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SUM DIVISOR CORDIAL LABELING ON SHELL RELATED GRAPHS

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Abstract

A sum divisor cordial labeling of a simple graph is a bijective function $f : V(G) \to \{1, 2, \ldots, |V(G)|\}$, such that when each edge uv is labelled 1 if $f(u)+f(v)$ is divisible by 2 and labelled 0 otherwise, then the absolute difference between the edges labelled as 0 and the edges labelled as 1 is at most 1. A graph is called a sum divisor cordial graph if it admits sum divisor cordial labeling. In this paper, we have proved that subdivided shell graphs, disjoint union of two subdivided shell graphs and uniform shell bow graphs are sum divisor cordial.

1. Introduction

Graph labeling [11] is an assignment of integers (usually non - negative) to vertices or to edges or to both, subject to certain conditions. Varatharajan et al. [15] introduced divisor cordial labeling. A divisor cordial labeling [15] of a simple graph has a bijection $f : V(G) \to \{1, 2, \ldots, |V(G)|\}$ such that an edge uv is assigned the label 1 if one $f(u)$ or $f(v)$ divides the other and 0 otherwise, then the number of edges labelled with 0 and 1 differ by at most 1. Lourdusamy and Patrick [9] defined a new concept of divisor cordial labeling known as sum divisor cordial labeling. A simple graph is said to admit sum divisor cordial labeling [9] if there exist a bijective function $f : V(G) \to \{1, 2, \ldots, |V(G)\}$, such that when each edge uv is labelled 1 if

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$f(u)+f(v)$ is divisible by 2 and labelled 0 otherwise, then the absolute difference between the edges labelled as 0 and the edges labelled as 1 is atmost 1.

Lourdusamy and Patrick [9] have proved that path, comb, star, complete bipartite $k_2 + mk_1$, bistar, crown, gear, subdivision of the star, $K_{1,3} * K_{1,n}$ are sum divisor cordial graphs. They have also proved that, P_n^2, the graph obtained by duplication of each vertex of P_n by an edge, the graph obtained by duplication of each vertex of C_n by an edge, $C_4^{(t)}$[10] are sum divisor cordial. Sugumaran and Rajesh [13] have proved that the joint of two paths P_n (n is odd), $P_n \odot K_{1,m}$, $C_n \odot K_{1,m}$ (n is odd) are sum divisor cordial graphs. They have also proved the path union of r copies of C_n (n is odd), complete binary tree, umbrella graphs [14] are sum divisor cordial. Adalja and Ghodasara [1] proved that quadrilateral snake, double quadrilateral snake, alternate quadrilateral snake, double alternate quadrilateral snake are sum divisor cordial graphs.

A shell graph [3], $C(n, n-3)$ where $n \geq 4$, is a cycle C_n with $(n-3)$ chords having a common end point called apex. A subdivision [2] of an edge is obtained by deleting it and replacing it by a path of length two which connects its ends, the internal vertex of this path being a new vertex.

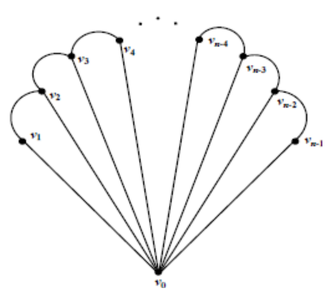

Figure 1. Shell graph $C(n, n-3)$.

A subdivided shell graph [5] is obtained by subdividing only the edges in the path of a shell graph. An uniform shell bow graph [8] is a one vertex union of two shell graphs of same order. A subdivided uniform shell bow graph [5] is a one vertex union of two subdivided shell graphs of same order.

Throughout this paper $e_f(0)$, $e_f(1)$ denote the number of edges labelled 0 and 1 respectively under $'f'$. The graphs considered here are simple, finite, connected and undirected graphs. One can refer Gallian survey [4] for further knowledge on various graph labelings.

2. Main Result

Theorem 2.1. *Subdivided shell graphs are Sum divisor cordial.*

Proof. Let G be a subdivided shell graph. Let us describe G as follows. The apex of G is denoted as v_0. The vertices in the path of G are denoted as $v_1, v_2, v_3, \ldots, v_m$. Note that G has $p = m+1$ vertices and $q = \frac{3m-1}{2}$ edges. Define $f : V(G) \to \{1, 2, \ldots, p\}$ as follows.

$$f(v_0) = 1$$

$$f(v_i) = \begin{cases} i+1, & \text{if } i \equiv 0, 1 (\text{mod } 4) \\ i+2, & \text{if } i \equiv 2 (\text{mod } 4), \ for\ 1 \le i \le m \\ i, & \text{if } i \equiv 3 (\text{mod } 4) \end{cases} \tag{1}$$

From the equations given in (1) we can see that the vertex labels are distinct. If any two vertex labels are equal we get a contradiction to the fact that $'m'$ is a positive integer.

