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THE UPPER BOUNDS OF ANNIHILATOR DOMINATION NUMBER OF ARITHMETIC GRAPHS $m = p_1^{a_1} \cdot p_2^{a_2} \cdot p_3^{a_3}$ AND $m = p_1^{a_1} \cdot p_2^{a_2} \cdot p_3^{a_3} \cdot p_4^{a_4}$

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Abstract

This paper focuses on the concept of Annihilator domination in Arithmetic Graphs. Kulli and Janakiram [7] introduced the split domination in Graphs and Suryanarayana Rao and Vangipuram [10] was introduced the Annihilator domination in graphs and obtained several interesting results in Standard graphs, Product Graphs and Arithmetic graphs. In this paper we have explored some interesting results on Annihilator domination in Arithmetic graphs.

1. Introduction

Many real-world situations can conveniently be described by means of a Diagram consisting of a set of points together with lines joining certain pairs of these points. For example, the points could represent people, with lines

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joining pairs of friends; or the points might be communication centres, with lines representing communication links. Notice that in such diagrams one is mainly interested in whether or not two given points are joined by a line; the manner in which they are joined is immaterial. A mathematical abstraction of situations of this type gives rise to the concept of a graph. Graph theory has a wide range of applications to many fields. Graph theory is one of the ever growing branch of Mathematics. Graph theory is intimately related to many branches of Mathematics, including group theory, Probability, numerical analysis, matrix theory, topology, operation research and many more. The concept of domination in graphs originated in 1850 with problem of placing minimum number of queens on a $n \times n$ chess board so as to cover or dominate every square. The problem of dominating the squares of chess board can be stated more generally as a problem of dominating the vertices of a graph. The theory of domination has been the nucleus of research activity in graph theory in recent times. This is largely due to a multiplicity of new parameters that can be developed from the basic definition of domination. In 1977 Cockayne and Hedetniemi [3] made an interesting and extensive survey of the results know at that time about dominating sets in graphs. They have used the notation (G) for the domination number of a graph, which has become very popular since then. The survey paper of Cockayane and Hedetniemi [3] has generated lot of interest in the study of domination in graphs. In a span of about twenty years after the survey, more than 1,200 research papers have been published on this topic, and the number of papers continued to be on the increase. Recent book on domination, has stimulated sufficient in spiration leading to the expansive growth of this field of study. Laskar and Walikar [4] developed various interesting results on domination related concepts in graph theory. The split domination in graphs was introduced by Kulli and Janakiram [7]. Domination parameters of an Arithmetic graph are introduced by Vasumathi and Vangipuram [6] and Vijayasaradhi and Vangipuram [9] and obtained an elegant method for the construction of a Arithmetic graph with the given domination parameter. Suryanarayana Rao and Sreenivasan [11, 12] obtained domination parameters of an arithmetic Graph and some product graphs and also they have obtained an elegant method for the construction of a arithmetic graph with the given domination parameter and investigated Annihilator domination number of Arithmetic graphs. Motivated by the study of domination and split domination, and new parameter on domination called the Annihilator dominating set and Annihilator dominating number we have investigated some more properties of the Annihilator domination number of Arithmetic graphs. The terminology and notations used in this paper are same as in Bondy and Murty [1].

Dominating set:

A subset D of V is said to be a dominating set of G if every vertex in $V \setminus D$ is adjacent to a vertex in D.

The dominating number $\gamma(G)$ of G is the minimum cardinality of a dominating set.

Split dominating set:

A dominating set D of graph G is called a split dominating set, if the induced subgraph $\langle V - D \rangle$ is disconnected.

The split dominating number $\gamma_s(G)$ of G is the minimum cardinality of the split dominating set.

Suryanarayana Rao and V. Sreeenivasan [4] obtained a new concept on Domination and t named it as Annihilator Domination and defined as follows.

Annihilator dominating set:

A dominating set D of graph G is said to be annihilator dominating set, if its induced subgraph $\langle V - D \rangle$ is a graph with isolated vertices or a graph with independent vertices.

Annihilator domination number:

The annihilator domination number $\gamma_a(G)$ of G is the minimum cardinality of an annihilator dominating set.

Arithmetic graph:

The Arithmetic graph V_m is defined as a graph with its vertex set as the set of all divisors of m (excluding 1), where m is a natural number and $m = p_1^{a_1}\ p_2^{a_2}, \ldots, p_r^{a_r}$, a canonical representation of m, where p_i's are distinct primes and a_i's≥ 1 and two distinct vertices a, b which are not of the same parity are adjacent in this graph if $(a, b) = p_i$, for $1 \leq i \leq r$.

The vertices a and b are said to be of the same parity if both a and b are the powers of the same prime, for instance $a = p^2$, $b = p^5$.

Split Domination-Arithmetic Graphs:

In most of the research in graph theory, the investigators are content with establishing the existence of a graph with a given graphical parameter. It is in this context the usage of elementary number theoretic principles will help in the constructions of such graphs. It is amazing to observe how such a graph with a given domination number can be enlarged to include more vertices and edges in a methodical, simple manner without affecting the domination number.

A similar method of construction using again elementary principles of number theory helped in the construction of a graph with the given annihilator domination number by Suryanarayana Rao and Vangipuram [11]. They developed a method of construction of graph with a given number as the split domination number of the graph. For this purpose we make use of an arithmetic graph V_m with its vertex set as the set of all devisors of m (except 1) and defining the adjancey property of the arithmetic graph suitably.

The split domination of these arithmetic graphs have been studied as it enables us to construct graphs with a given split domination number in a very simple way. They [11] obtained that the upper bound of split domination number of the V_m graph is $r+1$, where m is a positive integer and $m = p_1 a_1 \; p_2 a_2, \ldots, p_r a_r$ is the canonical representation $p_1 \; p_2, \ldots, p_r$ are distinct primes and a_i's>1.

2. Annihilator Domination-Arithmetic Graphs

Suryanarayana Rao and Vangipuram [11] obtained an interesting result on annihilator domination of the Arithmetic Graph which is given below and also they evolved a method of construction of the graph with the given annihilator domination number. The following are the two results on Arithmetic graphs.

Theorem 2.1. *If* $m = p_{11}^{a} \cdot p_{22}^{a}$, *where* p_1, p_2 *are distinct primes and* a_1, a_2 *are both* ≥ 1, *then*

(i) $\gamma_a[V_m] \leq 2a_i + 1$, *if* $a_i < a_2$

(ii) $\gamma_a[V_m] \leq 2a_1$, *if* $a_1 = a_2$.

Theorem 2.2. *If* $m = p_1^{a_1} \cdot p_2^{a_2} \cdot p_3^{a_3}$ *where* p_1, p_2 *are distinct primes and* a_1, a_2, a_3 *are* ≥ 1, *then* $\gamma_a[V_m] \leq 3a_1$, *if* $a_1 - a_2 = a_3$.

It is already established in the M.Phil. thesis of Susmitha [ ].

Now we have obtained the following interesting results related to Annihilator domination of an Arithmetic graph as mentioned below.

Theorem 2.3. *If* $m = p_1^{a_1} \cdot p_2^{a_2} \cdot p_3^{a_3}$ *where* p_1, p_2, p_3 *are distinct primes and* $a_1, a_2, a_3 \geq 1$ *then* $\gamma_a[V_m] \leq 3a_1 + 3$ *if* $a_1 < a_2 < a_3$.

