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GRAPHIC INTUITIONISTIC ALEXANDROFF TOPOLOGICAL SPACES

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Abstract

The focus of this paper is to introduce Intuitionistic concepts on Alexandroff topological space by using simple graphs. Some effective characterizations and properties are studied. Furthermore, proved some results, examples and counter examples are discussed by using simple graphs.

1. Introduction

Graph theory is most significant field of mathematical research fields. It has so much of application and plays vital role in various fields like operations research, social sciences, chemistry, computer science and etc.,

A graph $G = (V, E)$ is a pair of nodes denoted by V and set of arcs denoted by E. A simple graph has no self loops or multiple edges. The number of incident edges to v is called degree of V and is denoted by $d(v)$. An isolated vertex has zero degree.

Topology is a mathematical subject that studies shapes. A set becomes a topological space, when each element of the set is given a collection of neighbourhoods. Topology is the part of geometry has does not concern distance. Topology is one among four major areas of abstract mathematics:

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pure mathematics (equations), analysis (limits), foundations (set theory and logic), and topology. Topology is all told the weakest quite structure present in all usual geometrical areas, as well as projective and inversive areas. For now, allow us to solely offer intuitive descriptions instead of formal (mathematical) ones, since the doable formalisms are a small amount more durable to understand. Broadly speaking, topology is that the study of area and continuity. Since topology includes the study of continuous deformations of an area, it's typically popularly referred to as rubber sheet pure mathematics.

Topological graph theory deals with ways to represent the geometric realization of graphs. In this paper we study about an Alexandroff topological space associated with the graphs. An Alexandroff space is also a topological space with the condition arbitrary intersection of open sets are open. These spaces were first introduces by P. Alexandroff in 1937 [1].

S. M. Jafarian Amiri, A. Jafarzadeh and H. Khatibzadeh introduced an Alexandroff topology on graphs [6], in that paper they proved. An Alexandroff Topology was associated with graphs. The concept of intuitionistic set and intuitionistic topological space was introduced by [7]. In (2019) D. Sasikla, A. Divya introduced Alexandroff topological space on the vertex set of sum cordial graphs [11]. And also in (2020) they discussed Visualization of cordial graph in human excretion track [2]. T. Here we extended this concepts by using intuitionistics set, and investigate some properties of an Alexandroff topological space by simple graphs.

2. Preliminaries

The following brief summary of definitions are given which are we use in the subsequent sequel.

Definition 2.1 [11]. Let $G = (V(G), E(G))$ be a simple graph with sum cordial labeling and without isolated vertex. Define S_{0G} and S_{1G} as follows. $S_{0G} = \{A_{v(0)} \mid v \in V\}$ and $S_{1G} = \{A_{v(1)} \mid v \in V\}$ such that $A_{v(0)}$ and $A_{v(1)}$ is the set of all vertices adjacent to v of G having label 0 and 1 respectively. Since G has no isolated vertex, $S_{0G} \cup S_{1G}$ forms a sub basis for a topology τ_{CG} on V is called Cordial graphic topology of G and it is denoted by (V, τ_{CG}).

Definition 2.2 [11]**.** Let $G = (V(G), E(G))$ be a simple graph with sum cordial labeling and without isolated vertex. Define $S_{E(0G)}$ and $S_{E(1G)}$ as follows. $S_{E(0G)} = \{I_{e(0)} \mid e \in E\}$ and $S_{E(1G)} = \{I_{e(1)} \mid e \in E\}$ such that $I_{e(0)}$ and $I_{e(1)}$ is the incidence vertices having label 0 and 1 respectively. Since G has no isolated vertex, $S_{E(0G)} \cup S_{E(1G)}$ forms a subbasis for a topology τ_{CI} on V is called cordial incidence topology of G and it is denoted by (V, τ_{CI}).

Definition 2.3 [12]**.** Let $G = (V(G), E(G))$ be a simple graph with sum cordial labeling and without isolated vertex. Define S_{0G} and S_{1G} as follows. $S_{0G} = \{A_{v(0)} \mid v \in V\}$ and $S_{1G} = \{A_{v(1)} \mid v \in V\}$ such that $A_{v(0)}$ and A_{v1} is the set of all vertices adjacent to v of G having label 0 and 1 respectively. Since G has no isolated vertex. Hence $S_{0G} \cup S_{1G}$ forms a subbasis for a topology τ_{CG} on V is called Cordial graphic topology of G and it is denoted by (V, τ_{CG}).

