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INVARIANT CENTROIDAL MEAN GRAPH LABELING

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Abstract

A graph G is an ICM labeling graph (ICMLG), for a one to one mapping $z : V \to 1, 2, 3, \dots, p$ there exist an induced mapping $z^* : E(G) \to N$ given by

$$z^*(xy) = \left[\frac{3[z(x)z^2(y) + z^2(x)z(y)]}{2[z^2(x) + z(x)z(y) + z^2(y)]}\right] \text{ or } z^*(xy) = \left[\frac{3[z(x)z^2(y) + z^2(x)z(y)]}{2[z^2(x) + z(x)z(y) + z^2(y)]}\right]$$

for all distinct $xy \in E(G)$. This paper provides ICMLG of path, Broom, Triangular ladder T_n,

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Square graph P_2 and the graphs obtained by attaching both sides of each vertex of P_n, $P_n + S_{1,2}$, $P_n + S_{1,3}$, $P_n + S_{1,4}$, $P_n + S_3$, $P_n + S_1 + K_1$ and $P_n + 2S_3$ by paths of length 0 to $n-1$ are discussed.

1. Introduction

Georghe Toader and Silvia Toader [1] gave a brief collection of ten Greek means, comparison of Greek means, partial derivatives of means and related results. Somasundaram and Ponraj were introduced the labeling of graphs and its notions [14, 15]. A detailed survey on graph labeling is carried out by Gallian [2] and its various applications. Results on labeling were found [2, 3] and mean labeling in ([4]-[15]).

2. Main Results

Theorem 2.1. *Path is an ICM labeling graph.*

Proof. A path P_n with q edges to be an ICM labeling graph, if z from the vertices of G to N is injective and $z^* : E(G)$ to N induced function defined by;

$$z^*(x_i x_j) = \left\lceil \frac{3[z(x_i)z^2(x_j) + z^2(x_i)z(x_j)]}{2[z^2(x_i) + z(x_i)z(x_j) + z^2(x_j)]} \right\rceil$$

$$\text{or} \quad z^*(x_i x_j) = \left\lfloor \frac{3[z(x_i)z^2(x_j) + z^2(x_i)z(x_j)]}{2[z^2(x_i) + z(x_i)z(x_j) + z^2(x_j)]} \right\rfloor$$

for every $x_i, x_j \in V(G)$ and $x_i \neq x_j$. Then the resulting edges are distinct. Therefore the path is an ICM labeling graph.

Note that if the vertices are labeled with odd numbers, the edges admits even labeling with ICM labeling graph, if the vertices are labeled with even numbers, the edges admits odd labelling with ICM labeling graph.

Theorem 2.2. *The broom graph is an ICM labeling graph.*

Proof. The broom graph with $n + m$ vertices and $n + m - 1$ edges. Let u be vertex of P_n and $v_1, v_2, \ldots, v_m$ be the pendent vertices incident on one end of the path. The ordinary labeling of $v_1, v_2, \ldots, v_m$ is given in figure. Define a vertex labeling $z : V(P_n) \to 5, 11, 17, \ldots, n$ by $z(x) = 6n + 5$, for $n \geq 0$.

The pendent edges are labeled by the ICM,

$$z^*(v_i u_1) = \left[\frac{3[z(v_i) z^2(u_1) + z^2(v_i) z(u_1)]}{2[z^2(v_i) + z(v_i) z(u_1) + z^2(u_1)]} \right]$$

for every $v_i, u_1 \epsilon V(G)$.

The edges of the path are labeled by the ICM,

$$z^*(u_i u_j) = \left[\frac{3[z(u_i) z^2(u_j) + z^2(u_i) z(u_j)]}{2[z^2(u_i) + z(u_i) z(u_j) + z^2(u_j)]} \right]$$

for every $u_i, u_j \epsilon V(G)$ and $u_i \neq u_j$, are distinct. Thus, the broom graph is an ICM labeling graph.

