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INTERVAL VALUED FUZZY STRONG BIIDEALS OF NEARSUBTRACTION SEMIGROUPS

J. SIVA RANJINI[1] and V. MAHALAKSHMI[2]

[1]Research Scholar (Reg.No.19222012092003)
[2]Assistant Professor

[1,2]PG and Research Department of Mathematics
A. P. C. Mahalaxmi College for Women
Thoothukudi-628002, Tamilnadu, India
Affiliated to Manonmaniam Sundaranar University
Abishekapatti, Tirunelveli-627012 TamilNadu, India
E-mail: sivaranjini@apcmcollege.ac.in
mahalakshmi@apcmcollege.ac.in

Abstract

The main objective of the manuscript is to investigate and study the notion of $i-v$ fuzzy Strong biideals of near-subtraction semigroups. We already conceptualized the $i-v$ fuzzy biideals of Near-subtraction Semigroups. Interval valued functions commonly deals with the membership data. In this paper, we extend our study to strong bi-ideals. We can this concept to examine Union, Direct product etc. on them. Here we expand the permutable Set and Regularity.

1. Introduction

The Concepts of Fuzzy subsets, fuzzy logic and interval valued $(i-v)$ fuzzy subsets finds in the research work of L. A. Zadeh [18]. Interval valued fuzzy subsets basically defines the membership functions. These membership functions had closed intervals. Mostly all others have single members. The concept of fuzzy ideal found in the Research work of Lee and C. H. Park [5]. We have investigated these works in subtraction algebras. They also examine

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that fuzzy ideal have some conditions. Fuzzy has various applications in Medicine, Robotics, image processing, Decision making etc., J. Sivaranjini and V. Mahalakshmi [13] developed the research work of $i-v$ fuzzy biideal of near-Subtraction Semigroups(IVFBI). Through this, we conceptualize $i-v$ fuzzy Strong bi ideal of near-subtraction semigroup (IVFSBI) and have studied their related properties. The results obtained are entirely more beneficial to the researchers. Our aim of this article are given as follows

(i) To explore the new ideas in $i-v$ fuzzy Near-subtraction semigroups of said biideals and strong biideals.

(ii) To examine the some basic properties and fundamentals.

(iii) Also expand the direct product and regularity of Strong biideals.

2. Preliminaries

Definition 2.1 [6]**.** Consider X to be defined as set which is non empty along with the '-' and '•' is defined as a right near-subtraction semigroups if for each p, q, r in X

(i) With respect to '-' it defines as a subtraction algebra

(ii) With respect to '•' it defines as a semigroup

(iii) Right Distributive Law follows

Definition 2.2 [15]**.** Consider V as a nonempty subset of X is defined as sub algebra if for p-q in V.

Definition 2.3 [13]**.** Find I as an non-empty subset of X if

(1) If I is a sub algebra and pa-p(q-a) in X for each p, q in X and a in X then I is defines the left ideal.

(2) If I is a sub algebra and also IX is a subset of I then I defines as a right ideal.

(3) An ideal if it satisfies both (1) and (2).

Definition 2.4 [14]**.** A function $\overline{\psi} : X$ is maps to $D[0, 1]$ is defined as $i-v$ fuzzy sub set of X, and $\overline{\psi}(p) = [\psi^{-}(p), \psi^{+}(p)]$. Here the functions ψ^{-} and ψ^{+} are fuzzy subsets of X also $\overline{\psi}(p) \leq \psi^{+}(p)$ for each p in X.

Definition 2.5 [7]**.** An fuzzy sub algebra is defined to be fuzzy biideal of X if $\mu(pqr) \geq \min\{\mu(p), \mu(r)\}$ where p, q, r in X.

Definition 2.6 [13]**.** An $i-v$ fuzzy sub algebra is defined as $i-v$ fuzzy biideal (IVFBI) of X if p, q, r in X $\bar{\mu}(pqr) \geq \min\{\bar{\mu}(p), \bar{\mu}(r)\}$.

