1.2422 1.2422 1.2422 1.2162

It is observed that the performance of the estimator due to Nomura [19] is slightly better and then followed by suggested estimator k_{11} than all the other estimators considered in this article in terms of MSE over OLS, i.e., the other estimators either underperformed or little over estimated as compared to OLS estimator. Thus, we conclude the performance of the suggested estimator is satisfactory and comparable.

6. Simulation Study

It is herewith studied the performance of suggested estimator and the existing estimators through Monte-Carlo simulation technique in presence of various degree of multicollinearity (ρ), i.e., at low, moderate and at high degree of multicollinearity. The results were obtained by generating a random matrix *X* of size $(n \times p)$, using the relation:

$$x_{ij} = (1 - \rho^2)^{1/2} \xi_{ij} + \rho \xi_{ip}, \text{ where } i = 1, 2, \ldots, n \text{ and } j = 1, 2, \ldots, p.$$

Where ξ_{ij}'s are independent standard normal pseudo random number, ρ is fixed such that ρ^2 is the degree of correlation between any two variables. The predictor variables were standardized such that the matrix $X'X$, is in correlation form and it is used to generate *y* with $\beta = (1.03, 0.5, 1.3, 0.91, 0.59, 0.74, 0.3, 0.95, 0.83, 0.9, 1.5, 0.4, 1.3, 0.5, 0.3, 2.9)$. Firstly, performance of the estimators was evaluated when the error (u) distribution is normal $(0, \sigma^2 I)$, and secondly, the Student's $t_{(5)}$ d. f. was considered as error (u) distribution.

In the first case, i.e., $u \sim N(0, \sigma^2 I)$ performance of the proposed estimator was evaluated for different values of sample size $(n) : 20, 25, 100$ and 500; error variance $(\sigma^2) : 5, 25, 100$ and the degree of correlation $\rho : 0.3, 0.5, 0.8$ and 0.99. The simulation experiment was repeated for 10,000 times each and the average mean square error (AMSE) was computed using the relation:

$$AMSE(\hat{\beta}^*) = \frac{1}{10,000} \sum_{j=1}^{10000} (\hat{\beta}_j^* - \beta)'(\hat{\beta}_j^* - \beta),$$

where $\hat{\beta}^*$ is any estimator that was used in this study. Ridge estimates were computed for the suggested estimator and the other existing estimators defined in equations (9) through (19). Usually the estimator(s) leading to maximum ratio of AMSE of OLS estimator over AMSE of other ridge estimators were considered to be the best in terms of MSE, but this may not be valid always because some estimators such as Nomura [19], Lawless and Wang [13], Satish and Vidya [20] etc. have shown somewhat peculiar behaviour especially, when the error variance is either too small, or very large variance and high degree of correlation. In this context it is observed from Table 1, that performance of the proposed ridge estimator (k_{11}) is better than all the other estimators in terms of MSE, under either low or moderator high degree of multicollinearity $(\rho < 0.8)$. If we observe carefully the Table 1 that, when the sample size (n) is large along with smaller error variance (σ^2), and low to moderate degree of multicollinearity (ρ), all the estimators underperform than OLS estimator, and the estimators due to Hoerl et al. [9], Dorugade and Kashid [4], Dorugade [5], Satish and Vidya [20], [21] show slightly over estimated and thereby little away from the true parameter value as compared to that of the suggested estimator. Thus, from simulation study the performance of the suggested estimator is satisfactory.

Also, to study robustness we have assumed that the distribution for error term as Student's $t_{(5)}$ d. f., and then it is observed from the Table 2, that when degree of multicollinearity (ρ), is low to moderate, all the estimators underperform than the OLS estimator, which are considered under study, whereas when the degree of multicollinearity $(\rho > 0.9)$ is high, the proposed estimator out performed than all the other estimators in terms of MSE. Moreover, from simulation study it was observed that the suggested estimator produces estimates which are little closer to the true parameter value as compared to that of most of the existing estimators, which are considered under this study. Thus from empirical study, the performance of the suggested estimator is comparable and as well as satisfactory.

