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MATHEMATICAL MODELLING OF PROFIT FUNCTION FOR DETERIORATING INVENTORIES UNDER THE EFFECT OF REGULAR INSPECTION AND REVENUE FROM REJECTED PRODUCTS

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Abstract

The present research work focused on the inventories that are deteriorating in nature and needs regular inspection as well as separation of deteriorated inventory from good one so as to stop further deterioration. In present work regular inspection considered in a cycle and removed items are used to earn revenue. Our objective is to find the optimal number of inspections; selling price and cycle length to maximize profit. For our objective, it is considered price-stock dependent demand, shortage allowed, deterioration rate depends on time and number of inspection. Deteriorated items are sold out in the market to earn extra revenue. Deterioration can be reduced by removing deteriorating inventories at some level and deteriorated inventories can be used if removed initial level.

1. Introduction and Literature Review

Deterioration is a process that changes the state of a product with time due to that the product is no more usable for its original purpose. Some of the examples of deteriorating inventories are vegetables, fruits, flowers, blood, cereals, alcohol, and gasoline etc and these inventories are playing wide role in supply chain in global market. Due to deteriorating nature storage of these inventories are more costly and it needs special storage facilities such as

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preservation techniques, better maintenance and regular inspection. Therefore to maximize profit business firms need effective and special strategies for supply chain. Preservatives are one effective tool that increases the life of a product for longer period but use of such preservatives increases environmental pressure as well as health issues. Therefore, it is essential responsibility of organizations as well as researchers to reduce the environmental damage through effective policies. Academicians always show their keen interest to work on better policies for deteriorating inventories. Therefore, "continuous inspection" is one of the best policies to reduce the rate of deterioration. It is well-known the presence of the deteriorated items in the warehouse would increase the rate of deterioration of the entire inventory by contact. Therefore continuous inspection or monitoring can decrease the rate of deterioration. If these deteriorated products are inspect at the initial stage then it can be used to earn extra revenue by selling in market but a low price.

Many researchers worked and established effective policies and models for deteriorating inventories with different factors. Some of the recent works are as follows: Lo et al. [1], He et al. [2], Mahata [4], Alshamrani [5], Jiangtao et al. [6], Donselaar et al. [9], Zhang et al. [10], Adenso-Díaz et al. [11], Diabat et al. [14], Hsieh and Dye [16], Das and Roy [18], Taleizadeh et al. [22], Ferreira et al. [19], Ouaret et al. [21], Timajchi et al. [28], Lin et al. [26], Li et al. [25], Yavari et al. [29], Mishra et al. [34], etc. Sankar Sana [3] has developed a finite time-horizon deterministic EOQ model with quadratic decreasing price dependent demand. Jiangtao et al. [6] developed a stock-dependent multi-item inventory model with two-level trade credit. Tai et al. [8] developed a policy with consideration that fractions of deteriorated inventory can be sold to consumers together with fresh products to maximize profit. Ouaret et al. [21] worked and established production and replacement policies for a deteriorating manufacturing system. Ferreira et al. [19] established a policy for the management of deteriorating inventory used in long-term humanitarian operations by using "Markov Decision Processes".

Carbon emission is also one of the foremost factors that attract organizations as well as researchers. Organizations objective is not only to gain profit but also to control carbon emission for social welfare. The Government set some policies that reduce carbon emission like carbon cap, carbon tax, cap and tax, and carbon offset. But the major problem in adopting these policies is how to choose right policy at low cost, therefore many researchers worked in this area and established models with different factors. Some recent work in this area as follows: Gong et al. [23], Zhang et al. [33], Yu et al. [32], Wu and Kung [31], Mishra et al. [34], etc. Halat et al. [24] have developed the Stackelberg game approach for a multi-stage supply chain. Wu and Kung [31] have been developed a Cournot competition model considering asymmetric carbon emission technology and traditional supply chains. Yu et al. [32] have established an inventory optimization model under carbon tax policy. Mishra [34] developed a policy with price-dependent demand and carbon cap and tax regulation policy.

Some of the research work considered inspection policy during supply chain for deteriorating inventory. Some of these are Mohammadi et al. [7], Tai et al. [8], Banerjee and Agrawal [12], Tai et al. [27], Khakzad and Gholamian [30] studied the effect of inspection. Mohammadi et al. [7] studied a manufacturing inventory supply system for deteriorating products with imperfect inspection during the supply chain. Tai et al. [27] studied the combined effect of inventory control and inspection on the supply chain. They studied separate effect of one inspection and continuous monitoring for deteriorating inventories.