Definition 2.7 [15]**.** A near subtraction semigroup X is defined as left permutable if for all p, q, r in X then $pqr = qpr$ in X.

Definition 2.8 [14]**.** Define an $i-v$ fuzzy set $\bar{\mu}$ in X is called as an $i-v$ fuzzy X-sub algebra of X

(i) $\bar{\mu}(p-q) \geq \min\{\bar{\mu}(p), \bar{\mu}(q)\}$

(ii) $\bar{\mu}(pq) \geq \bar{\mu}(q)$

(iii) $\bar{\mu}(pq) \geq \bar{\mu}(q)$ for all p, q in X.

The Conditions (i) and (ii) defines $\bar{\mu}$ as an $i-v$ fuzzy left X-sub algebra of X and the Conditions (i) and (iii) defines $\bar{\mu}$ as an $i-v$ fuzzy right X-sub algebra of X.

3. Main Results

Definition 3.1. An IVFBI $\bar{\mu}$ of X is defined to be an $i-v$ Fuzzy Strong Biideal of X, (IVFSBI) if for p, q, r in X then $\bar{\mu}(pqr) \geq \min\{\bar{\mu}(q), \bar{\mu}(r)\}$.

Example 3.2. Consider $X = \{0, p, q, r\}$ in which '-' and '•' defined by

-	0	p	q	r
0	0	0	0	0
p	p	0	p	0
q	q	q	0	0
r	r	q	p	0

.	0	p	q	r
0	0	0	0	0
p	0	p	0	p

q	0	0	0	0
r	0	q	0	0

Now, Consider $i - v$ fuzzy $\overline{\mu} : X \to DX[0, 1]$ by $\overline{\mu}(0) = [.8, .9]$ $\overline{\mu}(p) = [.6, .7]\overline{\mu}(q) = [.4, .5]\overline{\mu}(r) = [.02, .1]$. Thence $\overline{\mu}$ is an IVFSBI of X.

Theorem 3.3. *If* $\{\overline{\rho}_j / j \in \gamma\}$ *is a family of an IVFSBI of X, then the set* $\bigcap_{j \in \gamma} \overline{\rho}_j$ *is also family of IVFSBI of X, where* γ *is an index set.*

Proof. Choose p, q, r in X. Also $\bigcap_{j \in \gamma} \overline{\rho}_j(p) = \inf_{j \in \gamma} \overline{\rho}_j(p)$. Also $\overline{\rho}_j$ defines as a family of IVFSBI of X. Now

(i) $\bigcap_{j \in \gamma} \overline{\rho}_j(p - q) = \inf_{j \in \gamma} \overline{\rho}_j(p - q)$

$$\geq \inf_{j \in \gamma} \min\{\overline{\rho}_j(p), \overline{\rho}_j(q)\}$$

$$= \min\{\inf_{j \in \gamma} \overline{\rho}_j(p), \inf_{j \in \gamma} \overline{\rho}_j(q)\}$$

$$= \min\{\bigcap_{j \in \gamma} \overline{\rho}_j(p), \bigcap_{j \in \gamma} \overline{\rho}_j(q)\}$$

(ii) $\bigcap_{j \in \gamma} \overline{\rho}_j(pqr) = \inf_{j \in \gamma} \overline{\rho}_j(pqr)$

$$\geq \inf_{j \in \gamma} \min\{\overline{\rho}_j(p), \overline{\rho}_j(r)\}$$

$$= \min\{\inf_{j \in \gamma} \overline{\rho}_j(p), \inf_{j \in \gamma} \overline{\rho}_j(r)\}$$

$$= \min\{\bigcap_{j \in \gamma} \overline{\rho}_j(p), \bigcap_{j \in \gamma} \overline{\rho}_j(r)\}$$