Definition 2.4 [11]**.** Cordial graphic topology of G is called Cordial Alexandroff topological space if and only if arbitrary intersection of members of $S_{0G} \cup S_{1G}$ is open in τ_{CG}.

Definition 2.5 [4]**.** Let X be a non-empty set, an intuitionistic set (IS in short). A is an object having the form $A = \langle X, A_1, A_2 \rangle$, where A_1 and A_2 are subsets of X satisfying $A_1 \cap A_2 = \emptyset$. The set A_1 is called the set of members of A, while A_2 is called the set of non-members of A.

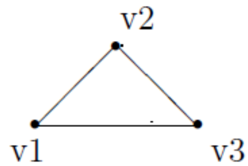
3. Graphic Intuitionistic Alexandroff Topological Spaces

Definition 3.1. Let $G = (V(G), E(G))$ be a simple graph without isolated vertex. Then A_v is called graph intuitionistic set (In short GIS) of V on G, if it is an object having the form $A_v = \langle V, A_1, A_2 \rangle$, where A_1 and A_2 are adjacent vertices and non-adjacent vertices of V respectively and satisfying $A_1 \cap A_2 = \emptyset$. The set A_1 is called the set of members of A and A_2 is called the set of non-members of A.

Definition 3.2. Let G be a simple graph without isolated vertex. Define

$S_{GIS} = \{A_v \mid v \in V\}$ such that A_v is the graphic Intuitionistic set of V on G. Since G has no isolated vertex, so S_{GIS} forms a subbasis for a topology τ_{GIS} on G is called graphic intuitionistic Alexandroff topological space and denoted by τ_{GIS}.

Example 3.3. Let us consider the Cycle C_3. Let v_1, v_2, v_3 be the vertices of C_3



Then we have from figure

$A_{v_1} = \langle V, \{v_2, v_3\}, \emptyset\rangle$

$A_{v_2} = \langle V, \{v_1, v_3\}, \emptyset\rangle$

$A_{v_3} = \langle V, \{v_1, v_2\}, \emptyset\rangle$

$S_{GIS} = \{V, \emptyset, \{v_2, v_3\}, \{v_1, v_3\}, \{v_1, v_2\}\}$

$\tau_{GIS} = \{V, \emptyset, \{v_2, v_3\}, \{v_1, v_3\}, \{v_1, v_2\}, \{v_1\}, \{v_2\}, \{v_3\}\}$.

Note. For any (V, τ_{CG}) graphic intuitionistic Alexandroff topological space, for each vertex v the intersection of all open sets containing v is the smallest open set containing v and denoted by $U_{GIS}(v)$ and the familiy, $MB_{GIS} = \{U_{CG}(v) \mid x \in V\}$ is the minimal basis for the graphic intuitionistic Alexandroff topological space (V, τ_{GIS}).

Theorem 3.4. *Suppose that* $G = (V, E)$ *is a simple graph* G. *Then* G *admits the graphic intuitionistic Alexandroff topological space*.

Proof. We have to prove that the condition arbitrary intersection of members of the union set S_{GIS} is open. Suppose for any vertex $u \in (\bigcap_{y \in S} A_y)$ for some subset $S \in V$, which implies that $u \in A_y$ for each

$y \in S$. Hence $y \in A_u$ for each $y \in S$ and so $S \subseteq A_u$. Since G is locally finite therefore A_u and so S are finite sets. Thus $(\bigcap_{y \in S} A_y)$ is the intersection of finitely many open sets and hence is open. □

Theorem 3.5. *If $G = (V, E)$ is graphic intuitionistic Alexandroff topological space, then we have $U_{GIS}(v) = \bigcap_{B_i \in S_{GIS}} B_i$ for any vertex such that $v \in B_i$ for all $i \geq 1$ and so $U_{GIS}(v)$ is finite for every $v \in V$.*

