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TOTALLY REGULAR PROPERTY OF UNION OF INTUITIONISTIC FUZZY GRAPHS

H. SHEIK MUJIBUR RAHMAN and A. NAGOOR GANI

P.G. & Research Department of Mathematics
Jamal Mohamed College (Autonomous)
(Affiliated to Bharathidasan University)
Tiruchirappalli-620020, India
E-mail: mujeebmaths@gmail.com
ganijmc@yahoo.co.in

Abstract

Totally regular property of union of two intuitionistic fuzzy graphs need not be a totally regular intuitionistic fuzzy graph. In this paper, the necessary conditions for the union of two totally regular intuitionistic fuzzy graphs to be totally regular under some restrictions are obtained.

1. Introduction

Intuitionistic Fuzzy Graph theory was introduced by Atanassov in [1]. In [6] A. Nagoor Gani and S. Shajitha Begum introduced degree, order and size in intuitionistic fuzzy graph. In [10] Radha and Vijaya introduced the totally regular property of the composition of some fuzzy graphs. A. Nagoor Gani and H. Sheik Mujibur Rahman introduced the Total degree of a vertex in union, join Cartesian product and Composition of some intuitionistic fuzzy graphs in [7]. In this paper we introduce totally regular property of the union of some intuitionistic fuzzy graph.

2. Preliminaries

Definition 2.1. An intuitionistic fuzzy graph (IFG) is of the form $G = \langle V, E \rangle$ where (i) $V = \{v_1, v_2, \ldots, v_n\}$ such that $\mu_1 : V \to [0, 1]$ and

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$\nu_1 : V \to [0, 1]$ denotes the degree of membership and non-membership of the element $v_i \in V$ respectively and $0 \le \mu_1(v_i) + \nu_1(v_i) \le 1$, for every $v_i \in V$.

(ii) $E \subseteq V \times V$ where $\mu_2 : V \times V \to [0, 1]$ and $\nu_2 : V \times V \to [0, 1]$ such that

$$\mu_2(v_i, v_j) \le \min(\mu_1(v_i), \mu_1(v_j))$$

$$\nu_2(v_i, v_j) \le \min(\nu_1(v_i), \nu_1(v_j))$$

and $0 \le \mu_2(v_i, v_j) + \nu_2(v_i, v_j) \le 1$, for every $(v_i, v_j) \in E$.

Here the triple $(v_i, \mu_{1i}, \nu_{1i})$ denotes the degree of membership and non-membership of the vertex v_i. The triple $(e_{ij}, \mu_{2ij}, \nu_{2ij})$ denotes the degree of membership and non-membership of the edge relation $e_{ij} = (v_i, v_j)$ on $V \times V$.

Definition 2.2. Let $G = \langle V, E\rangle$ be an IFG. Then the degree of a vertex v is defined by $d(v) = (d_\mu(v), d_\nu(v))$, where $d_\mu(v) = \sum_{u \ne v} \mu_2(v, u)$ and $d_\nu(v) = \sum_{u \ne v} \nu_2(v, u)$.

Definition 2.3. Let $G = \langle V, E\rangle$ be an IFG. If $(d_\mu(v), d_\nu(v)) = (k_1, k_2)$ for all $v \in V$ that is if each vertex has same membership degree k_1 and same nonmembership degree k_2 then G is said to be a regular intuitionistic fuzzy graph.

Definition 2.4. Let $G = \langle V, E\rangle$ be an IFG. Then the total degree of a vertex $u \in v$ is defined by

$$td(u) = (td_\mu(u), td_\nu(u)) = \left(\sum_{u \ne v} \mu_2(u, v) + \mu_1(u), \sum_{u \ne v} \nu_2(u, v) + \nu_1(u)\right)$$

$$= (d_\mu(u) + \mu_1(u), d_\nu(u) + \nu_1(u)).$$

If each vertex of G has same membership total degree k_1 and same nonmembership total degree k_2, then said to be a totally regular intuitionistic fuzzy graph.

Definition 2.5. Let $G = (V, E)$ be an intuitionistic fuzzy graph. Then the minimum degree of G is $\delta(G) = (\delta_\mu(G), \delta_\nu(G))$, where $\delta_\mu(G) = \min\{d_\mu(v)/v \in V\}$ and $\delta_\nu(G) = \min\{d_\nu(v)/v \in V\}$.

Definition 2.6. Let $G = (V, E)$ be an intuitionistic fuzzy graph. Then the maximum degree of G is $\Delta(G) = (\Delta_\mu(G), \Delta_v(G))$, where $\Delta_\mu(G) = \max\{d_\mu(v)/v \in V\}$ and $\Delta_\nu(G) = \max\{d_\nu(v)/v \in V\}$.