(iii) $\bigcap_{j \in \gamma} \overline{\rho}_j(pqr) = \inf_{j \in \gamma} \overline{\rho}_j(pqr)$

$$\geq \inf_{j \in \gamma} \min\{\overline{\rho}_j(p), \overline{\rho}_j(r)\}$$

$$= \min\{\inf_{j \in \gamma} \overline{\rho}_j(q), \inf_{j \in \gamma} \overline{\rho}_j(r)\}$$

$$= \min\{\bigcap_{j \in \gamma} \overline{\rho}_j(q), \bigcap_{j \in \gamma} \overline{\rho}_j(r)\}$$

Theorem 3.4. *Consider* $\overline{\mu}$ *be an IVFSBI of X if and only if* $X\overline{\mu}\overline{\mu} \subseteq \overline{\mu}$

Proof. Select $\overline{\mu}$ as an IVFSBI of X. Choose p, q, l, m, a in X. Consider $a = pq$ and $p = lm$

$$(\overline{\mu}X\overline{\mu})(\text{a}) = \sup_{a=pq}\{\min\{(\overline{\mu}X)\,(p),\ \overline{\mu}(q)\}\}$$

$$= \sup_{a=pq}\{\min\{\sup_{p=lm}\{\min\{\overline{\mu}(l),\ X\ (m)\},\ \overline{\mu}(q)\}\}$$

$$= \sup_{a=pq}\{\min\{\sup_{p=lm}\{\overline{\mu}(l)\},\ \overline{\mu}(q)\}\}$$

Since $\overline{\mu}$ is an IVFBI of X.

$$= \sup_{a=pq} \min\{\overline{\mu}(1),\ \overline{\mu}(q)\}$$

$$\leq \sup_{a=lmq}\{\overline{\mu}(1mq)\}$$

$$= \overline{\mu}(1mq) = \overline{\mu}(a)$$

We have, $\overline{\mu}X\overline{\mu} \subseteq \overline{\mu}$. Conversely, Assume that $\overline{\mu}X\overline{\mu} \subseteq \overline{\mu}$ If a cannot expressed as $a = pq$ then, $\overline{\mu}X\overline{\mu}(a) = 0 \leq \overline{\mu}(a)$. In both cases $\overline{\mu}X\overline{\mu} \subseteq \overline{\mu}$. Choose p, q, r, a, b, c in X so as $a = pqr$. Thence

$$\overline{\mu}(pqr) \geq \overline{\mu}(a) \qquad \geq \overline{\mu}X\overline{\mu}(a)$$

$$= \sup_{a=bc} \min\{(\overline{\mu}X)\,(b),\ \overline{\mu}(c)\}$$

$$\leq \min\{\overline{\mu}(p),\ X(q),\ \overline{\mu}(r)\}$$

$$= \min\{\overline{\mu}(p),\ \overline{\mu}(r)\}$$

Therefore, $\overline{\mu}(pqr) \leq \min\{\overline{\mu}(p),\ \overline{\mu}(r)\}$. Now to prove that $\overline{\mu}$ is an IVFSBI of X.

$$(X\overline{\mu}\overline{\mu})\,(a) = \sup_{a=pq}\{\min\{(\overline{\mu}X)\,(p),\ \overline{\mu}(q)\}\}$$

$$= \sup_{a=pq}\{\min\{\sup_{p=lm}\{\min\{X(1),\ \overline{\mu}(m)\}\},\ \overline{\mu}(q)\}\}$$

$$= \sup_{a=pq}\{\min\{\sup_{p=lm}\{\overline{\mu}(m)\},\ \overline{\mu}(q)\}\}$$

Since $\overline{\mu}$ is an IVFSBI of X.