Proof. Given $m = p_1^{a_1} \cdot p_2^{a_2} \cdot p_3^{a_3}$. The vertex set V_m is $\{p_1, p_1^2, \ldots, p_1^{a_1}$, $p_2, p_2^2, p_2^3, \ldots, p_2^{a_2}, p_3, p_3^2, p_3^3 \ldots p_3^{a_3}, p_1 p_2, p_1 p_2^2, p_1 p_2^3, \ldots, p_1 p_2^{a_2}, p_1^2 p_2, p_1^2 p_2^2$, $\ldots, p_1^2 p_2^{a_2}, p_1 p_3, p_1 p_3^2, p_1 p_3^3, \ldots, p_1 p_3^{a_3}, p_1^2 p_3, p_1^2 p_3^2, \ldots, p_1^2 p_3^{a_3}, p_2 p_3, p_2 p_3^2$, $p_2 p_3^3, \ldots, p_2 p_3^{a_3}, p_2^2 p_3, p_2^2 p_3^2, \ldots, p_2^2 p_3^{a_3}, p_2^3 p_3, p_2^3 p_3^2, \ldots, p_2^3 p_3^{a_3}, p_1 p_2, p_3, p_1$ $p_2 p_3^2, \ldots, p_1 p_2 p_3^{a_3}, p_1 p_2^2 p_3, p_1 p_2^2 p_3^2, \ldots, p_1 p_2^2 p_3^{a_3}, p_1 p_2^3 p_3, p_1, p_2^3 p_3^2, \ldots, p_1 p_2^3 p_3^{a_3}$, $p_1^2 p_2 p_3, p_1^2 p_2 p_3^2, \ldots, p_1^2 p_2 p_3^{a_3}, p_1^2 p_2^2 p_3, p_1^2 p_2^2 p_3^2, \ldots, p_1^2 p_2^2, p_3^{a_3}, p_1^2 p_2^3 p_3, p_1^2 p_2^3$ $p_3^2, \ldots, p_1^2 p_2^3 p_3^{a_3}\}$.

We get $(a_1 + 1)(a_2 + 1)(a_3 + 1) - 1$ vertices.

The set of vertices $D = \{p_1, p_1^2, \ldots, o_1^{a_1}, p_2, p_2^2, {}_2^3, \ldots, p_2^{a_2}, p_3, p_3^2, p_3^3, \ldots, p_3^{a_3}\}$ is an annihilator dominating set.

For if V is any vertex in $V - D$, then V is of the form

$p_1^{i_1}p_2^{i_2}$, $p_1^{i_1}p_3^{i_3}$, $p_2^{i_2}p_3^{i_3}$ and $p_1^{i_1}p_2^{i_2}p_3^{i_3}$ where $1 < i_1 < a_1, 1 < i_2 < a_2$ and $1 < i_3 < a_3$.

We observe that

(i) if $i_1 > 1$ (then for all i_2) the vertex set $V = p_1^{i_1}p_2^{i_2}$ is adjacent to p_1 and p_2 in D and if $i_1 = 0$ (then for all i_2) the vertex set V is adjacent to p_2 in D.

(ii) if $i_1 > 1$ (then for all i_3) the vertex set $V = p_1^{i_1}p_3^{i_3}$ is adjacent to p_1 and p_3 in D and if $i_1 = 0$ (then for all i_3) the vertex set V is adjacent to p_3 in D.

(iii) if $i_2 > 1$ (then for all i_3) the vertex set $V = p_2^{i_2}p_3^{i_3}$ is adjacent to p_2 and p_3 in D and if $i_2 = 0$ (then for all i_3) the vertex set V is adjacent to p_3 in D.

(iv) if $i_1 > 1$ (then for all i_2 and i_3) the vertex set $V = p_1^{i_1}p_2^{i_2}p_3^{i_3}$ is adjacent to p_1, p_2, p_3 in D and if $i_1 = 0$ (then for all i_2 and i_3) the vertex set V is adjacent to p_2 and p_3 in D.

(v) if $i_1 = 1$ (or) $i_2 = 1$ (or) $i_3 = 1$ respectively.

The corresponding vertex V is adjacent to p_1 and $p_1^{i_1}$ (for $1 < i_1 < a_1$); p_2 and $p_2^{i_2}$ (for $1 < i_2 < a_2$); p_3 and $p_3^{i_3}$ (for $1 < i_3 < a_3$); respectively.

In all the above cases it is proved that the vertices in $V - D$ are having adjacency with at least one vertex in D. Thus D is a dominating set.

In $V - D$ any vertices will be of the form $p_1^{d_1}p_2^{d_2}$, $p_1^{e_1}p_2^{e_2}$ and $p_1^{f_1}p_3^{f_2}$, $p_1^{g_1}p_3^{g_2}$ and $p_2^{k_1}p_3^{k_2}$, $p_2^{l_1}p_3^{l_2}$ and $p_1^{m_1}p_2^{m_2}p_3^{m_3}$, $p_1^{n_1}p_2^{n_2}p_3^{n_3}$. Now we have to verify the following cases.

Case (i). If $d_1, e_1, f_1, g_1, k_1, l_1, m_1, n_1 > 1$ then since $d_2, e_2, f_2, g_2, k_2, l_2, m_2, n_2 > 1$.

We have

(1) $(p_1^{d_1}p_2^{d_2},\ p_1^{e_1}p_2^{e_1}) = p_1^{b_1}p_2^{b_2}$ where $b_1, b_2 > 1$ and hence the vertices $p_1^{d_1}p_2^{d_2},\ p_1^{e_1}p_2^{e_2}$ of v_m are not adjacent in this case.

(2) $(p_1^{f_1}p_2^{f_2},\ p_1^{g_1}p_2^{g_1}) = p_1^{c_1}p_3^{c_2}$ where $c_1, c_2 > 1$ and hence the vertices $p_1^{f_1}p_3^{f_2},\ p_1^{g_1}p_3^{g_1}$ of v_m are not adjacent in this case.

(3) $(p_2^{k_1}p_3^{k_2},\ p_1^{l_1}p_3^{l_2}) = p_2^{h_1}p_3^{h_2}$ where $h_1, h_2 > 1$ and hence the vertices $p_2^{k_1}p_3^{k_2},\ p_2^{l_1}p_3^{l_2}$ of v_m are not adjacent in this case.

(4) $(p_1^{m_1}p_2^{m_2}p_3^{m_3},\ p_1^{n_1}p_2^{n_2}p_3^{n_3}) = p_1^{j_1}p_2^{j_2}p_3^{j_3}$ where $j_1, j_2, j_3 > 1$ and hence the vertices $p_1^{m_1}p_2^{m_2}p_3^{m_3},\ p_1^{n_1}p_2^{n_2}p_3^{n_3}$ of v_m are not adjacent in this case.

Case (ii). If $d_1 = 0, f_1 = 0, k_1 = 0, m_1 = 0$ and $e_1 > 0,\ g_1 > 0,\ l_1 > 0,\ n_1 > 0$ then since $d_2, e_2, f_2, g_2, k_2, l_2, m_2, n_2$ are > 1.

We have

(1) $(p_1^{d_1}p_2^{d_2},\ p_1^{e_1}p_2^{e_2}) = (p_2^{d_2},\ p_1^{e_1}p_2^{e_2}) = p_2^{b_2'}$ where $b_2' > 1$ and hence the vertices $p_1^{d_1}p_2^{d_2},\ p_1^{e_1}p_2^{e_2}$ of v_m are not adjacent in this case.

(2) $(p_1^{f_1}p_3^{f_2},\ p_1^{g_1}p_3^{g_2}) = (p_3^{f_2},\ p_1^{g_1}p_3^{g_2}) = p_3^{c_2'}$ where $c_2' > 1$ and hence the vertices $p_1^{f_1}p_3^{f_2},\ p_1^{g_1}p_3^{g_2}$ of v_m are not adjacent in this case.

(3) $(p_2^{k_1}p_3^{k_2},\ p_1^{l_1}p_3^{l_2}) = (p_3^{k_2},\ p_2^{l_1}p_3^{l_2}) = p_3^{h_2'}$ where $h_2' > 1$ and hence the vertices $p_2^{k_1}p_3^{k_2},\ p_2^{l_1}p_3^{l_2}$ of v_m are not adjacent in this case.

(4) $(p_1^{m_1}p_2^{m_2}p_3^{m_3},\ p_1^{n_1}p_2^{n_2}p_3^{n_3}) = (p_2^{m_2}p_3^{m_3},\ p_1^{n_1}p_2^{n_2}p_3^{n_3}) = p_2^{j_2'},\ p_3^{j_3'}$ where $j_2', j_3' > 1$ and hence the vertices $p_1^{m_1}p_2^{m_2}p_3^{m_3},\ p_1^{n_1}p_2^{n_2}p_3^{n_3}$ of v_m are not adjacent in this case.

Case (iii). If $d_1 > 0, f_1 > 0, k_1 > 0, m_1 > 0$ and $e_1 = 0, g_1 = 0, l_1 = 0, n_1 = 0$ then since $d_2, e_2, f_2, g_2, k_2, l_2, m_2, n_2$ are > 1.