The induced edge labels are

$$f^*(v_{2i-1}v_{2i}) = 1, \ for\ 1 \le i \le \frac{m-1}{2}$$

$$f^*(v_{2i}v_{2i+1}) = 0, \ for\ 1 \le i \le \frac{m-1}{2}$$

$$f^*(v_0 v_{2i-1}) = \begin{cases} 0, \text{ for } i = 2k-1, 1 \le k \le \left\lfloor \frac{m+3}{4} \right\rfloor \\ 1, \text{ for } i = 2k, 1 \le k \le \left\lceil \frac{m-1}{4} \right\rceil \end{cases} \tag{2}$$

From the above labeling pattern, we have

$$e_f(0) = \begin{cases} \frac{q+1}{2}, & \text{when } m \equiv 1(\text{mod } 4) \\ \frac{q}{2}, & \text{when } m \equiv 3(\text{mod } 4) \end{cases}$$

$$e_f(1) = \begin{cases} \frac{q-1}{2}, & \text{when } m \equiv 1(\text{mod } 4) \\ \frac{q}{2}, & \text{when } m \equiv 3(\text{mod } 4) \end{cases}$$

Thus the condition $| e_f(0) - e_f(1) | \leq 1$ holds true.

Hence subdivided shell graphs are sum divisor cordial.

An illustration for the above theorem when $m = 9,\ p = 10,\ q = 13$ is given below.

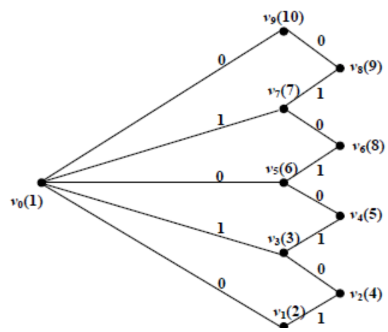

Figure 2. Sum divisor cordial subdivided shell graph, $m = 9,\ p = 10,\ q = 13$.

Theorem 2.2. *The disjoint union of two subdivided shell graphs of path orders* $'m'(m \geq 7)$ *and* $'l'(l \geq 7)$ *are sum divisor cordial, when* $m, l \equiv 3(\text{mod } 4)$.

Proof. Let G_1 and G_2 be two subdivided shell graphs of path orders $'m'$ and $'l'$ respectively $(m \geq 7,\ l \geq 7)$. Let G be the disjoint union of G_1 and G_2. The apex of G_1 is denoted as u_0 and the vertices in the path of G_1 are denoted as $u_1, u_2, u_3, \ldots, u_m$. The apex of G_2 is denoted as v_0 and the vertices in the path of G_2 are denoted as $v_1, v_2, v_3, \ldots, v_l$. Note that G has $p = m + l + 2$ vertices, $q = \frac{3m + 3l - 2}{2}$ edges.

Define $f : V(G) \to \{1, 2, \ldots, p\}$ as follows.

$$f(u_0) = 1$$

$$f(v_0) = m + 2$$

$$f(u_i) = \begin{cases} i+1, & \text{if } i \equiv 0, 1 (\text{mod } 4) \\ i+2, & \text{if } i \equiv 2 (\text{mod } 4), \text{ for } 1 \le i \le m \\ i, & \text{if } i \equiv 3 (\text{mod } 4) \end{cases}$$

$$f(v_i) = \begin{cases} m+2+i, & \text{if } i \equiv 0, 1 (\text{mod } 4) \\ m+3+i, & \text{if } i \equiv 2 (\text{mod } 4), \text{ for } 1 \le i \le l \\ m+1+i, & \text{if } i \equiv 3 (\text{mod } 4) \end{cases} \tag{3}$$

From the equations given in (3) we can see that the vertex labels are distinct. If any two vertex labels are equal we get a contradiction to the fact that 'm' and 'l' are positive integers.

The induced edge labels are

$$f^*(u_{2i-1}u_{2i}) = f^*(v_{2j-1}v_{2j}) = 1, \text{ for } 1 \le i \le \frac{m-1}{2}, 1 \le j \le \frac{l-1}{2}$$

$$f^*(u_{2i}u_{2i+1}) = f^*(v_{2j}v_{2j+1}) = 0, \text{ for } 1 \le i \le \frac{m-1}{2}, 1 \le j \le \frac{l-1}{2}$$

$$f^*(u_0u_{2i-1}) = \begin{cases} 0, \text{ for } i = 2k-1, 1 \le k \le \left\lfloor \frac{m+3}{4} \right\rfloor \\ 1, \text{ for } i = 2k, 1 \le k \le \left\lceil \frac{m-1}{4} \right\rceil \end{cases}$$

$$f^*(v_0v_{2i-1}) = \begin{cases} 0, \text{ for } i = 2k-1, 1 \le k \le \left\lfloor \frac{l+3}{4} \right\rfloor \\ 1, \text{ for } i = 2k, 1 \le k \le \left\lceil \frac{l-1}{4} \right\rceil \end{cases} \tag{4}$$

From the above labeling pattern, we have, $e_f(0) = e_f(1) = \frac{q}{2}$.