Theorem 2.3. *Every triangular ladder is an ICM labeling graph.*

Proof. The two paths with vertices $u_i; 1 \leq i \leq n$ and $v_i; 1 \leq i \leq n$ respectively, join the vertices $u_i v_i$ and $u_i v_{i+1}$ to obtain the triangular ladder T_n and $2n$ vertices and $4n - 1$ edges. Define a vertex $z = V(P_n) \to 4, 8, 12, \ldots, 4n$ by $z(u) = 4n$ for $n > 0$, and vertex labeling of another path with vertices v_i is $z = V(P_n) \to N$ by $z(v) = 4n - 2$; for $n > 0$. The pendent edges are labeled by the ICM labeling,

$$z^*(v_1 u_1) = \left[\frac{3[z(v_1) z^2(u_1) + z^2(v_1) z(u_1)]}{2[z^2(v_1) + z(v_1) z(u_1) + z^2(u_1)]} \right]$$

For every $v_1, u_1 \epsilon V(G)$. The edges of the path $\{u_i u_{i+1}\}, \{v_i v_{i+1}\}$ and $\{u_i v_{i+1}\}$ are labeled by the ICM labeling,

$$z^*(x_i y_j) = \left[\frac{3[z(x_i) z^2(y_j) + z^2(x_i) z(y_j)]}{2[z^2(x_i) + z(x_i) z(y_j) + z^2(y_j)]} \right];$$

For every $x_i, y_j \epsilon V(G)$ and $x_i \neq y$, are distinct. Thus, the T_n is an ICM labeling graph.

Theorem 2.4. *Square graph is an ICM labeling graph.*

Proof. If P_n has n vertices $x_1, x_2, x_3, \ldots, x_n$ and whenever $d(u, v) \leq 2$, then P_n^2 has n vertices and $(2n-3)$ edges. Define $z : V(P_n^2) \to N$ such that $z(x_j) = 2j-1, 1 \leq j \leq n$, then

$$z^*(x_i x_{i+1}) = \left[\frac{3[z(x_i)z^2(x_{i+1}) + z^2(x_i)z(x_{i+1})]}{2[z^2(x_i) + z(x_i)z(x_{i+1}) + z^2(x_{i+1})]}\right]$$

for every $\{x_i, x_{i+1}\} = e \epsilon E(P_n^2)$.

The edges $x_i x_{i+2}$ are labeled by

$$z^*(x_i x_{i+2}) = \left[\frac{3[z(x_i)z^2(x_{i+2}) + z^2(x_i)z(x_{i+2})]}{2[z^2(x_i) + z(x_i)z(x_{i+2}) + z^2(x_{i+2})]}\right],$$

for every $\{x_i, x_{i+2}\} = e \epsilon E(P_n^2)$, are distinct. Thus z^* is injective and P_n^2 is an ICM labeling graph.

Theorem 2.5. *The graph* $P_n + S_{1,2}$ *is an ICM labeling graph.*

Proof. The graph $P_n + S_{1,2}$ is a graph obtained by inserting a star graph $S_{1,2}$ for each vertices of a path P_n with vertex 1. Let $u_1, u_2, u_3, \ldots, u_n$ be the vertices of a path P_n. Let x_i and y_j be the pendant vertices of $S_{1,2}$. Then the graph $P_n + S_{1,2}$ has $3m+2$, vertices $m \geq 1$ and $n-1+2m$ edges. Define a vertex labeling $z : V(P_n + S_{1,2}) \to \{2, 4, 6, \ldots, 2n\}$ by $z(x_j) = 3j$, $j = 1, 2, \ldots, n; z(y_j) = 3j+1, j = 1, 2, \ldots, n; z(y_j) = 3j+2, j = 1, 2, \ldots, n$, clearly, labels of the edges received by the ICM labeling of the labels on end vertices using

$$z^*(x_i u_i) = \left[\frac{3[z(x_i)z^2(u_i) + z^2(x_i)z(u_i)]}{2[z^2(x_i) + z(x_i)z(u_i) + z^2(u_i)]}\right]; \text{ for every } x_i, u_i \epsilon V(G).$$

The edges $u_i y_i \in V(G); z^*(u_i y_i) = \left[\frac{3[z(u_i)z^2(y_i) + z^2(u_i)z(y_i)]}{2[z^2(u_i) + z(u_i)z(y_i) + z^2(y_i)]}\right]$ for every $x_i, y_i \epsilon V(G)$, and the edges $u_i u_{i+1}$ are labeled by

$$z^*(u_iu_{i+1}) = \left[\frac{3[z(u_i)z^2(u_{i+1}) + z^2(u_i)z(u_{i+1})]}{2[z^2(u_i) + z(u_i)z(u_{i+1}) + z^2(u_{i+1})]}\right];$$

for every $u_i, u_{i+1} \epsilon V(G)$. Such that for $i \neq j, z^*(e_i) \neq z^*(e_j)$. Thus, z^* is injective and $P_n + S_{1,2}$ is an ICM labeling.