We have

(1) $(p_1^{d_1} p_2^{d_2},\ p_1^{e_1} p_2^{e_2}) = (p_1^{d_1} p_2^{d_2},\ p_2^{e_2}) = p_2^{b_2''}$ where $b_2'' > 1$ and hence the vertices $p_1^{d_1} p_2^{d_2},\ p_1^{e_1} p_1^{e_2}$ of v_m are not adjacent in this case.

(2) $(p_1^{f_1} p_3^{f_2},\ p_1^{g_1} p_3^{g_2}) = (p_1^{f_1} p_3^{f_2},\ p_3^{g_2}) = p_3^{c_2''}$ where $c_2'' > 1$. Hence by the definition of arithmetic graph, the vertices $p_1^{f_1} p_3^{f_2},\ p_1^{g_1} p_3^{g_2}$ are not adjacent in this case.

(3) $(p_2^{k_1} p_3^{k_2},\ p_1^{l_1} p_3^{l_2}) = (p_2^{k_1} p_3^{k_2},\ p_3^{l_2}) = p_3^{h_2''}$ where $h_2'' > 1$ and hence by the definition of arithmetic graph the vertices $p_2^{k_1} p_3^{k_2},\ p_2^{l_1} p_3^{l_2}$ are not adjacent in this case.

(4) $(p_1^{m_1} p_2^{m_2} p_3^{m_3},\ p_1^{n_1} p_2^{n_2} p_3^{n_3}) = (p_1^{m_1} p_2^{m_2} p_3^{m_3},\ p_2^{n_2} p_3^{n_3}) = p_2^{j_2''},\ p_3^{j_3''}$ where $j_2'',\ j_3'' > 1$. Hence by the definition of v_m the vertices $p_1^{m_1} p_2^{m_2} p_3^{m_3}$, $p_1^{n_1} p_2^{n_2} p_3^{n_3}$ are not adjacent in this case.

Case (iv). If $d_1 = 0, e_1 = 0, f_1 = 0, g_1 = 0, k_1 = 0, l_1 = 0, m_1 = 0, n_1 = 0$ then since $d_2,\ e_2,\ f_2,\ g_2,\ k_2,\ l_2,\ m_2,\ n_2$ are > 1.

We have the vertices

(1) $(p_1^{d_1} p_2^{d_2},\ p_1^{e_1} p_2^{e_2}) = (p_2^{d_2},\ p_2^{e_2}) = p_2^{b_2'''}$ where $b_2''' > 1$ and hence the vertices $p_1^{d_1} p_2^{d_2},\ p_1^{e_1} p_2^{e_2}$ of v_m are not adjacent in this case.

(2) $(p_1^{f_1} p_3^{f_2},\ p_1^{g_1} p_3^{g_2}) = (p_3^{f_2},\ p_3^{g_2}) = p_3^{c_2'''}$ where $c_2''' > 1$. Hence by the definition of arithmetic graph, the vertices $p_1^{f_1} p_3^{f_2},\ p_1^{g_1} p_3^{g_2}$ are not adjacent in this case.

(3) $(p_2^{k_1} p_3^{k_2},\ p_1^{l_1} p_3^{l_2}) = (p_3^{k_2},\ p_1^{l_2}) = p_3^{h_2'''}$ where $h_2''' > 1$ and hence by the definition of arithmetic graph the vertices $p_2^{k_1} p_3^{k_2},\ p_1^{l_1} p_3^{l_2}$ are not adjacent in this case.

(4) $(p_1^{m_1} p_2^{m_2} p_3^{m_3},\ p_1^{n_1} p_2^{n_2} p_3^{n_3}) = (p_2^{m_2} p_3^{m_3},\ p_2^{n_2} p_3^{n_3}) = p_2^{j_2''''},\ p_3^{j_3''''}$ where $j_2'''', j_3'''' > 1$. Hence by the definition of v_m the vertices $p_1^{m_1} p_2^{m_2} p_3^{m_3},\ p_1^{n_1} p_2^{n_2} p_3^{n_3}$ are not adjacent in this case.

Then from all the above cases the vertices $p_1^{d_1} p_2^{d_2},\ p_1^{e_1} p_2^{e_2}$; $p_1^{f_1} p_3^{f_2},\ p_1^{g_1} p_3^{g_2};\ p_2^{k_1} p_3^{k_2},\ p_2^{l_1} p_3^{l_2};\ p_1^{m_1} p_2^{m_2} p_3^{m_3},\ p_1^{n_1} p_2^{n_2} p_3^{n_3}$ are not adjacent in the induced subgraph $\langle V - D \rangle$ so that D is an annihilator dominating set of V_m. D is also minimal annihilator dominating set.

If we remove any vertex V in D then $D - \{V\}$ is not annihilator dominating set.

If V is of the form $p_1^{i_1}$, where $1 \leq i_1 \leq a_1$;

(or) $p_2^{i_2}$, where $1 \leq i_2 \leq a_2$;

(or) $p_3^{i_3}$, where $1 \leq i_3 \leq a_3$;

(i) Then the vertex $p_1^{i_1} (1 \leq i_1 \leq a_1)$ is having adjacency with all the vertices $p_1 p_2^{b_2};\ p_1 p_3^{b_3};\ p_1 p_2^{b_2} p_3^{b_3};$ where $0 \leq b_2 \leq a_2$ and $0 \leq b_3 \leq a_3$ in the induced subgraph of $\langle V - \{D - \{V\}\} \rangle$.

(ii) Then the vertex $p_2^{i_2} (1 \leq i_2 \leq a_2)$ is having adjacency with all the vertices $p_1^{b_1} p_2,\ p_2 p_3^{b_3},\ p_1^{b_1} p_2 p_3^{b_3}$ where $0 \leq b_1 \leq a_1,\ 0 \leq b_3 \leq a_3$ in the induced subgraph of $\langle V - \{D - \{V\}\} \rangle$.

(iii) Then the vertex $p_3^{i_3} (1 \leq i_3 \leq a_3)$ is having adjacency with all the vertices $p_1^{b_1} p_3,\ p_2^{b_2} p_3,\ p_1^{b_1} p_2^{b_2} p_3$ where $0 \leq b_1 \leq a_1,\ 0 \leq b_2 \leq a_2$ in the induced subgraph of $\langle V - \{D - \{V\}\} \rangle$.

Then $D - \{V\}$ is not an annihilator dominating set.

Hence D is a minimal annihilator dominating set. Then $\gamma_a[V_m] \leq |D| = 3a_1 + 3$.

Construction of a Graph whose Annihilator Domination number Does not Exceed a given number *n*

The construction of a graph whose cardinality of an annihilator dominating set is known. For this construction we have the following two cases.

If n is any number, which is the sum of a_1, a_2, a_3 where $a_1 < a_2 < a_3$; we have $p_1^{\frac{n-3}{3}} \cdot p_2^{a_2} \cdot p_3^{a_3}$; where p_1, p_2, p_3 are distinct primes and $\frac{n-3}{3} < a_2 < a_3$. Then $D = \{p_1, p_1^2, \ldots, p_1^{\frac{n-3}{3}}; p_2, p_2^2, \ldots, p_2^{a_2}; p_3, p_3^2, \ldots, p_3^{a_3}\}$ is an annihilator dominating set.

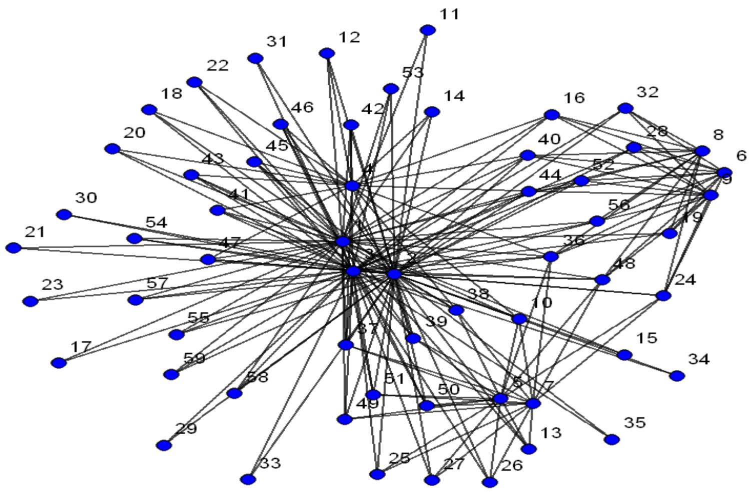
Illustration:

Given $n = 9$(odd); choose any three primes p_1, p_2, p_3 and let $V_m\, m = p_1^2 \cdot p_2^3 \cdot p_3^4$ (with $a_1 = 2$, $a_2 = 3$, $a_3 = 4$ such that $a_1 < a_2 < a_3$).