Definition 2.7. Let $G = (V, E)$ be an intuitionistic fuzzy graph. Then G is an irregular intuitionistic fuzzy graph, if there is a vertex which is adjacent to vertices with distinct degrees.

Definition 2.8. Let $G_1 : (V_1, E_1)$ and $G_2 : (V_2, E_2)$ be two intuitionistic fuzzy graphs and $G = G_1 \cup G_2 = (V_1 \cup V_2, E_1 \cup E_2)$ be the union of G_1 and G_2. Then the union of intuitionistic fuzzy graphs G_1 and G_2 is an intuitionistic fuzzy graph defined by

$$(\mu_1 \cup \mu_1')(v) = \begin{cases} \mu_1(v), & \text{if } v \in V_1 \\ \mu_1'(v), & \text{if } v \in V_2 \\ \mu_1(v) \vee \mu_1'(v), & \text{if } v \in V_1 \cap V_2 \end{cases}$$

$$(\nu_1 \cup \nu_1')(v) = \begin{cases} \nu_1(v), & \text{if } v \in V_1 \\ \nu_1'(v), & \text{if } v \in V_2 \\ \nu_1(v) \wedge \nu_1'(v), & \text{if } v \in V_1 \cap V_2 \end{cases}$$

$$(\mu_2 \cup \mu_2')(v_i, v_j) = \begin{cases} \mu_{2ij}, & \text{if } e_{ij} \in E_1 \\ \mu_{2ij}', & \text{if } e_{ij} \in E_2 \\ \mu_{2ij} \vee \mu_{2ij}', & \text{if } e_{ij} \in E_1 \cap E_2 \end{cases}$$

$$(\nu_2 \cup \nu_2')(\nu_i, \nu_j') = \begin{cases} \nu_{2ij}, & \text{if } e_{ij} \in E_1 \\ \nu_{2ij}', & \text{if } e_{ij} \in E_2 \\ \nu_{2ij} \wedge \nu_{2ij}', & \text{if } e_{ij} \in E_1 \cap E_2 \end{cases}$$

Where (μ_1, ν_1) and (μ_1', ν_1') refer the vertex membership and nonmembership of G_1 and G_2 respectively, (μ_2, ν_2) and (μ_2', ν_2') refer the edge membership and nonmembership of G_1 and G_2 respectively.

3. Total Degree of a Vertex in Union of Intuitionistic Fuzzy Graph

Let $G_1 : (V, E)$ and $G_2 : (V', E')$ be two intuitionistic fuzzy graphs

(i) $td_{\mu(G_1 \cup G_2)}(u) = td_{\mu G_1}(u)$; $td_{\mu(G_1 \cup G_2)}(u) = td_{\mu G_2}(u)$ and $td_{\nu(G_1 \cup G_2)}(u) = td_{\nu G_1}(u)$; $td_{\upsilon(G_1 \cup G_2)}(u) = td_{\nu G_2}(u)$, if $u \in V_1$ or $u \in V_2$ but not both.

(ii) $td_{\mu(G_1 \cup G_2)}(u) = td_{\mu G_1}(u) + td_{\mu G_2}(u) - \mu_1(u) \wedge \mu_1'(u)$ and $td_{\nu(G_1 \cup G_2)}(u) = td_{\nu G_1}(u) + td_{\nu G_2}(u) - \nu_1(u) \vee \nu_1'(u)$ if $u \in V_1 \cap V_2$ but no edge incident at u lies in $E_1 \cap E_2$.

(iii)

$$td_{\mu(G_1 \cup G_2)}(u) = td_{\mu G_1}(u) + td_{\mu G_2}(u) - \mu_1(u) \wedge \mu_1'(u) - \sum_{uv \in E_1 \cap E_2} \mu_2(uv) \wedge \mu_2'(uv)$$

and

$$td_{\nu(G_1 \cup G_2)}(u) = td_{\nu G_1}(u) + td_{\nu G_2}(u) - \nu_1(u) \vee \nu_1'(u) - \sum_{uv \in E_1 \cap E_2} \nu_2(uv) \vee \nu_2'(uv)$$

if $u \in V_1 \cap V_2$ and some edges incident at u are in $E_1 \cap E_2$.

4. Properties of Totally Regular Intuitionistic Fuzzy Graphs

Proposition 4.1. *In any intuitionistic fuzzy graph G, if* $(\mu_1, \nu_1)(v) > 0$, *for every vertex* $v \in V$, *then* $td_\mu(v) > 0$, $td_\nu(v) > 0$ *for every vertex* $v \in V$.