7. Conclusion

The suggested estimator k_{11}, performed better in terms of MSE in most of the cases as compared to other ridge estimators which are considered under this study. Since all the ridge estimators considered here are compared and studied for a wide range of sample size (u) error variance (σ^2) and the degree of correlations (ρ), we conclude that the performance of the suggested estimator is satisfactory. Moreover, the robustness of the proposed estimator was also studied under the assumption of non normality for the disturbance term (u) and thus it is reasonable to conclude that the performance of the suggested estimator is comparable and satisfactory.

However, there is always a scope for research in real life situations and one may think about studying the inaccuracy of estimates, test of significance, and confidence interval of the estimators.

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Table 1. AMSE ratio of OLSE over different Ridge estimator's when error $u \sim N(0, \sigma^2 I)$.

Degree of Correlation (ρ)	Method of Estimation	$n = 20$			$n = 25$		
		$\sigma^2 = 5$	$\sigma^2 = 25$	$\sigma^2 = 100$	$\sigma^2 = 5$	$\sigma^2 = 25$	$\sigma^2 = 100$
		AMSE Ratio	AMSE Ratio	AMSE Ratio	AMSE Ratio	AMSE Ratio	AMSE Ratio
0.3	k_1	2.6272	5.6793	5.7862	1.3335	4.3057	4.7571
	k_2	2.5908	5.3978	5.6304	1.3314	4.2927	4.7415
	k_3	0.9754	1.2285	1.2383	0.6926	1.1113	1.1400
	k_4	1.6182	2.4465	2.5001	0.8982	1.8176	1.9014
	k_5	5.7648	7.3560	9.0760	2.3206	6.7547	9.2055
	k_6	1.2432	1.6455	1.6645	0.8988	1.6174	1.6660
	k_7	2.6273	5.6794	5.7862	1.3335	4.3057	4.7571
	k_8	2.6272	5.6793	5.7862	1.3335	4.3057	4.7571
	k_9	2.6065	5.5386	5.7086	1.3325	4.2992	4.7493
	k_{10}	2.6057	5.4915	5.7016	1.3325	4.2992	4.7492
	k_{11}	2.4027	4.7094	4.9120	1.2583	3.7791	4.1342
		$n = 20$			$n = 25$		
0.5	k_1	2.8674	5.6475	6.0316	1.4346	4.4125	4.8256
	k_2	2.8438	5.5254	5.9405	1.4328	4.4015	4.8129
	k_3	1.0332	1.2634	1.2808	0.7344	1.1378	1.1668
	k_4	1.7393	2.4715	2.6205	0.9710	1.9522	2.0319
	k_5	6.2556	7.6968	9.5326	2.4424	6.7656	8.6465
	k_6	1.1934	1.4862	1.5128	0.8779	1.4503	1.5019
	k_7	2.8675	5.6475	6.0316	1.4346	4.4125	4.8256
	k_8	2.8674	5.6475	6.0316	1.4346	4.4125	4.8256
	k_9	2.8556	5.5862	5.9851	1.4337	4.4070	4.8193