2. Concept and Methods

2.1 Assumptions and notations

1. The average deterioration rate of the inventory is linear function of the deterioration rate of the each item in each period, and it is dependent function of the number of inspections. Mathematically, rate of deterioration is calculated as follows: $\ddot{\theta} = \theta' + \frac{g}{n+1}$.

2. Demand depends on hybrid-price-stock, which is as follows: $D(p, I(t)) = D(p) + \delta(I(t))$, where $0 < \delta << 1$, is a product based utilization (consume) rate and $D(p) = \tau(x_1 - yp) + (1 - \tau)x_2 p^{-\gamma}$ is a hybrid price both for linear and nonlinear price which satisfying following condition $0 \leq \tau \leq 1$;

$x_1 > 0; x_1 > 0 y > 0; \frac{x_1}{y} \geq p$ and $\gamma > 1$. Mishra et al. [34] used such type of demand in their work.

3. Shortages are allowed and inspection time ignored or not considered.

4. Lead time zero and deteriorated items to be use to earn revenue.

6. Carbon tax policy considered for producing each unit carbon emission.

7. Rate of deterioration assumed to be $\left(\ddot{\theta} + \frac{g}{n+1}\right)$ (Used in Khakzad and Gholamian [30]).

Notations

Symbol	Description
D(P)	price-stock dependent demand
θ	The fix rate of deterioration of each product
$\ddot{\theta}$	The effective rate of deterioration during each cycle
δ	Inventory dependent consumption rate
h_1	Holding cost of inventory coming to supply chain
h_2	Holding cost of rejected inventory
p	Price of good products
p_1	Price of rejected products during inspection
O_c	Ordering cost
Q	Inventory coming to supply chain
c	Purchasing cost
$I(t)$	On-hand inventory level T at time t
$I_1(t)$	Inventory level of rejected products at time t
n	Number of inspection during cycle

r	Hiring cost of inspector
g	Multiple constant of the effect of deteriorated product on desirable product
χ	Tax/unit emission production
υ	Emission/unit due to disposal of a deteriorating product
λ	Constant rate of emission reduction, $\lambda \in (0,1)$
T	Cycle length
T	Time
S_p	Selling price per unit
R_d	Revenue from deteriorated inventory
O_c	Ordering cost
P_c	Production cost
HC_1	Holding cost for on hand inventory
HC_2	Holding cost for deteriorated inventory
E^r	Emission cost
C_{ins}	Cost of inspection per cycle
Decision variables	
	T, p, n

2.2. Problem statement and mathematical modeling

In this work, it is considered deteriorating inventories that need continuous inspection. It is considered that the whole seller can earn revenue by selling deteriorating products (rejected from supply chain) at a low price. If the deteriorating inventory is removed from the storage it will not lead to deterioration by contact and the rate of the deterioration slow down. Therefore, the supplier must inspect deteriorating inventory at regular time intervals for removing deteriorating inventory so that it cannot spoil other inventory and removed inventory can be utilized at the initial level and profit can be maximized. In this work, sell price, cycle length and number of inspections are decision variables for discussed supply chain. Figure 1a representing the behavior of inventories with the time. According to figure 1a, the stock level reduces with time because of customer's demand and rate of deterioration. As a result, at time *t* the present inventory can be calculated by following differential equations.

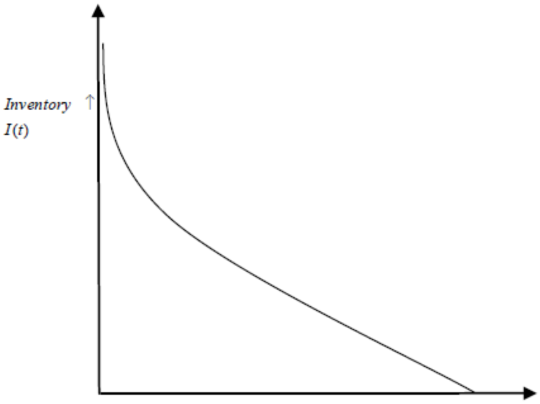

Figure 1a. Behavior of inventory with.