The vertices of V_m are the divisors of m (except 1). The numbers are given for every vertex from 1 to 59.

$\{1-p_1, 2-p_2, 3-p_3, 4-p_1^2, 5-p_2^2, 6-p_3^2, 7-p_2^3, 8-p_3^3, 9-p_3^4, 10-p_1p_2, 11-p_1^2 p_2^2, 12-p_1p_2^2, 13-p_1^2p_2, 14-p_1p_2^3, 15-p_1^2p_2^3, 16-p_1p_3, 17-p_1^2p_3^2, 18-p_1p_3^2, 19-p_1^2p_3, 20-p_1p_3^4, 21-p_1^2p_3^3, 22-p_1p_3^3, 23-p_1^2p_3^4, 24-p_2p_3, 25-p_2p_3^2, 26-p_2p_3^3, 27-p_2 p_3^4, 28-p_2^2p_3, 29-p_2^2p_3^2, 30-p_2^2p_3^3, 31-p_2^2p_3^4, 32-p_2^3p_3, 33-p_2^3p_3^2, 34-p_2^3p_3^3, 35-p_2^3p_3^4, 36-p_1p_2p_3, 37-p_1p_2p_3^2, 38-p_1p_2p_3^3, 39-p_1p_2p_3^4, 40-p_1p_2^2p_3, 41 - p_1p_2^2p_3^2, 42-p_1p_2^2p_3^3, 43-p_1p_2^2p_3^4, 44-p_1p_2^3p_3, 45-p_1p_2^3p_3^2, 46-p_1p_2^3p_3^3, 47-p_1p_2^3p_3^4, 48-p_1^2p_2p_3, 49-p_1^2p_2p_3^2, 50-p_1^2p_2p_3^3, 51-p_1^2p_2p_3^4, 52-p_1^2p_2^2p_3, 53-p_1^2p_2^2p_3^2, 54-p_1^2p_2^2p_3^3, 55-p_1^2p_2^2p_3^4, 56-p_1^2p_2^3p_3, 57-p_1^2p_2^3, 58-p_1^2p_2^3p_3^2, 59-p_1^2p_2^3p_3^4\}$.

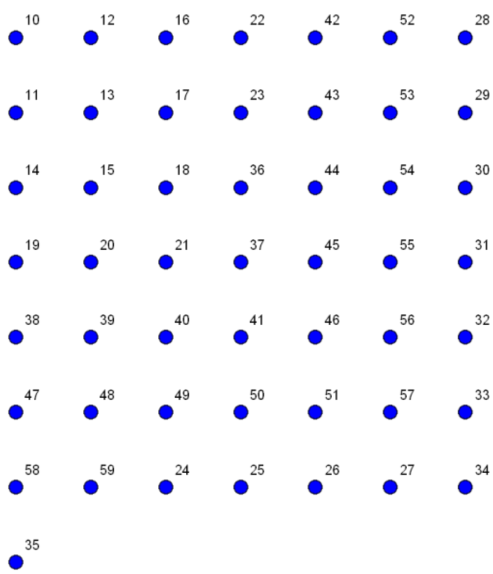
There are 60 – 1 = 59 vertices.



The graph of V_m with $m = p_1^2 \cdot p_2^3 \cdot p_3^4$.

The Annihilator Dominating set

$D = \{1 - p_1,\ 2 - p_2,\ 3 - p_3,\ 4 - p_1^2,\ 5 - p_2^2,\ 6 - p_3^2,\ 7 - p_2^3,\ 8 - p_3^3,\ 9 - p_3^4\}$



The induced sub graph $\langle V - D \rangle$.

Hence $\gamma_a[V_m] \leq |D| = 3a_1 + 3 = 3 \times 2 + 3 = 9.$

Theorem 2.4. *If* $m = p_1^{a_1} \cdot p_2^{a_2} \cdot p_3^{a_3} \cdot p_4^{a_4}$ *where* p_1, p_2, p_3, p_4 *are distinct primes and* $a_1, a_2, a_3, a_4 \geq 1$ *then* $\gamma[v_m] \leq 4a_1$ *if* $a_1 = a_2 = a_3 = a_4$.

Proof. Given $m = p_1^{a_1} \cdot p_2^{a_2} \cdot p_3^{a_3} \cdot p_4^{a_4}$ where p_1, p_2, p_3, p_4 are distinct primes and $a_1, a_2, a_3, a_4 \geq 1$.

Now we have to prove that $\gamma[v_m] \leq 4a_1$ if $a_1 = a_2 = a_3 = a_4$.

The Vertex set of v_m is $\{p_1, p_1^2, \ldots, p_1^{a_1}; p_2, p_2^2, \ldots, p_2^{a_2}; p_3 p_3^2, \ldots, p_3^{a_3};$ $p_4, p_4^2, \ldots, p_4^{a_4} p_1, p_2, p_1p_2^2, \ldots, p_1p_2^{a_2}; p_1^2p_2, p_1^2p_2^2, \ldots, p_1^2p_2^{a_2}; \ldots; p_1^{a_1}p_2,$ $p_1^{a_1}p_2^2, \ldots, p_1^{a_1}p_2^{a_2}; p_1p_3, p_1p_3^2, \ldots, p_1p_3^{a_3}; p_1^2p_3, p_1^2p_3^2, \ldots, p_1^2p_3^{a_3}; \ldots; p_1^{a_1}p_3,$ $p_1^{a_1}p_3^2, \ldots, p_1^{a_1}p_3^{a_3}; p_1p_4, p_1p_4^2, \ldots, p_1p_4^{a_4}; p_1^2p_4, p_1^2p_4^2, \ldots, p_1^2p_4^{a_4}; \ldots; p_1^{a_1}p_4,$ $p_1^{a_1}p_4^2, \ldots, p_1^{a_1}p_4^{a_4}; p_2p_3, p_2p_3^2, \ldots, p_2p_3^{a_3}; p_2^2p_3, p_2^2p_3^2, \ldots, p_2^2p_3^{a_3}; \ldots; p_2^{a_2}$ $p_3, p_2^{a_2}p_3^2, \ldots, p_2^{a_2}p_3^{a_3}; p_2p_4, p_2p_4^2, \ldots, p_2p_4^{a_4}; p_2^2p_4, p_2^2p_4^2, \ldots, p_2^2p_4^{a_4}; \ldots;$ $p_2^{a_2}p_4, p_2^{a_2}p_4^2, \ldots, p_2^{a_2}p_4^{a_4}; p_3p_4, p_3p_4^2, \ldots, p_3p_4^{a_4}; p_3^2p_4, p_3^2p_4^2, \ldots, p_3^2p_4^{a_4};$ $\ldots; p_3^{a_3}p_4, p_3^{a_3}p_4^2, \ldots, p_3^{a_3}p_4^{a_4}; p_1p_2p_3, p_1p_2p_3^2, \ldots, p_1p_2p_3^{a_3}; p_1p_2^2p_3, p_1p_2^2$ $p_3, p_1p_2^2p_3^2, \ldots, p_1p_2^2p_3^2, \ldots, p_1p_2^2p_3^{a_3}; \ldots; p_1p_2^{a_2}p_3, p_1p_2^{a_2}p_3^2, \ldots, p_1^2p_2^{a_2}p_3^{a_3};$ $\ldots; p_1p_2^{a_2}p_3^{a_3}; p_1^2p_2p_3, p_1^2p_2p_3^2, \ldots, p_1^2p_2p_3^{a_3}; p_1^2p_2^2p_3, p_1^2p_2^2p_3^2, \ldots, p_1^2p_2^2p_3^{a_3}; \ldots; p_1^2$ $p_2^{a_2}p_3, p_1^2p_2^{a_2}p_3^2, \ldots, p_1^2p_2^{a_2}p_3^{a_3}; \ldots, p_1^{a_1}p_2p_3, p_1^{a_1}p_2p_3^2, \ldots, p_1^{a_1}p_2p_3^{a_3}; p_1^{a_1}p_2^2p_3 p_1^{a_1}$ $p_2^2p_3^2, \ldots, p_1^{a_1}p_2^2p_3^{a_3}; \ldots; p_1^{a_1}p_2^{a_2}p_3, p_1^{a_1}p_2^{a_2}p_3^2, \ldots, p_1^{a_1}p_2^{a_2}p_3^{a_3} p_1p_2p_4, p_1p_2p_4^2, \ldots, p_1$ $p_2p_4^{a_4}; p_1p_2^2p_4, p_1p_2^2p_4^2, \ldots, p_1p_2^2p_4^{a_4}; \ldots; p_1p_2^{a_2}p_4, p_1p_2^{a_2}p_4^2, \ldots, p_1p_2^{a_2}p_4^{a_4}; p_1^2p_2p_4,$ $p_1^2p_2p_4^2, \ldots, p_1^2p_2p_4^{a_4}; p_1^2p_2^2p_4, p_1^2p_2^2p_4^2, \ldots, p_1^2p_2^2p_4^{a_4}; \ldots; p_1^2p_2^{a_2}p_4, p_1^2p_2^{a_2}p_4^2, \ldots,$ $p_1^2p_2^{a_2}p_4^{a_4}, \ldots, p_1^{a_1}p_2p_4, p_1^{a_1}p_2p_4^2, \ldots, p_1^{a_1}p_2p_4^{a_4}; p_1^{a_1}p_2^2p_4, p_1^{a_1}p_2^2p_4^2, \ldots, p_1^{a_1}p_2^2p_4^{a_4};$ $\ldots; p_1^{a_1}p_2^{a_2}p_4, p_1^2p_2^{a_2}p_4^2, \ldots, p_1^2p_2^{a_2}p_4^{a_4}; p_1p_3p_4, p_1p_3p_4^2, \ldots, p_1p_3p_4^2; p_1p_3^2p_4, p_1p_3^2$ $p_4^2, \ldots, p_1p_3^2p_4^{a_4}; \ldots; p_1p_3^{a_3}p_4, p_1^2p_3^{a_3}p_4^2, \ldots, p_1^2p_2^{a_2}p_4^{a_4} p_1^2p_3p_4, p_1^2p_3p_4^2, \ldots, p_1^2p_3p_4^{a_4};$