$$= \sup_{a=pq} \min\{\overline{\mu}(m),\ \overline{\mu}(q)\}$$

$$\leq \sup_{p=lmq}\{\overline{\mu}(1mq)\}$$

$$= \overline{\mu}(1mq)$$

$$= \overline{\mu}(a)$$

We have, $X\bar{\mu}\bar{\mu} \subseteq \bar{\mu}$. Conversely, Assume that $X\bar{\mu}\bar{\mu} \subseteq \bar{\mu}$. If a cannot expressed as $a = pq$ then, $X\bar{\mu}\bar{\mu}(a) = 0 \leq \bar{\mu}(a)$. In both cases $X\bar{\mu}\bar{\mu} \subseteq \bar{\mu}$. Select p, q, r, a, b, c in X so as $a = pqr$. Thences

$$= \sup_{a=bc} \min\{(X\bar{\mu})(b), \bar{\mu}(c)\}$$

$$\leq \min\{X(p), \bar{\mu}(q), \bar{\mu}(r)\}$$

$$= \min\{\bar{\mu}(q), \bar{\mu}(r)\}$$

Therefore, $\bar{\mu}(pqr) \leq \min\{\bar{\mu}(q), \bar{\mu}(r)\}$

Theorem 3.5. *Consider X as a Strongly regular Near–Subtraction Semigroup. Also* $\bar{\mu}$ *defines as an IVFSBI of X, thence* $X\bar{\mu}\bar{\mu} = \bar{\mu}$.

Proof. Let $\bar{\mu}$ be an IVFSBI of X.

Choose p in X. We have consider X is a strongly regular there exists a $\in X$ then $p = ap^2$. We have, $(X\bar{\mu}\bar{\mu})(p) = X\bar{\mu}\bar{\mu}(ap^2)$. Now

$$(X\bar{\mu}\bar{\mu})(a) = \sup_{p=app}\{\min\{(X\bar{\mu})(ap), \bar{\mu}(q)\}\}$$

$$\geq \min\{X\bar{\mu}(ap), \bar{\mu}(q)\}$$

$$= \min\{\sup_{ap=lm}\{\min\{X(l), \{\bar{\mu}(m)\}, \bar{\mu}(p)\}\}$$

$$\geq \min\{\min\{X(a), \{\bar{\mu}(p)\}, \bar{\mu}(p)\}$$

$$= \min\{\bar{\mu}(p), \{\bar{\mu}(p)\} = \bar{\mu}(p)$$

Also we know that $X\bar{\mu}\bar{\mu} \subseteq \bar{\mu}$

From that, $X\bar{\mu}\bar{\mu} = \bar{\mu}$

Theorem 3.6. *Consider* $\bar{\rho}$ *as* $i - v$ *fuzzy right X-sub algebra of X. Thence every left permutable* $i - v$ *fuzzy right X-sub algebra of X is again an IVFSBI of X.*

Proof. Consider $\bar{\rho}$ be an $i - v$ fuzzy right X-sub algebra of X.

First we prove $\bar{\rho}$ is an IVFBI of X. Choose a, p, q, l, m in X. Take

$a = pq,\ p = lm$

$$\overline{\rho}X\overline{\rho}(a) = \sup_{p=pq}\{\min\{(\overline{\rho}X)\ (p),\ \overline{\rho}(q)\}\}$$

$$= \sup_{a=pq}\{\min\{\sup_{p=lm}\{\min\{\overline{\rho}(l),\ X(m)\},\ \overline{\rho}(q)\}$$

$$= \sup_{a=pq}\{\min\{\sup_{p=lm}\{\overline{\rho}(l)\},\ \overline{\rho}(q)\}$$

$$= \sup_{a=pq}\min\{\overline{\rho}\ (l)\},\ \overline{\rho}(q)\}$$

Also $\overline{\rho}$ defines as an $i-v$ fuzzy right X-sub algebra

$$\overline{\rho}(pq) = \overline{\rho}((lm)q) \geq \overline{\rho}(l)$$

$$= \sup_{a=pq}\min\{\overline{\rho}(pq),\ \overline{\rho}(q)\} \text{ since } X(q) = 1$$

$$= \overline{\rho}(pq) = \overline{\rho}(a)$$

Therefore, $\overline{\rho}X\overline{\rho} \subseteq \overline{\rho}$. Now, to prove for IVFSBI of X.