	k_{10}	2.8551	5.5823	5.9837	1.4337	4.4070	4.8193
	k_{11}	2.6615	4.9197	5.2675	1.3647	3.9364	4.2719
		$n = 20$			$n = 25$		
	k_1	3.9377	5.8942	6.1242	2.2381	4.7436	4.8891
	k_2	3.8916	5.7969	5.9917	2.2353	4.7332	4.8771
	k_3	1.3368	1.5096	1.5127	1.0033	1.3107	1.3200
	k_4	2.1185	2.6434	2.6030	1.4146	2.1357	2.1644
	k_5	6.1973	8.1112	11.0331	4.4797	6.6180	7.7497
0.8	k_6	1.1691	1.2969	1.2986	1.0020	1.2930	1.3096
	k_7	3.9377	5.8942	6.1242	2.2381	4.7436	4.8891
	k_8	3.9377	5.8942	6.1242	2.2381	4.7436	4.8891
	k_9	3.9148	5.8457	6.0591	2.2367	4.7384	4.8831
	k_{10}	3.9140	5.8442	6.0552	2.2367	4.7384	4.8831
	k_{11}	3.6327	5.2582	5.4176	2.1213	4.3006	4.4155
		$n = 20$			$n = 25$		
	k_1	5.9572	6.1019	6.0796	4.6608	4.9624	4.9977
	k_2	3.9777	4.5853	4.4333	4.5394	4.8310	4.8593
	k_3	6.8829	7.1274	7.0080	5.3747	5.6917	5.7508
	k_4	2.7237	2.5946	2.7055	2.1730	2.2029	2.2202
	k_5	7.1839	9.8345	10.3606	6.1583	6.7364	7.0909
0.99	k_6	1.2200	1.2374	1.2284	1.2284	1.2430	1.2417
	k_7	5.9573	6.1019	6.0796	4.6608	4.9624	4.9977
	k_8	5.9572	6.1019	6.0796	4.6608	4.9624	4.9977
	k_9	4.2030	4.9155	4.7349	4.5996	4.8961	4.9280
	k_{10}	4.1594	4.8447	4.6662	4.5982	4.8949	4.9266
	k_{11}	3.7184	4.2206	4.0913	4.1599	4.4063	4.4316

Degree of correlation (ρ)	Method of Estimation	$n = 100$			$n = 500$		
		$\sigma^2 = 5$	$\sigma^2 = 25$	$\sigma^2 = 100$	$\sigma^2 = 5$	$\sigma^2 = 25$	$\sigma^2 = 100$
		AMSE Ratio	AMSE Ratio	AMSE Ratio	AMSE Ratio	AMSE Ratio	AMSE Ratio
0.3	k_1	0.2051	2.0827	3.6531	0.0737	0.6591	2.8722
	k_2	0.2051	2.0825	3.6528	0.0737	0.6591	2.8722
	k_3	0.1785	0.8249	1.0114	0.0717	0.4323	0.9236
	k_4	0.1803	0.8599	1.0610	0.0718	0.4339	0.9289
	k_5	0.2549	5.9139	7.8253	0.0776	1.0681	6.2642
	k_6	0.2031	1.1476	1.4994	0.0741	0.4912	1.1490
	k_7	0.2051	2.0827	3.6531	0.0737	0.6591	2.8722
	k_8	0.2051	2.0827	3.6531	0.0737	0.6591	2.8722
	k_9	0.2051	2.0826	3.6530	0.0737	0.6591	2.8722
	k_{10}	0.2051	2.826	3.6530	0.0737	0.6591	2.8722
	k_{11}	0.2022	1.9369	3.2783	0.0735	0.6414	2.6758
		$n = 100$			$n = 500$		
0.5	k_1	0.2254	2.2055	3.7139	0.0780	0.7247	2.9977
	k_2	0.2254	2.2054	3.7137	0.0780	0.7247	2.9977
	k_3	0.1967	0.8511	1.0192	0.0760	0.4703	0.9381
	k_4	0.1992	0.9363	1.1402	0.0761	0.4755	0.9552
	k_5	0.2710	6.7660	6.7188	0.0817	1.1961	9.5701
	k_6	0.2149	1.0824	1.3469	0.0777	0.5141	1.0887
	k_7	0.2254	2.2055	3.7139	0.0780	0.7247	2.9977
	k_8	0.2254	2.2055	3.7139	0.0780	0.7247	2.9977