Proof. Since $U_{GIS}(v)$ is the intersection of all open sets containing v, and S_{GIS} is the subbasis of τ_{GIS}, so we have $U_{GIS}(v) = \bigcap_{B_i \in S_{GIS}} B_i$ such that $i \geq 1$. This implies that $v \in B_i$ for all $i \geq 1$. Then by the definition of $U_{GIS}(v)$ the proof is complete. □

Theorem 3.6. *Let $G = (V, E)$ be a simple graph admits graphic intuitionistic Alexandroff topological space (V, τ_{GIS}) then for every vertex v in V, $U_{GIS}(v) \subseteq B_i$ and so $\overline{U_{GIS}(v)} \subseteq \overline{B_i}$ for all i such that $v \in B_i$ for all i.*

Proof. Since $U_{GIS}(v) = \bigcap_{B_i \in S_{GIS}} B_i$ such that $v \in B_i$ for all $i \geq 1$. Which implies that $U_{GIS}(v) \subseteq B_i$ for all i. Now the remaining part of the proof is $\overline{U_{GIS}(v)} \subseteq \overline{B_i}$ for all i. Let any vertex $u \in \overline{U_{GIS}(v)}$ this implies that $U_{GIS} \cap U_{GIS}(v) \neq \emptyset$ for all open set U_{GSI} containing u. From first part we have $U_{GIS}(v) \subseteq B_i$. Therefore $U_{GIS}(v) \cap B_i \neq \emptyset$ for all open set U_{GIS} containing u. Hence $u \in \overline{B_i}$ and so we have $\overline{U_{GIS}(v)} \subseteq \overline{B_i}$ for all i. □

Corollary 3.7. *Let $G = (V, E)$ be a simple graph admits graphic intuitionistic Alexandroff topological space (V, τ_{GIS}). Then for every vertex v in V, $\overline{\{v\}} \subseteq \overline{U_{GIS}(v)} \subseteq \overline{B_i}$ for all i such that $v \in B_i$ for all i.*

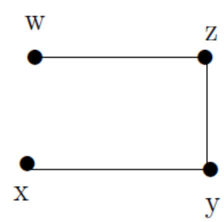
Proof. Let $s \in \overline{\{v\}}$ this implies that $U_{GIS} \cap \{v\} \neq \emptyset$ for every open set U_{GIS} containing s. Since $\{v\} \subseteq U_{GIS}(v)$, thus we have $U_{GIS} \cap U_{GIS}(v) \neq \emptyset$ for every open set containing s. Hence $s \in \overline{U_{GIS}(v)}$, so we have $\overline{\{v\}}$

$\subseteq \overline{U_{GIS}(v)}$. Since $\overline{U_{GIS}(v)} \subseteq \overline{B_i}$ for all i such that $v \in B_i$ for all i, so $\overline{\{v\}} \subseteq \overline{U_{GIS}(v)} \subseteq \overline{B_i}$ for all i. □

Theorem 3.8. *Let G be a graphic intuitionistic Alexandroff topological space (V, τ_{GIS}) and $U_{GIS}(u)$ be the intersection of all open sets containing u and smallest open set, for each u in V. Then $\overline{U_{GIS}(v)} \neq V$, in particulr for each u, $\overline{\{u\}} \neq V$.*

Proof. Let G be a simple graph without isolated vertex. Then from the definition graphic intuitionistic Alexandroff topological space (V, τ_{GIS}), we have $\tau_{GIS} = \tau$ on V. Let us take $u \in V$. Since G has no isolated vertex, thus we have $A_u \neq \emptyset$ and $A_u^c \neq V$. Since $\overline{\{u\}} \subseteq \overline{U_{GIS}(u)} \subseteq A_u^c$ and $\overline{A_u} \subseteq U_{GIS}(x)$, which implies that $\overline{U_{GIS}(u)} \neq V$. □

Example 3.9. Let $G = (V, E)$ be a simple graph $V = \{w, x, y, z\}$ and $E = \{e_1, e_2, e_3\}$.