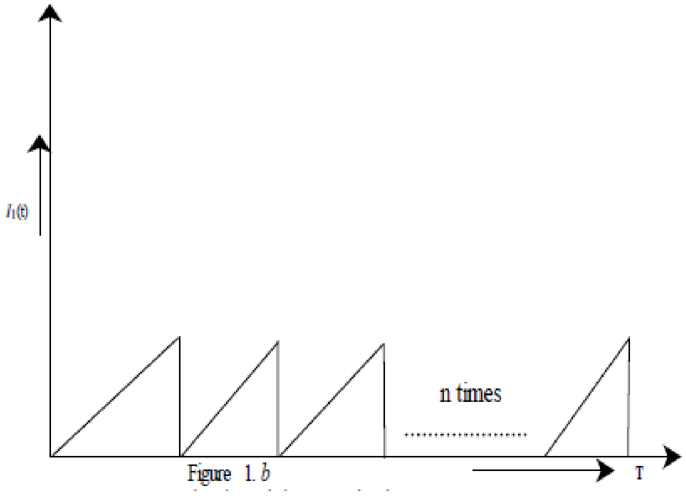

Figure 1b. Deteriorated inventory level.

$$\frac{dI(t)}{dt} + \ddot{\theta} I(t) = -D(p,\ I(t)),\ 0 \le t \le T \tag{1}$$

Now with the help of figure 1a, it is observed that at time T inventory level $I(T) = 0$.

$$I(t) = \frac{D(p)}{(\ddot{\theta} + \delta)} (e^{(\ddot{\theta}+\delta)(T-t)} - 1) \tag{2}$$

Let at the initial point when $t = 0$, the inventory level be Q then by using above result in equation (2), we get

$$Q = \frac{D(p)}{(\ddot{\theta} + \delta)} (e^{(\ddot{\theta}+\delta)T} - 1)$$

$$= D(P)\left(T + \frac{(\ddot{\theta} + \delta)T^2}{2}\right) \tag{3}$$

This result gives initial inventory level. Now we calculate each cost function of supply chain as follows:

(I) Holding cost of desirable products

$$HC_1 = h_1 \int_0^T I(t) = h_1 \int_0^T \frac{D(P)}{(\ddot{\theta} + \delta)} (e^{(\ddot{\theta}+\delta)(T-t)} - 1)dt$$

$$D(P)h_1\left(\frac{T^2}{2}\right) \tag{4}$$

(II) Holding cost of rejected products from inventory that got deteriorated but can be use to earn revenue.

The products that deteriorated and removed from fresh inventory during inspection stored in different place and will be used for earning revenue by selling at low price. For this, first we calculate the inventory level of the rejected items. After every inspection deteriorated inventory removed from fresh inventory. As shown in figure 1b inventory level of deteriorated items increased at the rate of as follows:

$$\frac{dI_1(t)}{dt} = \ddot{\theta} I(t)$$

$$I_1(t) = \int_0^t \ddot{\theta} I(t)dt = \int_0^t \ddot{\theta} \frac{D(P)}{(\ddot{\theta} + \delta)} (e^{(\ddot{\theta}+\delta)(T-t)} - 1)dt$$

$$= D(P)\ddot{\theta}(t^2 - 2Tt)$$

If there are n inspections then total cycle is divided into $n + 1$ small cycle when deteriorated items will be collected. Let $Ti = (n + 1)$

$$h_2 \int_0^T I_1(t)dt = h_2 \int_0^{T_i} \ddot{\theta} I_1(t)dt = -\frac{D(P)\ddot{\theta}}{(\ddot{\theta} + \delta)} \left(\frac{e^{(\ddot{\theta}+\delta)(T_i+t)}}{(\ddot{\theta} + \delta)^2} + \frac{t^2}{2} - \frac{te^{(\ddot{\theta}+\delta)T_i}}{(\ddot{\theta} + \delta)} \right)_0^{T_i}$$

$$= \frac{(n+1)D(P)h_2}{2} \left(\frac{T}{(n+1)} \right)^3 \left(\theta' + \frac{g}{n+1} \right)$$

If there is only one inspection during one cycle then by using $n = 1$ (by figure 1b)

$$= \frac{2D(P)\ddot{\theta}h_2}{2} \left(\frac{T}{2} \right)^3 \tag{5}$$

(III) Ordering cost $= O_c$

(IV) Purchasing cost $PC = cQ = cD(P)\left(T + \frac{(\ddot{\theta} + \delta)T^2}{2} \right)$ (7)

(V) Cost of inspection.