$p_1p_3^2p_4^2, p_1^2p_3^2p_4^2, \ldots, p_1^2p_3^{a_3}p_4, p_1^2p_3^{a_3}p_4^2, \ldots, p_1^2p_3^{a_3}p_4^{a_4}, \ldots, p_1^{a_1}p_3p_4, p_1^{a_1}p_3p_4^2, \ldots,$
$p_1^{a_1}p_3p_4^{a_4}; p_1^{a_1}p_3^2p_4, p_1^{a_1}p_3^2p_4^2, \ldots, p_1^{a_1}p_3^2p_4^{a_4}; \ldots; p_1^{a_1}p_3^{a_3}p_4, p_1^{a_1}p_3^{a_3}p_4^2, \ldots, p_1^{a_1}p_3^{a_3}p_4^{a_4};$
$p_2p_3p_4, p_2p_3p_4^2, \ldots, p_2p_3p_4^{a_4}; p_2p_3^2p_4, p_2p_3^2p_4^2, \ldots, p_2p_3^2p_4^{a_4}; \ldots; p_2p_3^{a_3}p_4, p_2^2p_3^{a_3}p_4^2,$
$\ldots, p_2p_3^{a_3}p_4^{a_4}; p_2^2p_3p_4, p_2^2p_3p_4^2, \ldots, p_2^2p_3p_4^{a_4}; p_2^2p_3^2p_4, p_2^2p_3^2p_4^2, \ldots, p_2^2p_3^2p_4^{a_4}; \ldots; p_2^2$
$p_3^{a_3}p_4, p_2^2p_3^{a_3}p_4^2, \ldots, p_2^2p_3^{a_3}p_4^{a_4}; \ldots, p_2^{a_2}p_3p_4, p_2^{a_2}p_3p_4^2, \ldots, p_2^{a_2}p_3p_4^{a_4}; p_2^{a_2}p_3^2p_4, p_2^{a_2}$
$p_3^2p_4^2, \ldots, p_2^{a_2}p_3^2p_4^{a_4}; \ldots; p_2^{a_2}p_3^{a_3}p_4, p_2^{a_2}p_3^{a_3}p_4^2, \ldots, p_2^{a_2}p_3^{a_3}p_4^{a_4}; p_1p_2p_3p_4, p_1p_2p_3p_4^2,$
$\ldots, p_1p_2p_3p_4^{a_4}; p_1p_2^2p_3p_4, p_1p_2^2p_3p_4^2, \ldots, p_1p_2^2p_3p_4^{a_4}; \ldots; p_1p_2^{a_2}p_3p_4, p_1p_2^{a_2}p_3p_4^2, \ldots,$
$p_1p_2^{a_2}p_3p_4^{a_4}; p_1p_2p_3^2p_4, p_1p_2p_3^2p_4^2, \ldots, p_1p_2p_3^2p_4^{a_4}; p_1p_2^2p_3^2p_4, p_1p_2^2p_3^2p_4^2, \ldots, p_1p_2^2p_3^2$
$p_4^{a_4}; \ldots; p_1p_2^{a_2}p_3^2p_4, p_1p_2^{a_2}p_3^2p_4^2, \ldots, p_1p_2^{a_2}p_3^2p_4^{a_4}, \ldots, p_1p_2p_3^{a_3}p_4, p_1p_2p_3^{a_3}p_4^2, \ldots, p_1p_2$
$p_3^{a_3}p_4^{a_4}; p_1p_2^2p_3^{a_3}p_4^2, \ldots, p_1p_2^2p_3^{a_3}p_4^{a_4}; \ldots; p_1p_2^{a_2}p_3^{a_3}p_4, p_1p_2^{a_2}p_3^{a_3}p_4^2, \ldots, p_1p_2^{a_2}p_3^{a_3}p_4^{a_4}; \ldots$
$p_1^{a_1}p_2p_3p_4, p_1^{a_1}p_2p_3p_4^2, \ldots, p_1^{a_1}p_2p_3p_4^{a_4}; p_1^{a_1}p_2^2p_3p_4, p_1^{a_1}p_2^2p_3p_4^2, \ldots, p_1^{a_1}p_2^2p_3p_4^{a_4}; \ldots;$
$p_1^{a_1}p_2^{a_2}p_3p_4, p_1^{a_1}p_2^{a_2}p_3p_4^2, \ldots, p_1^{a_1}p_2^{a_2}p_3p_4^{a_4}; p_1^{a_1}p_2p_3^2p_4, p_1^{a_1}p_2p_3^2p_4^2, \ldots, p_1^{a_1}p_2p_3^2p_4^{a_4};$
$p_1^{a_1}p_2^2p_3^2p_4, p_1^{a_1}p_2^2p_3^2p_4^2, \ldots, p_1^{a_1}p_2^2p_3^2p_4^{a_4}; \ldots; p_1^{a_1}p_2^{a_2}p_3^2p_4, p_1^{a_1}p_2^{a_2}p_3^2p_4^2, \ldots, p_1^{a_1}p_2^{a_2}p_3^2p_4^{a_4}; \ldots$
$p_1^{a_1}p_2p_3^{a_3}p_4, p_1^{a_1}p_2p_3^{a_3}p_4^2, \ldots, p_1^{a_1}p_2p_3^{a_3}p_4^{a_4}; p_1^{a_1}p_2^2p_3^{a_3}p_4, p_1^{a_1}p_2^2p_3^{a_3}p_4^2, \ldots, p_1^{a_1}p_2^2p_3^{a_3}p_4^{a_4};$
$\ldots; p_1^{a_1}p_2^{a_2}p_3^{a_3}p_4, p_1^{a_1}p_2^{a_2}p_3^{a_3}p_4^2, \ldots, p_1^{a_1}p_2^{a_2}p_3^{a_3}p_4^{a_4}\}.$

The number of vertices of v_m is $[(a_1+1)\cdot(a_2+1)\cdot(a_3+1)\cdot(a_4+1)-1]$.