Theorem 2.6. *The graph* $P_n + S_{1,3}$ *is an invariant centroidal mean graph.*

Proof. The graph $P_n + S_{1,3}$ has $4n$ vertices $4n - 1$ edges. Let the path P_n has u_i vertices and x_i, y_i, z_i are the pendant vertices of the star $S_{1,3}$. On every vertices of a path P_n place the vertex 1 of star $S_{1,3}$ to get the graph $P_n + S_{1,3}$. Define a vertex labeling $z : V(P_n + S_{1,3}) \to N$ by

$$z(u_j) = 4j + 4; 1 \leq j \leq n; z(x_j) = 4j + 1; 1 \leq j \leq n;$$

$$z(y_j) = 4j + 2; 1 \leq j \leq n; z(z_j) = 4j + 3; 1 \leq j \leq n;$$

The edges $\{u_iu_{i+1}\}$ are labeled with

$$z^*(\{u_iu_{i+1}\}) = \left[\frac{3[z(u_i)z^2(u_{i+1}) + z^2(u_i)z(u_{i+1})]}{2[z^2(u_i) + z(u_i)z(u_{i+1}) + z^2(u_{i+1})]}\right]$$

The edges $\{u_ix_i\}$ are labeled with

$$z^*(\{u_ix_i\}) = \left[\frac{3[z(u_i)z^2(x_i) + z^2(u_i)z(x_i)]}{2[z^2(u_i) + z(u_i)z(x_i) + z^2(x_i)]}\right]$$

The edges $\{u_iy_i\}$ are labeled with

$$z^*(\{u_iy_i\}) = \left[\frac{3[z(u_i)z^2(y_i) + z^2(u_i)z(y_i)]}{2[z^2(u_i) + z(u_i)z(y_i) + z^2(y_i)]}\right]$$

for every in $u_i, y_i \in V(G)$ and $u_i \neq y_i$; and the edges in $\{u_iz_i\}$ are labeled by

$$z^*(\{u_iz_i\}) = \left[\frac{3[z(u_i)z^2(z_i) + z^2(u_i)z(z_i)]}{2[z^2(u_i) + z(u_i)z(z_i) + z^2(z_i)]}\right]$$

for every in $u_i, z_i \in V(G)$ and $u_i \neq z_i$ are all distinct. Hence the function z^* admits the invariant centriodal mean labeling. Therefore the graph $P_n + S_{1,3}$ is an invariant centroidal mean graph.

Theorem 2.7. *The graph* $P_n + S_{1,4}$ *is an invariant centriodal mean graph.*

Proof. The graph $P_n + S_4$ has $5n$ vertices $5n - 1$ edges. Let the path P_n has u_i vertices and x_i, y_i, z_i are the pendant vertices of the star $S_{1,4}$. On every vertices of a path P_n place the vertex 1 of star $S_{1,4}$ to get the graph $P_n + S_{1,4}$. Define a vertex labeling $z : V(P_n + S_{1,4}) \to N$ by

$$z(u_j) = 5j + 3; 1 \leq j \leq n; z(x_j) = 5j - 1; 1 \leq j \leq n; z(x_j) = 5j - 1; 1 \leq j \leq n;$$

$$z(h_j) = 5j + 1; 1 \leq j \leq n; z(w_j) = 5j + 2; 1 \leq j \leq n;$$

The edges $\{u_i u_{i+1}\}$ are labeled with

$$z^*(\{u_i u_{i+1}\}) = \left[\frac{3[z(u_i)z^2(u_{i+1}) + z^2(u_i)z(u_{i+1})]}{2[z^2(u_i) + z(u_i)z(u_{i+1}) + z^2(u_{i+1})]}\right]$$

The edges $\{u_i x_i\}$ are labeled with

$$z^*(\{u_i x_i\}) = \left[\frac{3[z(u_i)z^2(x_i) + z^2(u_i)z(x_i)]}{2[z^2(u_i) + z(u_i)z(x_i) + z^2(x_i)]}\right]$$

The edges $\{u_i y_i\}$ are labeled with

$$z^*(\{u_i y_i\}) = \left[\frac{3[z(u_i)z^2(y_i) + z^2(u_i)z(y_i)]}{2[z^2(u_i) + z(u_i)z(y_i) + z^2(y_i)]}\right]$$

for every in $u_i, y_i \in V(G)$ and $u_i \neq y_i$ and the edges in $\{u_i z_i\}$ are labeled by

$$z^*(\{u_i z_i\}) = \left[\frac{3[z(u_i)z^2(z_i) + z^2(u_i)z(z_i)]}{2[z^2(u_i) + z(u_i)z(z_i) + z^2(z_i)]}\right]$$

for every in $u_i, z_i \in V(G)$ and $u_i \neq z_i$.