Here $A_x = \langle V, \{y\}, \{w, z\}\rangle$, $A_y = \langle V, \{x, z\}, \{w\}\rangle$, $A_z = \langle V, \{w, y\}, \{x\}\rangle$,

$A_w = \langle V, \{z\}, \{x, y\}\rangle$

$S_{GIS} = \{V, \emptyset, \{y\}, \{w, z\}, \{x, z\}, \{w\}, \{w, y\}, \{x\}, \{z\}, \{x, y\}\}$.

$\tau_{GIS} = \{V, \emptyset, \{x, y\}, \{w, y\}, \{w, z\}, \{x, z\}, \{x\}, \{y\}, \{z\}, \{w\}, \{y, z\}, \{x, w\},$

$\{x, y, z\}, \{x, y, w\}, \{w, y, z\}, \{w, x, z\}\}$ and

$\tau_{GIS}^c = \{V, \emptyset, \{z, w\}, \{x, z\}, \{x, y\}, \{y, w\}, \{w, y, z\}, \{w, x, z\}, \{x, y, w\},$

$\{x, y, z\}, \{x\}, \{y\}, \{z\}, \{w\}, \{y, z\}, \{x, w\}\}$.

Thus we have,

$$U_{GIS}(x) = \{x\} \Rightarrow \overline{U_{GIS}(x)} = \{x\}$$

$$U_{GIS}(y) = \{y\} \Rightarrow \overline{U_{GIS}(y)} = \{y\}$$

$$U_{GIS}(z) = \{z\} \Rightarrow \overline{U_{GIS}(z)} = \{z\}$$

$$U_{GIS}(w) = \{w\} \Rightarrow \overline{U_{GIS}(w)} = \{w\}$$

Hence, $\overline{\{w\}} = \{w\}$

$$\overline{\{x\}} = \{x\}$$

$$\overline{\{y\}} = \{y\}$$

$$\overline{\{z\}} = \{z\}$$

Hence G satisfies the condition.

References

[1] P. Alexandroff, Diskrete Raume, Mat. Sb. (N.S.) 2 (1937), 501-518.

[2] Dr. S. J. Aravindan, D. Sasikala and A. Divya, Visualization of cordial graph in human excretion track, Italian Journal of Pure and Applied Mathematics 44 (2020), 460-469.

[3] F. G. Arenas, Alexandroff Spaces, Acta Math. Univ. Comenian. 68(1) (1999), 17-25.

[4] N. Bhuvaneswari and L. Vidyarani, On intuitionistic supra generalized α-closed sets in intuitionistic supra topological spaces, International Journal of Scientific Research and Review 7(7) (2018), 388-395.

[5] Dogan Coker, An introduction to intuitionistic topological spaces, Busefal 81 (2000), 51-56.

[6] J. Dugundji, Topology, Allyn and Bacon Inc., Boston, (1966).

[7] S. M. Jafarian Amiri, A. Jafarzadeh and H. Khatibzadeh, An Alexandroff topology on graphs, Bull. Iranian Math. Soc. 39 (2013), 647-662.

[8] Khalid Abdulkalek Abdu and Adem Kilicman, Bitopological spaces on undirected graphs, J. Math. Computer Sci. 18 (2018), 232-241.

[9] Khalid Abdulkalek Abdu and Adem Kilicman, Topologies on the edges of directed graphs, International Journal of Mathematical Analysis 12(2) (2018), 71-84.

[10] A. Kilicman and K. Abdulkalek, Topological spaces associated with simple graphs, Journal of Mathematical Analysis 9 (2018), 44-52.

[11] D. Sasikala and A. Divya, An Alexandroff topological space on vertex set of sum cordial graphs, Journal of Advances Research in Dynamical and control systems 11(2) (2019), 1551-1555.

[12] D. Sasikala and A. Divya, An Alexandroff bitopological space on undirected graphs, International Journal of Scientific Research and Reviews 8(2) (2019), 1-9.

[13] D. Sasikala and A. Divya, Behavior open sets in bi Alexandroff topological space, Malaya Journal of Matematik 8(1) (2020), 48-53.

[14] D. Sasikala, A. Divya and S. Jafari, The role of interior and closure operator in medical application, TWMS Journal of Applied and Engineering Mathematics, (Yet to publish).