Let us consider the set of vertices $D = \{p_1, p_1^2, \ldots, p_1^{a_1}; p_2, p_2^2, \ldots, p_2^{a_2}; p_3, p_3^2, \ldots, p_3^{a_3}; p_4, p_4^2, \ldots, p_4^{a_4}\}$ is an annihilator dominating set of v_m.

Now we have to prove that D is an annihilator dominating set.

For any vertex in $V - D$ is of the form $p_1^{i_1}p_2^{i_2}, p_1^{i_1}p_2^{i_2}p_3^{i_3}, p_1^{i_1}p_2^{i_2}p_3^{i_3}p_4^{i_4}$ where $1 \le \{i_1, i_2, i_3, i_4\} \le a_1$.

These vertices are adjacent with p_1, p_2, p_3, p_4 in D then D is a dominating set.

Moreover, if u_1, v_1, w_1 are any three vertices in $V - D$ then u_1, v_1, w_1

are of the form

$$u_1 = p_1^{i_1} p_2^{i_2} \text{ (or) } p_1^{i_1} p_2^{i_2} p_3^{i_3} \text{ (or) } p_1^{i_1} p_2^{i_2} p_3^{i_3} p_4^{i_4}$$

$$v_1 = p_1^{j_1} p_2^{j_2} \text{ (or) } p_1^{j_1} p_2^{j_2} p_3^{j_3} \text{ (or) } p_1^{j_1} p_2^{j_2} p_3^{j_3} p_4^{j_4}$$

$$w_1 = p_1^{k_1} p_2^{k_2} \text{ (or) } p_1^{k_1} p_2^{k_2} p_3^{k_3} \text{ (or) } p_1^{k_1} p_2^{k_2} p_3^{k_3} p_4^{k_4},$$

where $1 \leq \{i_1, i_2, i_3, i_4, f_1, f_2, f_3, f_4, k_1, k_2, k_3, k_4\} \leq a_1$.

But u_1, v_1 and w_1 are not adjacent in the induced sub graph $\langle V - D \rangle$.

Since $(p_1^{i_1} p_2^{i_2}, p_1^{j_1} p_2^{j_2}) = p_1^{b_1} p_2^{b_2}$ where $b_1, b_2 \geq 1$

$(p_1^{j_1} p_2^{j_2}, p_1^{k_1} p_2^{k_2}) = p_1^{b_1'} p_2^{b_2'}$ where $b_1', b_2' \geq 1$

$(p_1^{i_1} p_2^{i_2}, p_1^{k_1} p_2^{k_2}) = p_1^{b_1'} p_2^{b_2'}$ where $b_1'', b_2'' \geq 1$

(or)

$(p_1^{i_1} p_2^{i_2} p_3^{i_3}, p_1^{j_1} p_2^{j_2} p_3^{j_3}) = p_1^{c_1} p_2^{c_2} p_3^{c_3}$ where $c_1, c_2, c_3 \geq 1$

$(p_1^{j_1} p_2^{j_2} p_3^{j_3}, p_1^{k_1} p_2^{k_2} p_3^{k_3}) = p_1^{c_1'} p_2^{c_2'} p_3^{c_3'}$ where $c_1', c_2', c_3' \geq 1$

$(p_1^{i_1} p_2^{i_2} p_3^{i_3}, p_1^{k_1} p_2^{k_2} p_3^{k_3}) = p_1^{c_1''} p_2^{c_2''} p_3^{c_3''}$ where $c_1'', c_2'', c_3'' \geq 1$ (or)

$(p_1^{i_1} p_2^{i_2} p_3^{i_3} p_4^{i_4}, p_1^{j_1} p_2^{j_2} p_3^{j_3} p_4^{j_4}) = p_1^{d_1} p_2^{d_2} p_3^{d_3} p_4^{d_4}$ where $d_1, d_2, d_3, d_4 \geq 1$

$(p_1^{j_1} p_2^{j_2} p_3^{j_3} p_4^{j_4}, p_1^{k_1} p_2^{k_2} p_3^{k_3} p_4^{k_4}) = p_1^{d_1'} p_2^{d_2'} p_3^{d_3'} p_4^{d_4'}$ where $d_1', d_2', d_3', d_4' \geq 1$

$(p_1^{i_1} p_2^{i_2} p_3^{i_3} p_4^{i_4}, p_1^{k_1} p_2^{k_2} p_3^{k_3} p_4^{k_4}) = p_1^{d_1''} p_2^{d_2''} p_3^{d_3''} p_4^{d_4''}$ where $d_1'', d_2'', d_3'', d_4'' \geq 1$.

Thus D is an annihilator dominating set. Further, it is an annihilator dominating set of minimal cardinality.

For, if we remove any vertex v_r in D, then v_r is of the form $p_1^{i_1}$ (or) $p_2^{i_2}$ (or) $p_3^{i_3}$ (or) $p_4^{i_4}$ where $1 \leq \{i_1, i_2, i_3, i_4\} \leq a_1$.

If v_r is of the form $p_1^{i_1}$, for $1 \le i_1 \le a_1$ then v_r is adjacent with $p_1p_2,\, p_1p_2^2, \ldots, p_1p_2^{a_2};\, p_1p_3,\, p_1p_3^2, \ldots, p_1p_3^{a_3};\, p_1p_4,\, p_4^2,\, \ldots,\, p_1p_4^{a_4};\, p_1p_2p_3,$ $p_1p_2,\, p_3^2,\, \ldots,\, p_1p_2p_3^{a_3};\, p_1p_2^2p_3,\, p_1p_2^2p_3^2,\, \ldots,\, p_1p_2^2p_3^{a_3};\, \ldots p_1p_2^{a_2}p_3^2,\, \ldots,\, p_1$ $p_2^{a_2}p_3^{a_3};\, p_1p_2p_4,\, p_1p_2p_4^2,\, \ldots,\, p_1p_2p_4^{a_4};\, p_1p_2^2p_4,\, p_1p_2^2p_4^2,\, \ldots,\, p_1p_2^2p_4^{a_4};$ $\ldots,\, p_1p_2^{a_2}p_4,\, p_1p_2^{a_2}p_4^2,\, \ldots,\, p_1p_2^{a_2}p_4^{a_4};\, \ldots;\, p_1p_3p_4,\, p_1p_3p_4^2,\, \ldots,\, p_1p_3p_4^{a_4};\, p_1$ $p_3^2p_4,\, p_1p_3^2p_4^2,\, \ldots,\, p_1p_3^2p_4^{a_4};\, p_1p_3^{a_3}p_4,\, p_1p_3^{a_3}p_4^2,\, \ldots,\, p_1p_3^{a_3}p_4^{a_4};\, \ldots;\, p_1p_3^{a_2}p_4,$ $p_1p_3^{a_3}p_4^2,\, \ldots,\, p_1p_3^{a_3}p_4^{a_4};\, p_1p_2p_3p_4,\, p_1p_2p_3p_4^2, \ldots, p_1p_2p_3p_4^{a_4};\, p_1p_2^2p_3p_4,\, p_1$ $p_2^2p_3p_4^2,\, \ldots,\, p_1p_2^2p_3p_4^{a_4};\, \ldots;\, p_1p_2^{a_2}p_3p_4,\, p_1p_2^{a_2}p_3p_4^2,\, \ldots,\, p_1p_2^{a_2}p_3p_4^{a_4};$ $p_1p_2p_3^2p_4,\, p_1p_2p_3^2p_4^2,\, \ldots,\, p_1p_2p_3^2p_4^{a_4};\, p_1p_2^2p_3^2p_4,\, p_1p_2^2p_3^2p_4^2,\, \ldots,\, p_1p_2^2p_3^2$ $p_4^{a_4};\, \ldots;\, p_1p_2^{a_2}p_3^2p_4,\, p_1p_2^{a_2}p_3^2p_4^2,\, \ldots,\, p_1p_2^{a_2}p_3^2p_4^{a_4},\, \ldots,\, p_1p_2p_3^{a_3}p_4,\, p_1p_2$ $p_3^{a_3}p_4^2,\, \ldots,\, p_1p_2p_3^{a_3}p_4^{a_4};\, p_1p_2^2p_3^{a_3}p_4,\, p_1p_2^2p_3^{a_3}p_4^2,\, \ldots,\, p_1p_2^2p_3^{a_3}p_4^{a_4};\, \ldots;\, p_1$ $p_2^{a_2}p_3^{a_3}p_4,\, p_1p_2^{a_2}p_3^{a_3}p_4^2,\, \ldots,\, p_1p_2^{a_2}p_3^{a_3}p_4^{a_4}$; and so on in $\langle V - \{D - v_r\}\rangle$.