Proof. Since $(\mu_1, \nu_1)(v) > 0$ for every vertex $v \in V$, $td_\mu(v) > 0$, $td_\nu(v) > 0$ for every vertex $v \in V$.

Proposition 4.2. *The total degree of a vertex v is* $(\mu_1, \nu_1)(v)$ *if and only if the degree of v is zero.*

Proof. The total degree of a vertex v is $(td_{\mu(G)}(v), td_{\nu(G)}(v)) = (\mu_1, \nu_1)(v)$

$$\Leftrightarrow \sum_{uv \in E} \mu_2 \nu_2(uv) + (\mu_1, \nu_1)(v) = (\mu_1, \nu_1)(v)$$

$$\Leftrightarrow \sum_{uv \in E} \mu_2 \nu_2(uv) = 0$$

$$\Leftrightarrow (d_{\mu(G)}(v), d_{\nu(G)}(v)) = 0$$

$$\Rightarrow d_G(v) = 0.$$

Proposition 4.3. *The total degree of a vertex v is $(\mu_1, \nu_1)(v)$ for every vertex v in G if and only if G is a null intuitionistic fuzzy graph.*

Proof. $(td_{\mu(G)}(v), td_{\nu(G)}(v)) = (\mu_1, \nu_1)(v)$ for every vertex $v \in V$

$$\Leftrightarrow d_G(v) = 0, \text{ for every vertex } v \in V$$

$$\Leftrightarrow G \text{ is a null intuitionistic fuzzy graph.}$$

5. Totally Regular Property of Union of Intuitionistic Fuzzy Graphs

Theorem 5.1. *If G_1 and G_2 are two disjoint (k_1, k_2)-totally regular intuitionistic fuzzy graphs, then $G_1 \cup G_2$ is a (k_1, k_2)-totally regular intuitionistic fuzzy graph.*

Proof. Since G_1 and G_2 are disjoint intuitionistic fuzzy graphs.

$$td_{\mu(G_1 \cup G_2)}(u) = \begin{cases} td_{\mu(G_1)}(u), \text{ if } u \in V_1 \\ td_{\mu(G_2)}(u), \text{ if } u \in V_2 \end{cases}$$

$$= k_1, \text{ for every } u \in V_1 \cup V_2$$

$$td_{\nu(G_1 \cup G_2)}(u) = \begin{cases} td_{\nu(G_1)}(u), \text{ if } u \in V_1 \\ td_{\nu(G_2)}(u), \text{ if } u \in V_2 \end{cases}$$

$$= k_2, \text{ for every } u \in V_1 \cup V_2$$

$\therefore G_1 \cup G_2$ is (k_1, k_2)-totally regular intuitionistic fuzzy graph.

Example 5.2.

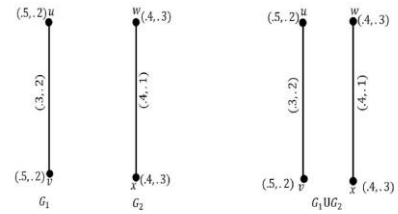

Figure 5.1.

In the above Example the graphs G_1 and G_2 are (0.8, 0.4) totally regular intuitionistic fuzzy graphs, $G_1 \cup G_2$ is also (0.8, 0.4) totally regular intuitionistic fuzzy graph.

Remark 5.3. The above theorem does not hold when G_1 and G_2 are edge disjoint but not vertex disjoint intuitionistic fuzzy graphs. For example, consider the union of two intuitionistic fuzzy graphs G_1 and G_2 in the following figure

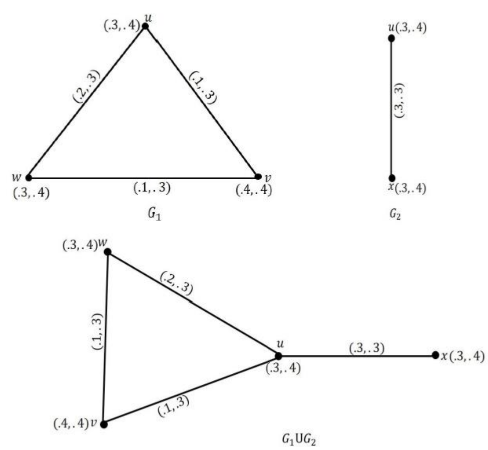

Figure 5.2.

In figure 5.2 G_1 and G_2 are totally regular fuzzy graphs but $G_1 \cup G_2$ is not totally regular. Here G_1 and G_2 are edge disjoint but they have the vertex u in common.

Conclusion

In this paper we have showed that the Union of two totally regular intuitionistic fuzzy graphs to be totally regular intuitionistic fuzzy graph in some particular cases and also discussed some basic properties of totally regular intuitionistic fuzzy graphs.

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