Cost of inspection is linearly dependent on number of inspection per cycle and cost of hiring number of inspector that can be given by

$$C_{ins} = nr \tag{8}$$

(VI) Total revenue sale price

$$= p\int_0^T D(P, I(t))dt = D(p)p\left(T + \frac{\delta T^2}{2} \right) \tag{9}$$

(VII) Revenue from rejected products

Rejected inventory during inspection can be used for revenue. These rejected inventory sell at lower price. Total earn revenue can be calculated as follows:

$$p_1 I_1(t) = D(p) p_1 \ddot{\theta}(t^2 - 2Tt) \tag{10}$$

(VIII) Emission cost

If the total emission due to disposal of a deteriorating product be ν and emission due to use of deteriorating product be $\lambda\nu$, where $\lambda \in (1 - \lambda)$, depends on use of rejected products. Use of deteriorating product reducing emission by $(1 - \lambda)$.

Total emission reduction $= (1 - \lambda)\nu D(p)\ddot{\theta}(t^2 - 2Tt)$.

Let for each unit generation of emission, firm charged by a constant tax $\chi > 0$. Therefore, total environmental impact cost incurred is given by

$$\chi(1 - \lambda)\nu D(p)\ddot{\theta}(t^2 - 2Tt) \tag{11}$$

(IX) Total profit function

Total profit can be calculated as follows by using equations (1)-(11)

Total profit function $S_p + R_d - O_c - P_c - HC_1 - HC_2 - C_{ins} - E^r$ (12)

Theorem. *The profit function is maximize with respect to* n, p *and* T *if the Hessian matrix* H *of second order derivatives positive semi definite.*

Proof. In order to prove the above theorem, we calculate first order derivatives of profit function with respect to n, p and T and then by putting $\frac{\partial TP}{\partial n} = 0$; $\frac{\partial TP}{\partial p} = 0$ and $\frac{\partial TP}{\partial T} = 0$, optimum values of n, p and T are calculated and then second order derivative of profit function TP with respect to n, p and T will be calculated and Hessian matrix H of second order derivatives will be find, if this matrix is positive semi definite i.e. $|H| \geq 0$ for all n, p and T then profit function is convex and give maximum value of TP for given n, p and T.

$$H(TP)_{n,\,p,\,T} = \begin{bmatrix} \dfrac{\partial^2 TP}{\partial n^2} & \dfrac{\partial^2 TP}{\partial p \partial n} & \dfrac{\partial^2 TP}{\partial T \partial n} \\ \dfrac{\partial^2 TP}{\partial n \partial p} & \dfrac{\partial^2 TP}{\partial p^2} & \dfrac{\partial^2 TP}{\partial T \partial p} \\ \dfrac{\partial^2 TP}{\partial n \partial T} & \dfrac{\partial^2 TP}{\partial p \partial T} & \dfrac{\partial^2 TP}{\partial T^2} \end{bmatrix}$$

So profit function TP is convex and giving maximum value of profit at calculated values of n, p and T.

3. Results

3.1 Numerical

$O_c = 20,\ g = 4,\ \theta' = 0.04,\ \gamma = 2,\ \tau = 0.4,\ c = 4,\ x_1 = 400,$

$x_2 = 300,\ y = 40,\ h_1 = 5,$

$h_2 = 2,\ \delta = 0.01,\ \nu = 1.25,\ \chi = 0.5,\ \lambda = 0.4,\ r = 2,\ p_1 = \frac{p}{4}.$

Optimal values calculated by above values are $n = 4,\ T = 0.4486$, $p = 8.00152$ and optimal value of profit is $TP = 18.0031$.

Table1. Sensitivity analysis of profit *TP* with respect to *T* and *n* when price *p* is constant.

p	T	n	TP
8.0015	0.4496	4	18.0246
8.0015	0.4586	4	18.1935
8.0015	0.4786	4	18.4223
8.0015	0.4986	4	18.4707
8.0015	0.5786	4	17.2886
8.0015	0.5786	4	13.7797
8.0015	0.4486	4	18.0031
8.0015	0.4486	5	16.7926
8.0015	0.4486	6	14.6344

Table 2. Sensitivity analysis of total profit TP with price p.

p	8.0025	8.0045	8.0065	8.0095	8.0135	8.0185
TP	17.522	17.517	17.512	17.503	17.493	17.477

Table 3. Sensitivity analysis of total profit by changing different values of different parameter in numerical 1.