$$X\overline{\rho}\overline{\rho}(a) = \sup_{p=pq}\{\min\{(X\overline{\rho})\ (p),\ \overline{\rho}(q)\}\}$$

$$= \sup_{a=pq}\{\min\{\sup_{p=lm}\{\min\{X(l),\ \overline{\rho}(m)\},\ \overline{\rho}(q)\}$$

$$= \sup_{a=pq}\{\min\{\sup_{p=lm}\{\overline{\rho}(m)\},\ \overline{\rho}(q)\}$$

Also $\overline{\rho}$ defines as a left permutable $i-v$ Fuzzy right X-Sub algebra of X.

$$\overline{\rho}(pq) = \overline{\rho}((lm)q) = \overline{\rho}(mlq) \geq \overline{\rho}(m)$$

$$= \sup_{p=lmq}\min\{\overline{\rho}(pq),\ X(q)\}. \text{ Since } X(q) = 1$$

$$= \overline{\rho}(pq) = \overline{\rho}(a)$$

Theorem 3.7. *Consider $\overline{\rho}$ as $i-v$ fuzzy left X-sub algebra of X. Thence every left permutable $i-v$ fuzzy left X-sub algebra of X is again an IVFSBI of X.*

Proof. Consider $\overline{\rho}$ as an $i-v$ fuzzy left X-sub algebra of X.

First we prove $\overline{\rho}$ is an IVFBI of X. Choose a, p, q, l, m in X. Take $a = pq,\ p = lm$

$$\overline{\rho}X\overline{\rho}(a) = \sup_{p=pq}\{\min\{\overline{\rho}(p),\ X\overline{\rho}(q)\}\}$$

$$= \sup_{a=pq}\{\min \overline{\rho}(p),\ \{\sup_{p=lm}\{\min\{X(l),\ \overline{\rho}(m)\}\}$$

$$= \sup_{a=pq}\{\min\{\overline{\rho}(p),\ \sup_{p=lm} \overline{\rho}(m)\}$$

$$= \sup_{a=pq} \min\{\overline{\rho}(p)\},\ \overline{\rho}(m)\}$$

Also $\overline{\rho}$ defines as an $i-v$ fuzzy left X-sub algebra

$$\overline{\rho}(pq) = \overline{\rho}((pl)m) \geq \overline{\rho}(m)$$

$$\geq \sup_{a=pq} \min\{\overline{\rho}(p),\ \overline{\rho}(pq)\} \text{ since } X(q) = 1$$

$$= \overline{\rho}(pq) = \overline{\rho}(a)$$

Therefore, $\overline{\rho}X\overline{\rho} \subseteq \overline{\rho}$ Now, to prove for IVFSBI of X.

$$X\overline{\rho}\overline{\rho}(a) = \sup_{p=pq}\{\min\{\overline{\rho}(p),\ \overline{\rho}\overline{\rho}(q)\}\}$$

$$= \sup_{a=pq}\{\min\{\overline{\rho}(p),\ \sup_{p=lm} \min\{\overline{\rho}(l),\ \overline{\rho}(m)\}\}$$

Also $\overline{\rho}$ defines as a left permutable $i-v$ fuzzy left X-Sub algebra of X.

$$\overline{\rho}(pq) = \overline{\rho}(plm) = \overline{\rho}((lp)m) \geq \overline{\rho}(m)$$

$$\geq \sup_{a=pq} \min\{X(1),\ \overline{\rho}(pq)\}. \text{ Since } X(q) = 1$$

$$= \overline{\rho}(pq) = \overline{\rho}(a)$$

4. Conclusion

In this manuscript, we derived the new type of $i-v$ sets in Strong biideals. We will discuss about the permutable function. Here, we defined the some of the basic concept of IVFSBI and their related properties. This research work can be extended to other types of ideals and other algebraic structures of Near-Subtraction semigroups.

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