The edges $\{u_i w_i\}$ are labeled with

$$z^*(\{u_i w_i\}) = \left[\frac{3[z(u_i)z^2(w_i) + z^2(u_i)z(w_i)]}{2[z^2(u_i) + z(u_i)z(w_i) + z^2(w_i)]}\right]$$

for every in $u_i, w_i \in V(G)$ and $u_i \neq w_i$ are all distinct. Hence the function z^* admits the invariant centriodal mean labeling. Therefore the graph $P_n + S_{1,3}$ is an invariant centroidal mean graph.

Theorem 2.8. *For any integer* $n \geq 1$, *the graph* $P_n + S_3$ *is an invariant centriodal mean graph*.

Proof. Consider $u_1, u_2, u_3, \ldots, u_n$ are the vertices of a path. Then $P_n + S_3$ is obtained by connecting one pendant vertex of star S_3 to every vertices of a path.

Define a vertex labeling $z : V(P_n + S_3) \to N$ by

$$z(x_j) = 4j - 3; 1 \leq j \leq n; z(y_j) = 4j - 2; 1 \leq j \leq n;$$

$$z(h_j) = 4j - 1; 1 \leq j \leq n; z(u_j) = 4j; 1 \leq j \leq n;$$

The edges $\{u_i u_{i+1}\}$ are labeled with

$$z^*(\{u_i u_{i+1}\}) = \left[\frac{3[z(u_i)z^2(u_{i+1}) + z^2(u_i)z(u_{i+1})]}{2[z^2(u_i) + z(u_i)z(u_{i+1}) + z^2(u_{i+1})]}\right]$$

The edges $\{x_i z_i\}$ are labeled with

$$z^*(\{x_i z_i\}) = \left[\frac{3[z(x_i)z^2(z_i) + z^2(x_i)z(z_i)]}{2[z^2(x_i) + z(x_i)z(z_i) + z^2(z_i)]}\right]$$

The edges $\{y_i z_i\}$ are labeled with

$$z^*(\{y_i z_i\}) = \left[\frac{3[z(y_i)z^2(z_i) + z^2(y_i)z(z_i)]}{2[z^2(y_i) + z(y_i)z(z_i) + z^2(z_i)]}\right]$$

for every in $y_i, z_i \in V(G)$ and $y_i \neq z_i$; and the edges in $\{u_i z_i\}$ are labeled by

$$z^*(\{u_i z_i\}) = \left[\frac{3[z(u_i)z^2(z_i) + z^2(u_i)z(z_i)]}{2[z^2(u_i) + z(u_i)z(z_i) + z^2(z_i)]}\right]$$

for every in $u_i, z_i \in V(G)$ and $u_i \neq z_i$.

Such that for $i \neq j, z^*(e_i) \neq z^*(e_j)$ for $i \neq j$ therefore z^* admits the invariant centroidal mean labeling. Hence the graph $P_n + S_3$ is an invariant centroidal mean graph.

Theorem 2.9. *For any integer* $n \geq 1$, *the graph* $P_n + S_3 + K_1$ *is an invariant centriodal mean graph.*

Proof. Consider $u_1, u_2, u_3, \ldots, u_n$ are the vertices of a path. Then $P_n + S_3 + K_1$ is obtained by connecting one pendant vertex of star S_3 and each vertex K_1 to every vertices of a path P_n. Define a vertex labeling $z : V(P_n + S_3 + K_1) \to N$ by

$$z(x_i) = 5j - 4; i = 1, 2, \ldots, n; z(y_j) = 5j - 3; j = 1, 2, \ldots, n;$$

$$z(h_j) = 5j - 2; j = 1, 2, \ldots n; z(u_j) = 5j - 1; j = 1, 2, \ldots n;$$

$$z(u_j) = 5j; j = 1, 2, \ldots n$$

The edges $\{u_i u_{i+1}\}$ are labeled with

$$z^*(\{u_i u_{i+1}\}) = \left[\frac{3[z(u_i)z^2(u_{i+1}) + z^2(u_i)z(u_{i+1})]}{2[z^2(u_i) + z(u_i)z(u_{i+1}) + z^2(u_{i+1})]}\right]$$

The edges $\{x_i z_i\}$ are labeled with

$$z^*(\{x_i z_i\}) = \left[\frac{3[z(x_i)z^2(z_i) + z^2(x_i)z(z_i)]}{2[z^2(x_i) + z(x_i)z(z_i) + z^2(z_i)]}\right]$$

The edges $\{y_i z_i\}$ are labeled with

$$z^*(\{y_i z_i\}) = \left[\frac{3[z(y_i)z^2(z_i) + z^2(y_i)z(z_i)]}{2[z^2(y_i) + z(y_i)z(z_i) + z^2(z_i)]}\right]$$

for every in $y_i, z_i \in V(G)$ and $y_i \neq z_i$; and the edges in $\{u_i z_i\}$ are labeled by

$$z^*(\{u_iz_i\}) = \left[\frac{3[z(u_i)z^2(z_i) + z^2(u_i)z(z_i)]}{2[z^2(u_i) + z(u_i)z(z_i) + z^2(z_i)]}\right]$$

for every in $u_i, z_i \in V(G)$ and $u_i \neq z_i$.