On the other hand, if v_r is of the form $p_2^{i_2}$ for $1 \le i_2 \le a_2$, then v_r is adjacent with $p_2p_3,\, p_2p_3^2, \ldots, p_2p_3^{a_3};\, p_2p_4,\, p_2p_4^2, \ldots, p_2p_4^{a_4};\, p_1p_2p_3,\, p_1p_2$ $p_3^2,\, \ldots,\, p_1p_2p_3^{a_3};\, p_1^2p_2p_3,\, p_1^2p_2p_3^2,\, \ldots,\, p_1^2p_2p_3^{a_3};\, p_1^{a_1}p_2p_3,\, p_1^{a_1}p_2p_3^2,\, \ldots,\, p_1^{a_1}$ $p_2p_3^{a_3};\, p_1p_2p_4,\, p_1p_2p_4^2,\, \ldots,\, p_1p_2p_4^{a_4};\, p_1^2p_2p_4,\, p_1^2p_2p_4^2,\, \ldots,\, p_1^2p_2p_4^{a_4};\, p_1^{a_1}$ $p_2p_4,\, p_1^{a_1}p_2p_4^2,\, \ldots,\, p_1^{a_1}p_2p_4^{a_4};\, p_2p_3p_4,\, p_2p_3p_4^2,\, \ldots,\, p_2p_3p_4^{a_4};\, p_2p_3^2p_4,\, p_2p_3^2$ $p_4^2,\, \ldots,\, p_2p_3^2p_4^{a_4};\, \ldots;\, p_2p_3^{a_3}p_4,\, p_2p_3^{a_3}p_2p_3^{a_3}p_4^2,\, \ldots,\, p_2p_3^{a_3}p_4^{a_4};\, p_1p_2p_3p_4,\, p_1$ $p_2p_3p_4^2, \ldots, p_1p_2p_3p_4^{a_4}; p_1p_2p_3^2p_4, p_1p_2p_3^2p_4^2, \ldots, p_1p_2p_3^2p_4^{a_4}; p_1p_2p_3^{a_3}p_4, p_1$ $p_2p_3^{a_3}p_4^2,\, \ldots,\, p_1p_2p_3^{a_3}p_4^{a_4};\, {}^{a_1}_{1}p_2p_3p_4,\, p_1^{a_1}p_2p_3p_4^2,\, \ldots,\, p_1^{a_1}p_2p_3p_4^{a_4};\, p_1^{a_1}p_2p_3^2p_4,$ $p_1^{a_1}p_2p_3^2p_4^2,\, \ldots,\, p_1^{a_1}p_2p_3^2p_4^{a_4};\, p_1^{a_1}p_2p_3^{a_3}p_4,\, p_1^{a_1}p_2p_3^{a_3}p_4^2,\, \ldots,\, p_1^{a_1}p_2p_3^{a_3}p_4^{a_4};$ and so on in $\langle V - \{D - v_r\}\rangle$.

Also if v_r is of the form $p_3^{i_3}$ for $1 \le i_3 \le a_3$, then v_r is adjacent with

$p_1p_3, p_1^2p_3, \ldots, p_1^{a_1}p_3, p_2p_3, p_2^2p_3, \ldots, p_2^{a_2}p_3, p_3p_4, p_3p_4^2, \ldots, p_3p_4^{a_4}; p_1p_2p_3, p_1p_2^2$ p_3, $p_1p_2^{a_2}p_3$; $p_1^2p_2p_3$, $p_1^2p_2^2p_3, \ldots, p_1^2p_2^{a_2}p_3$; $p_1^{a_1}p_2p_3$, $p_1^{a_1}p_2^2p_3, \ldots, p_1^{a_1}p_2^{a_2}p_3$; $p_1p_3p_4$, $p_1p_3p_4^2, \ldots, p_1p_3p_4^{a_4}$; $p_1^2p_3p_4$, $p_1^2p_3p_4^2, \ldots, p_1^2p_3p_4^{a_4}$; $p_1^{a_1}p_3p_4$, $p_1^{a_1}p_3p_4^2$, $\ldots$, $p_1^{a_1}p_3p_4^{a_4}$; $p_2p_3p_4$, $p_2p_3p_4^2, \ldots, p_2p_3p_4^{a_4}$; $p_2^2p_3p_4$, $p_2^2p_3p_4^2, \ldots, p_2^2p_3p_4^{a_4}$; $p_2^{a_2}p_3p_4$, $p_2^{a_2}p_3p_4^2, \ldots, p_2^{a_2}p_3p_4^{a_4}$; $p_1p_2p_3p_4$, $p_1p_2p_3p_4^2, \ldots, p_1p_2p_3$; $p_1p_2^2p_3p_4$, $p_1p_2^2p_3p_4^2, \ldots, p_1p_2^2p_3p_4^{a_4}$; $\ldots$; $p_1p_2^{a_2}p_3p_4$, $p_1p_2^{a_2}p_3p_4^2, \ldots, p_1p_2^{a_2}p_3p_4^{a_4}$; $p_1^{a_1}p_2p_3$ p_4, $p_1^{a_1}p_2p_3p_4^2, \ldots, p_1^{a_1}p_2p_3p_4^{a_4}$; $p_1^{a_1}p_2^2p_3p_4$, $p_1^{a_1}p_2^2p_3p_4^2, \ldots, p_1^{a_1}p_2^2p_3p_4^{a_4}$; $\ldots$; $p_1^{a_1}$ $p_2^{a_2}p_3p_4$, $p_1^{a_1}p_2^{a_2}p_3p_4^2, \ldots, p_1^{a_1}p_2^{a_2}p_3p$ and so on in $\langle V - \{D - v_r\}\rangle$.

Also if v_r is of the form $p_4^{i_4}$ for $1 \le i_4 \le a_4$, then v_r is adjacent with $p_1p_4, p_1^2p_4, \ldots, p_1^{a_1}p_4; p_2p_4, p_2^2p_4, \ldots, p_2^{a_2}p_4; p_3p_4, p_3^2p_4; \ldots, p_3^{a_3}p_4; p_1p_2p_4, p_1p_2^2$ $p_4, \ldots, p_1p_2^{a_2}p_4; p_1^2p_2p_4, p_1^2p_2^2p_4, \ldots, p_1^2p_2^{a_2}p_4; p_1^{a_1}p_2p_4, p_1^{a_1}p_2^2p_4, \ldots, p_1^{a_1}p_2^{a_2}p_4; p_1$ $p_3p_4, p_1p_3^2p_4, \ldots, p_1p_3^{a_3}p_4; p_1^2p_3p_4, p_1^2p_3^2p_4, \ldots, p_1^2p_3^{a_3}p_4; p_1^{a_1}p_3p_4, p_1^{a_1}p_3^2p_4, \ldots, p_1^{a_1}$ $p_3^{a_3}p_4; p_2p_3p_4, p_2p_3^2p_4, \ldots, p_2p_3^{a_3}p_4; p_2^2p_3p_4, p_2^2p_3^2p_4, \ldots, p_2^2p_3^{a_3}p_4; p_2^{a_2}p_3^2p_4,$ $p_2^{a_2}p_3^2p_4, \ldots, p_2^{a_2}p_3^{a_3}p_4; p_1p_2p_3p_4, p_1p_2^2p_3p_4, \ldots, p_1p_2^{a_2}p_3p_4; p_1p_2p_3^{a_3}p_4, p_1p_2^2$ $p_3^{a_3}p_{4, \ldots,}\ p_1p_2^{a_2}p_3^{a_3}p_4; p_1^{a_1}p_2p_3p_4, p_1^{a_1}p_2^2p_3p_4, \ldots, p_1^{a_1}p_2^{a_2}p_3p_4; p_1^{a_1}p_2p_3^2p_4, p_1^{a_1}$ $p_2^2p_3^2p_4, \ldots, p_1^{a_1}p_2^{a_2}p_3^2p_4; p_1^{a_1}p_2p_3^{a_3}p_4, p_1^{a_1}p_2^2p_3^{a_3}p_4, \ldots, p_1^{a_1}p_2^{a_2}p_3^{a_3}p_4;$ and so on in $\langle V - \{D - v_r\}\rangle$.