Parameters	+10%	+20%	-10%	-20%
h_1	13.6235 (-22.2620)	9.7221 (-44.5240)	21.4264 (+22.3070)	25.3278 (+44.5240)
h_2	17.5009 (-0.1369)	17.4768 (-0.2744)	17.549 (+0.1375)	17.5731 (+0.2750)
C	0.894586 (-94.888)	15.7358 (-179.58)	34.1553 (+94.8958)	50.7857 (+189.791)
T	17.9642 (+2.506)	17.5835 (+0.3343)	15.9504 (-8.9843)	12.7677 (-27.1453)
θ’	17.3815 (-0.8182)	17.2381 (-1.6365)	17.6684 (+0.8188)	17.8118 (+1.6370)
O_c	13.0666 (-25.4398)	8.60832 (-50.877)	21.9833 (+23.7556)	26.4416 (+50.8801)

3.2. Sensitivity analysis

1. With the help of Table 1, it can be noticed that if price and cycle length are constant then with increase in number of inspection; total profit decreases. Likewise enhancement in the values of cycle length when number of inspection and price is constant results profit increases but after some values profit starts decline. Similarly with the help of table 2 it is noticed that if price is increases; total profit is decreases that are approving result in our work as we have considered price based demand so with increase in price demand should decrease and as result profit should decrease.

2. It is observed with the help of table 3 and numerical 1 that as we have increases holding cost h_1, h_2, deterioration rate θ, ordering cost and cost by 10%, 20% respectively the total profit decreases and vise versa.

3. Similarly using numerical 1, it is observed that as with enhancement in the values of total cycle length T, holding cost $h_1, h_2,$ and cost c; total profit decreases and vise versa.

4. Figure 2 to 4 are convex plot of profit function with respect to price p and cycle length T, number of inspection n and cycle length T and price p and number of inspection n, respectively. Figure 5 shows that if cycle length is constant then increase in number of inspection decreases profit. Figure 6 shows convexity of profit with respect to cycle length.

3.3. Observation

It is observed that if price and cycle length are constant then with increase in number of inspection; total profit decreases. Likewise enhancement in the values of cycle length when number of inspection and price is constant results profit increases but after some values profit starts decline. By sensitivity analysis we can understand that total cycle length T and number of inspection directly affecting the total profit. If cycle length T is constant and number of inspections is increases then it results decline in profit. Numerical values of table 1 shows that number of inspections is depend on cycle length. It should increase with cycle length T. Likewise if number of inspection is constant with other parameters and cycle length increases then profit first increase but after some values profit starts decline with increase in cycle length T. This shows that there exist only one optimal value of T and n that optimize profit. By table 3, it is observed that holding costs $h_1, h_2,$ deterioration rate θ', ordering cost O_c and purchasing cost c are inversely proportional to total profit TP. On the bases of results of numerical 1; it is noticed that enhancement in the values of holding costs $h_1, h_2,$ the deterioration rate θ', ordering cost O_c and purchasing cost c declines the total profit and vise versa. Similarly as price is increases; total profit is decreases that is approving result in our work as we considered demand as price dependent so if price increases demand should decrease (at practical point of view) and hence profit should decrease and this result is also showing same. Similarly if price is increases; total profit is decreases that are approving result in our work as we have considered price based demand so with increase in price demand should decrease and as result profit should decrease. Figure 2, figure 3 and figure 4 are convex plot of profit function with respect to price p and cycle length T, number of inspection n and cycle length T and price p and number of inspection n, respectively. It is observed by figure 5 and numerical values of table 1 that number of inspection depends on cycle length. If for a fix cycle length number of inspections increases, it decline profit. It is observed by figure 6 that profit is convex with respect to cycle length.

4. Conclusion

Regular inspection is one of effective tool to maximize profit of deteriorating inventory supply chain. It is concluded that regular inspection during each cycle reduces loss due to deterioration. It is decreasing the rate of deterioration, wastage, carbon emission that leads to profit. By regular inspection deteriorating inventories can be removed at initial stage and can be utilized to earn revenue, that increases profit. Number of inspection can be fixed up according to inventory level and nature of inventory. Regular inspection is also reducing carbon emission by using deteriorated inventories at initial stage. It is also concluded that market demand depends on price as price increase demand and profit decreases. This work concluded that regular inspection increase profit and reduce carbon emission, which fulfil the both objectives of profit maximize and emission minimize. In future, this work can be extended with distinct demands and different emission reducing policies. It can be also extended with preservation technique.