The edges $\{u_iv_i\}$ are labeled with

$$z^*(\{u_iv_i\}) = \left[\frac{3[z(u_i)z^2(v_i) + z^2(u_i)z(v_i)]}{2[z^2(u_i) + z(u_i)z(v_i) + z^2(v_i)]}\right]$$

for every in $u_i, v_i \in V(G)$ and $u_i \neq v_i$. Such that for $i \neq j$, $z^*(e_i) \neq z^*(e_j)$ for $i \neq j$ therefore z^* admits the invariant centroidal mean labeling. Hence the graph $P_n + S_3 + K_1$ is an invariant centroidal mean graph.

Theorem 2.10. *For any integer* $n \geq 1$, *the graph* $P_n + 2S_3$ *is an invariant centroidal mean graph.*

Proof. Consider $u_1, u_2, u_3, \ldots, u_n$ are the vertices of a path. Then $P_n + 2S_3$ is obtained by connecting one pendant vertex of star S_3 on either side of every vertices of a path P_n.

Define a vertex labeling $z : V(P_n + 2S_3) \to N$ by

$$z(x_i) = 7i - 6; z(y_i) = 7i - 5; z(z_i) = 7i - 4; z(u_i) = 7i - 3; z(v_i) = 7i - 1;$$

$$z(t_i) = 7i - 2; z(s_i) = 7i; i = 1, 2, \ldots, n,$$

The edges $\{u_1u_2\}$ are labeled with $z^*(\{u_1u_2\}) = 7$.

The edges $\{u_iu_{i+1}\}$ are labeled with

$$z^*(\{u_iu_{i+1}\}) = \left[\frac{3[z(u_i)z^2(u_{i+1}) + z^2(u_i)z(u_{i+1})]}{2[z^2(u_i) + z(u_i)z(u_{i+1}) + z^2(u_{i+1})]}\right] \text{ for } i \geq 2$$

The edges $\{x_iz_i\}$ are labeled with

$$z^*(\{x_iz_i\}) = \left[\frac{3[z(x_i)z^2(z_i) + z^2(x_i)z(z_i)]}{2[z^2(x_i) + z(x_i)z(z_i) + z^2(z_i)]}\right]$$

The edges $\{y_i z_i\}$ are labeled with

$$z^*(\{y_i z_i\}) = \left[\frac{3[z(y_i)z^2(z_i) + z^2(y_i)z(z_i)]}{2[z^2(y_i) + z(y_i)z(z_i) + z^2(z_i)]}\right]$$

for every in $y_i, z_i \in V(G)$ and $y_i \neq z_i$.

The edges $\{u_i v_i\}$ are labeled with

$$z^*(\{u_i v_i\}) = \left[\frac{3[z(u_i)z^2(v_i) + z^2(u_i)z(v_i)]}{2[z^2(u_i) + z(u_i)z(v_i) + z^2(v_i)]}\right]$$

for every in $u_i, v_i \in V(G)$ and $u_i \neq v_i$.

The edges $\{t_i v_i\}$ are labeled with

$$z^*(\{t_i v_i\}) = \left[\frac{3[z(t_i)z^2(v_i) + z^2(t_i)z(v_i)]}{2[z^2(t_i) + z(t_i)z(v_i) + z^2(v_i)]}\right]$$

for every in $t_i, v_i \in V(G)$ and $t_i \neq v_i$.

The edges $\{s_i v_i\}$ are labeled with

$$z^*(\{s_i v_i\}) = \left[\frac{3[z(s_i)z^2(v_i) + z^2(s_i)z(v_i)]}{2[z^2(s_i) + z(s_i)z(v_i) + z^2(v_i)]}\right]$$

For every in $s_i, v_i \in V(G)$ and $u_i \neq v_i$. Such that $z^*(e_i) \neq z^*(e_j)$ for $i \neq j$ therefore z^* admits the invariant centroidal mean labeling. Hence the graph $P_n + 2S_3$ is an invariant centroidal mean graph.

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