Thus $\{D - v_r\}$ is not an annihilator dominating set.

So D is a minimal annihilator dominating set.

Therefore hence $\gamma_a[v_m] \le |D| = 4a_1$, if $a_1 = a_2 = a_3 = a_4$.

Construction of a graph whose annihilator domination number does not exceed a given number $4n$ for $n = 1, 2, 3, \ldots,$

With the help of the above theorem, we now construct a graph with the given annihilator domination number.

These constructions are quite useful in the applications of domination theory in real life situations.

If we are required to construct a graph with a given annihilator domination number $'4n'$, we proceed as follows:

If $4n$ is the given annihilator domination number for $n = 1, 2, 3, \ldots$, choose $m = p_1^n,\ p_2^n,\ p_3^n,\ p_4^n$ where $p_1,\ p_2,\ p_3,\ p_4$ are four distinct primes.

By the above theorem, the graph V_m has a minimal annihilator dominating set $D = \{p_1, p_1^2, \ldots, p_1^{a_1}; p_2, p_2^2, \ldots, p_2^{a_2}; p_3, p_3^2, \ldots, p_3^{a_3}; p_4, p_4^2, \ldots, p_4^{a_4}\}$ Which is of cardinality $4n$.

Illustration

Let V_m be an arithmetic graph with $m = p_1^2 \cdot p_2^2 \cdot p_3^2 \cdot p_4^2$. Choose any four primes p_1, p_2, p_3, p_4 such that $a_1 = a_2 = a_3 = a_4 = 2$. The vertices of V_m are the divisors of m (except 1). The numbers are given for every vertex from 1 to 80.

$\{1 - p_1, 2 - p_2, 3 - p_3, 4 - p_4, 5 - p_1^2, 6 - p_2^2, 7 - p_3^2, 8 - p_4^2, 9 - p_1p_2, 10 - p_1p_3,$

$11 - p_1p_4, 12 - p_2p_3, 13 - p_2p_4, 14 - p_3p_4, 15 - p_1^2p_2, 16 - p_1^2p_3, 17 - p_1^2p_4,$

$18 - p_2^2p_3, 19 - p_2^2p_4, 20 - p_3^2p_4, 21 - p_1p_2^2, 22 - p_1p_3^2, 23 - p_1p_4^2, 24 - p_2p_3^2,$

$25 - p_2p_4^2, 26 - p_3p_4^2, 27 - p_1^2p_2^2, 28 - p_1^2p_3^2, 29 - p_1^2p_4^2, 30 - p_2^2p_3^2, 31 - p_2^2p_4^2,$

$32 - p_3^2p_4^2, 33 - p_1p_2p_3, 34 - p_1p_2p_4, 35 - p_2p_3p_4, 36 - p_3p_4p_1, 37 - p_1^2p_2p_3,$

$38 - p_1^2p_2p_4, 39 - p_2^2p_3p_4, 40 - p_3^2p_4p_1, 41 - p_1p_2^2p_3, 42 - p_1p_2^2p_4, 43 - p_2p_3^2$

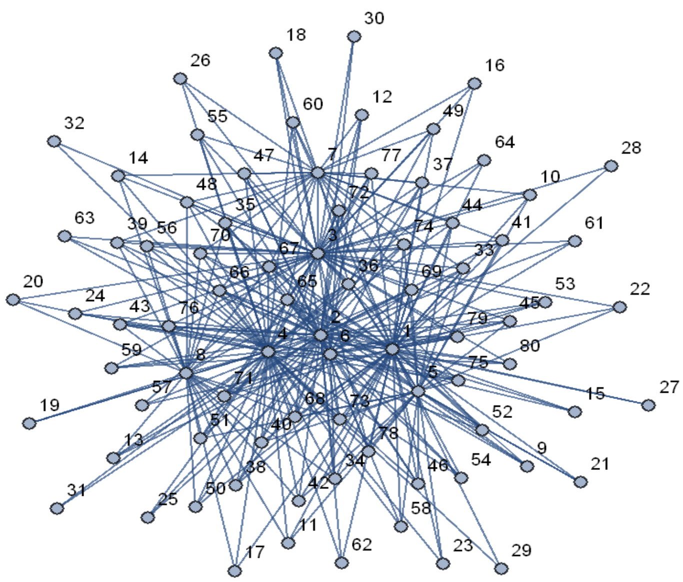
$p_4, 44 - p_3p_4^2p_1, 45 - p_1p_2p_3^2, 46 - p_1p_2p_4^2, 47 - p_2p_3p_4^2, 48 - p_3p_4p_1^2, 49 - p_1^2$

$p_2^2p_3, 50 - p_1^2p_2^2p_4, 51 - p_2^2p_3^2p_4, 52 - p_3^2p_4^2p_1, 53 - p_1^2p_2p_3^2, 54 - p_1^2p_2p_4^2,$

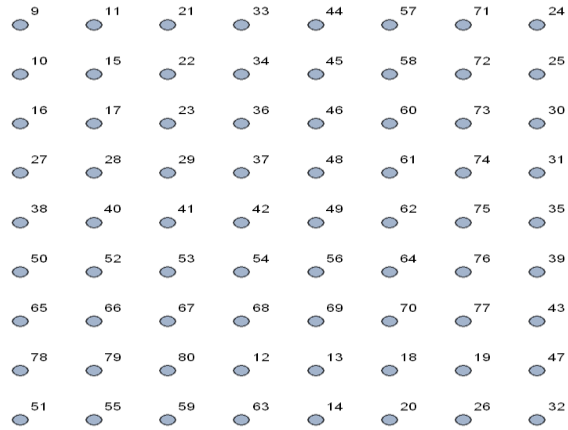
$55 - p_2^2p_3p_4^2, 56 - p_3^2p_4p_1^2, 57 - p_1p_2^2p_3^2, 58 - p_1p_2^2p_4^2, 59 - p_2p_3^2p_4^2, 60 - p_3p_4^2$

$$p_1^2, 61 - p_1^2 p_2^2 p_3^2, 62 - p_1^2 p_2^2 p_4^2, 63 - p_2^2 p_4^2 p_1^2, 64 - p_3^2 p_4^2 p_1^2, 65 - p_1 p_2 p_3 p_4, 66 - p_1^2 p_2 p_3 p_4, 67 - p_1 p_2^2 p_3 p_4, 68 - p_1 p_2 p_3^2 p_4, 69 - p_1 p_2 p_3 p_4^2, 70 - p_1^2 p_2^2 p_3 p_4, 71 - p_1^2 p_2 p_3^2 p_4, 72 - p_1^2 p_2 p_3 p_4^2, 73 - p_1 p_2^2 p_3^2 p_4, 74 - p_1 p_2^2 p_3 p_4^2, 75 - p_1 p_2 p_3^2 p_4^2, 76 - p_1^2 p_2^2 p_3^2 p_4, 77 - p_1^2 p_2^2 p_3 p_4^2, 78 - p_1 p_2^2 p_3^2 p_4^2, 79 - p_1^2 p_2 p_3^2 p_4^2, 80 - p_1^2 p_2^2 p_3^2 p_4^2\}.$$

There are $81 - 1 = 80$ vertices.



The graph of V_m with $m = p_1^2, p_2^2, p_3^2, p_4^2$; $D = \{p_1, p_2, p_3, p_4, p_1^2, p_2^2, p_3^2, p_4^2\}$ which is the minimal annihilator dominating set of V_m with cardinality $4 \times 2 = 8$. By removing D from the vertex set V of V_m, the induced sub graph $\langle V - D \rangle$ is an independent graph with isolated vertices.



Hence $\gamma_a[V_m] = 8$.

3. Conclusion

The implements of the number theory facilitate us to grow a simple method of constructing a graph with a given cardinality of the annihilator dominating set with astonishing ease. We can apply the concept of Annihilator domination in various real-life situations. One application among several applications of Annihilator domination set is Epidemiology: Vertices represent individuals and directed edges the transfer of an infectious disease from one individual to another. Analyzing such graphs has become an important component in understanding and controlling the spread of diseases.

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