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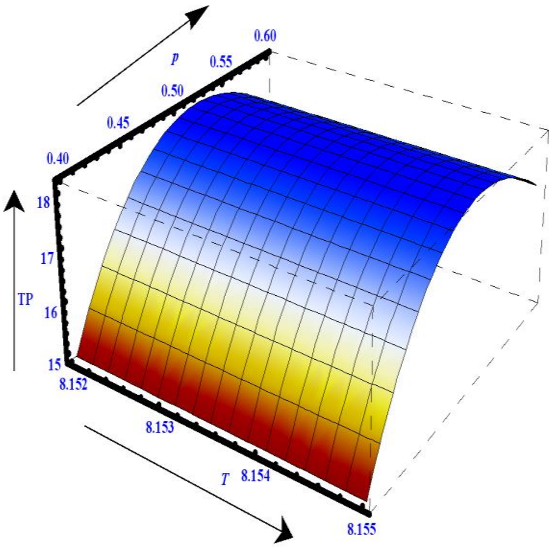

Figure 2 profit function with respect to price p and cycle length T

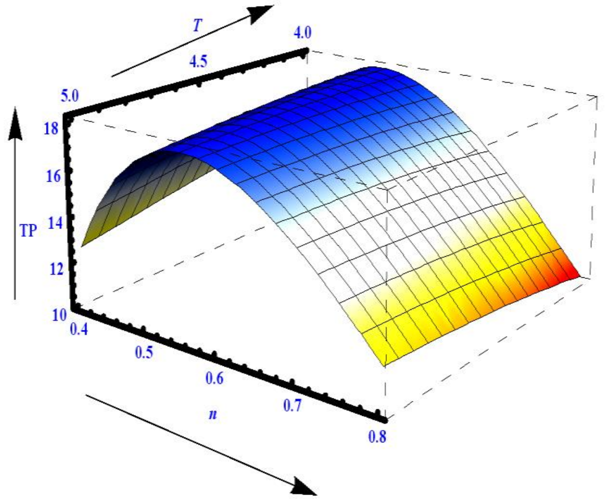

Figure 3 profit function with respect
to number of inspection n and cycle length T

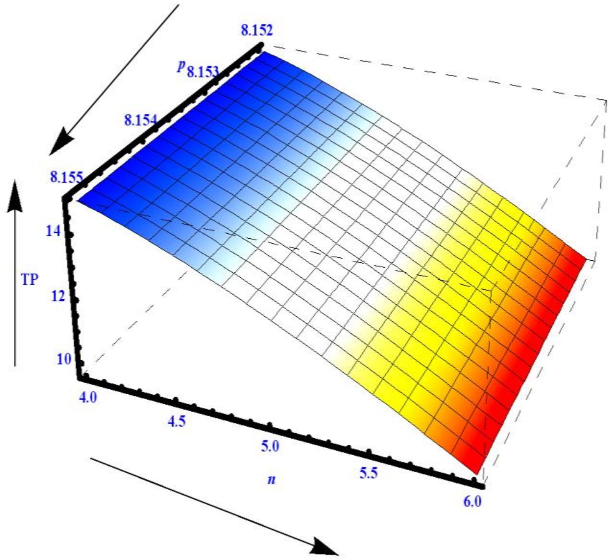

Figure 4 profit function with respect to number of inspection n and price p

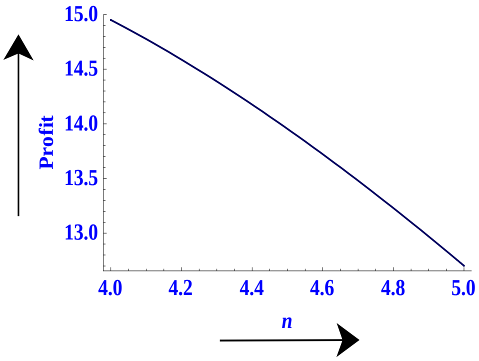

Figure 5
Change in profit with respect to *n*

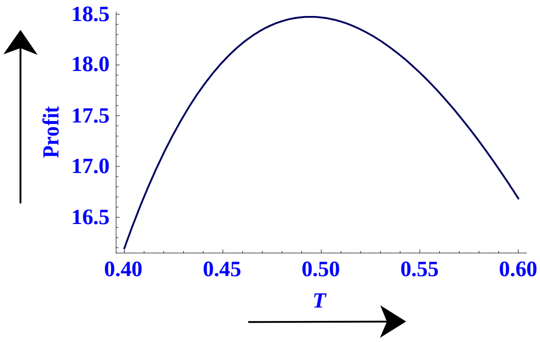

Figure 6
Change in profit with respect to *T*