This implies $|e_f(0) - e_f(1)| = 0$. Thus the condition $|e_f(0) - e_f(1)| \le 1$ holds true.

Hence the disjoint union of two subdivided shell graphs are sum divisor cordial.

An illustration for the above theorem when $m = 11$, $l = 11$, $p = 24$, $q = 32$ is given below.

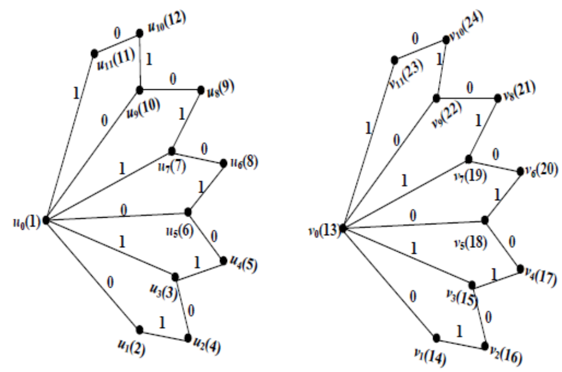

Figure 3. Sum divisor cordial disjoint union of two subdivided shell graphs, $m = 11$, $l = 11$, $p = 24$, $q = 32$.

Theorem 2.3. *Uniform shell bow graphs of path order 'm' are sum divisor cordial, when 'm' is odd.*

Proof. Let G be a uniform shell bow graph. The apex of G is denoted as v_0. The vertices in the left shell of G are denoted as $v_1, v_2, v_3, \ldots, v_m$. Let $v_{m+1}, v_{m+2}, v_{m+3}, \ldots, v_{2m}$ denotes the vertices in the right shell of G. Note that the graph G has $p = 2m+1$ vertices and $q = 4m-2$ edges.

Define $f : V(G) \to \{1, 2, \ldots, p\}$ as follows.

$$f(v_0) = 1$$

$$f(v_i) = \begin{cases} i+1, & \text{if } i \equiv 1, 2 \pmod 4 \\ i+2, & \text{if } i \equiv 3 \pmod 4, \text{ for } 1 \le i \le 2m \\ i, & \text{if } i \equiv 0 \pmod 4 \end{cases} \tag{5}$$

From the equations given in (5) we can see that the vertex labels are distinct. If any two vertex labels are equal we get a contradiction to the fact that 'm' is a positive integer.

The induced edge labels are

$$f^*(v_{2i-1}v_{2i}) = 0, \text{ for } 1 \le i \le \frac{m-1}{2}, \frac{m+3}{2} \le i \le m$$

$$f^*(v_{2i}v_{2i+1}) = 1, \text{ for } 1 \le i \le m-1$$

$$f^*(v_0 v_{2i-1}) = \begin{cases} 0, \text{ for } i = 2k-1, 1 \leq k \leq \dfrac{m+1}{2} \\ 1, \text{ for } i = 2k, 1 \leq k \leq \dfrac{m-1}{2} \end{cases}$$

$$f^*(v_0 v_{2i}) = \begin{cases} 1, \text{ for } i = 2k-1, 1 \leq k \leq \dfrac{m+1}{2} \\ 0, \text{ for } i = 2k, 1 \leq k \leq \dfrac{m-1}{2} \end{cases} \tag{6}$$

From the above labeling pattern, we have, $e_f(0) = e_f(1) = \dfrac{q}{2}$.

This implies $| e_f(0) - e_f(1) | = 0$. Thus the condition $| e_f(0) - e_f(1) | \leq 1$ holds true.

Hence uniform shell bow graphs are sum divisor cordial.

An illustration for the above theorem when $m = 5$, $p = 11$, $q = 18$ is given below.

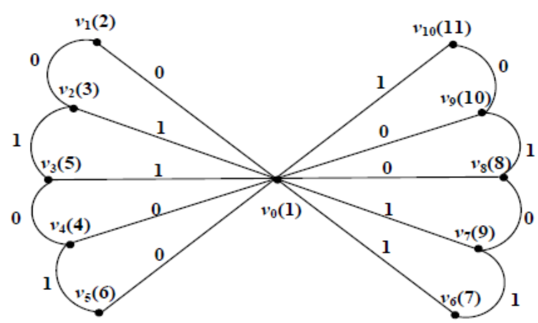

Figure 4. Sum divisor cordial uniform shell bow graph, $m = 5$, $p = 11$, $q = 18$.

Corollary 1. *Subdivided uniform shell bow graphs are sum divisor cordial.*

3. Conclusion

In this paper, we have proved that subdivided shell graphs, disjoint union of two subdivided shell graphs and uniform shell bow graphs are sum divisor cordial. This is an interesting concept and can be investigated for path related graphs also.

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