	k_9	0.2254	2.2055	3.7138	0.0780	0.7247	2.9977
	k_{10}	0.2254	2.2055	3.7138	0.0780	0.7247	2.9977
	k_{11}	0.2227	2.0685	3.3781	0.0778	0.7072	2.8063
		$n = 100$			$n = 500$		
	k_1	0.3908	2.8432	3.8806	0.1097	1.1974	3.4136
	k_2	0.3908	2.8431	3.8804	0.1097	1.1974	3.4136
	k_3	0.3140	0.9642	1.0593	0.1048	0.6432	0.9790
	k_4	0.3241	1.2533	1.4161	0.1053	0.6804	1.0641
	k_5	0.4881	6.6101	8.3686	0.1180	2.2127	9.6302
0.8	k_6	0.3342	1.0968	1.2186	0.1068	0.6912	1.0811
	k_7	0.3908	2.8432	3.8806	0.1097	1.1974	3.4136
	k_8	0.3908	2.8432	3.8806	0.1097	1.1974	3.4136
	k_9	0.3908	2.8431	3.8805	0.1097	1.1974	3.4136
	k_{10}	0.3908	2.8431	3.8805	0.1097	1.1974	3.4136
	k_{11}	0.3851	2.6612	3.5643	0.1094	1.1623	3.1920
		$n = 100$			$n = 500$		
	k_1	2.5499	3.9257	3.9891	0.9445	3.4522	3.8688
	k_2	2.5490	3.9239	3.9872	0.9445	3.4522	3.8687
	k_3	1.7027	2.1868	2.2032	0.6281	1.1790	1.2246
	k_4	1.4695	1.8136	1.8294	0.6779	1.5687	1.6457
	k_5	6.0391	8.7766	8.2697	1.3766	5.7561	6.7620
0.99	k_6	1.0218	1.1829	1.1847	0.5970	1.0636	1.0987
	k_7	2.5499	3.9257	3.9891	0.9445	3.4522	3.8688
	k_8	2.5499	3.9257	3.9891	0.9445	3.4522	3.8688

	k_9	2.5495	3.9248	3.9882	0.9445	3.4522	3.8687
	k_{10}	2.5495	3.9248	3.9882	0.9445	3.4522	3.8687
	k_{11}	2.4082	3.6173	3.6711	0.9227	3.2321	3.5985

Table 2. AMSE ratio of OLSE over different Ridge estimator's when error $u \sim t_{(5)}$ d. f.

Method of Estimation	$n = 20$			
	Degree of Correlation			
	$\rho = 0.3$	$\rho = 0.5$	$\rho = 0.8$	$\rho = 0.99$
	AMSE ratio	AMSE ratio	AMSE ratio	AMSE ratio
k_1	0.3403	0.3642	0.6636	4.2025
k_2	0.3388	0.3632	0.6592	3.4245
k_3	0.2784	0.2989	0.5037	4.8122
k_4	0.3009	0.3173	0.5406	2.2331
k_5	0.4093	0.4180	0.7924	6.3635
k_6	0.3144	0.3253	0.4882	1.1399
k_7	0.3403	0.3642	0.6636	4.2031
k_8	0.3403	0.3642	0.6636	4.2025
k_9	0.3395	0.3637	0.6611	3.5765
k_{10}	0.3395	0.3637	0.6606	3.5532
k_{11}	0.3324	0.3575	0.6471	3.2233
		$n = 25$		
k_1	0.1609	0.1698	0.2925	2.5103
k_2	0.1607	0.1696	0.2923	2.4728
k_3	0.1467	0.1553	0.2566	2.8227
k_4	0.1495	0.1567	0.2575	1.5265
k_5	0.1875	0.1873	0.3298	4.5346

k_6	0.1646	0.1688	0.2638	1.0128
k_7	0.1609	0.1698	0.2925	2.5105
k_8	0.1609	0.1698	0.2925	2.5103
k_9	0.1608	0.1697	0.2924	2.4915
k_{10}	0.1608	0.1697	0.2924	2.4911
k_{11}	0.1588	0.1679	0.2889	2.3468

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