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PRODUCT BINARY L-CORDIAL LABELING

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Abstract

Here, we introduced the Binary $\mathcal{L}$-Cordial labeling and Product Binary $\mathcal{L}$-Cordial labeling. We have shown that Path, $P_n \circ K_1$, Dumbell graph, Flag graph, Crown graph and some other simple graphs admit Product Binary $\mathcal{L}$-Cordial labeling.

1. Introduction

$\mathcal{L}$-Cordial labeling was first presented in [1]. In [2, 3] they discussed about the $\mathcal{L}$-Cordial labeling of standard graphs. Here, we utilize the graph $\mathcal{G}$ on p vertices and q edges which are undirected, finite and simple. A detailed review of graph labeling is given in [5]. The basic definitions needed for this assessment is given in [2]. In this work we introduced the definition of Binary $\mathcal{L}$-Cordial labeling and Product Binary $\mathcal{L}$-Cordial labeling. We have studied that some simple graphs admit Product Binary $\mathcal{L}$-Cordial labeling.

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2. Preliminaries

Definition 2.1. "Let P_n be a path graph with n vertices. The comb graph is defined as $P_n \circ K_1$. It has $2n$ vertices and $2n - 1$ edges".

Definition 2.2. "The graph obtained by joining disjoint cycles $u_1, u_2, \ldots, u_n, u_1$ and $v_1, v_2, \ldots, v_n, v_1$ with an edge u_1, v_1 is called *dumbbell graph* Db_n".

3. We Characterize another Sort of Labeling

Definition 3.1. A graph $\mathcal{G}$ with vertex set $\mathcal{V}$ and edge set $\mathcal{E}$, together with the binary operation is said to have *Binary $\mathcal{L}$-Cordial labeling* if it has a bijection $h : E(G) \to \{1, 2, \ldots, |E|\}$ and each vertex u is given the label 1 if $h(uv_i) * h(uv_j)$ is odd and 0 otherwise for any binary operation $*$, where uv_i, uv_j have the smallest and the greatest h values respectively. Now $V_h(1)$ represents the total number of u's labeled as 1 and $V_h(0)$ represents the total number of u's labeled as 0 then h is called binary $\mathcal{L}$-Cordial labeling of a graph if $|V_h(0) - V_h(1)| \leq 1$ and it is denoted by B_{LCL}. A graph is called binary $\mathcal{L}$-Cordial graph on the off chance that concedes the above labeling.

Definition 3.2. If a graph $\mathcal{G}$ has a bijection $h : E(G) \to \{1, 2, \ldots, |E|\}$ and for each vertex u, assign the label as 1 if $h(uv_i)h(uv_j)$ is odd and 0 otherwise where uv_i, uv_j have the smallest and the greatest h values. This graph is said to be Product B_{LCL} if it also satisfies the condition that $|V_h(0) - V_h(1)| \leq 1$ where $V_h(0)$ denote the number of vertices labeled with 0 and $V_h(1)$, the number of vertices labeled with 1. The graph is called Product Binary $\mathcal{L}$-Cordial graph on the off chance that admits product B_{LCL}.

4. Main Results

Theorem 4.1. *Any Path P_n admits product B_{LCL} for $n \geq 3$.*

Proof. Let us consider a Path of length n.

Let $\{v_i\}_{i=1}^{n}$ denote the vertices of the path P_n and $\{e_i\}_{i=1}^{n-1}$ denote the edges of the path P_n. When $n = 3$,

The result is obvious.

When $n > 3$,

For even n,

$$h(e_i) = 2i - 1; i = 1, 2, 3, \dots, \left\lceil \frac{n}{2} \right\rceil$$

$$h(e_{\left\lceil \frac{n}{2} \right\rceil + i}) = 2i; i = 1, 2, 3, \dots, \left\lfloor \frac{n-1}{2} \right\rfloor$$

We get, $|V_h(0)| = |V_h(1)| = \frac{n}{2}$.

For odd n,

$$h(e_i) = 2i - 1; i = 1, 2, \dots, \left\lfloor \frac{n}{2} \right\rfloor$$

$$h(e_{\left\lfloor \frac{n}{2} \right\rfloor + i}) = 2i; i = 1, 2, \dots, \left\lfloor \frac{n}{2} \right\rfloor$$

We get, $|V_h(0)| = |V_h(1)| + 1$.

Hence the path P_n admits Product B_{LCL}.

Theorem 4.2. *Odd Cycles admit Product* B_{LCL}.

Proof. Let $V = \{v_i\}_{i=1}^{n}$ denote the vertices and $E = \{e_i\}_{i=1}^{n-1}$ denote the edges of the cycle.

Define $h : E(G) \to \{1, 2, \dots, |E|\}$ as follows.

$$h(e_i) = 2i - 1; i = 1, 2, \dots, \left\lceil \frac{n}{2} \right\rceil$$

$$h(e_{\left\lceil \frac{n}{2} \right\rceil + i}) = 2i; i = 1, 2, \ldots, \frac{n-1}{2}$$

We get, $|V_h(0)| = \left\lceil \frac{n}{2} \right\rceil$

$|V_h(1)| = \left\lfloor \frac{n}{2} \right\rfloor$

Hence, $|V_h(0) - V_h(1)| \leq 1$.

Theorem 4.3. $K_{1, n}$ *satisfies Product* B_{LCL} *for even values of* n.

Proof. Label the pendant edges as $1, 2, \ldots, n$.

We get,

$|V_h(0)| = \frac{n}{2} + 1$

$|V_h(1)| = \frac{n}{2}$

Hence the theorem.

Theorem 4.4. $P_n \circ K_1$ *admits Product* B_{LCL}.

Proof. Let $\{v_i\}_{i=1}^{n}$ and $\{e_i\}_{i=1}^{n}$ denote the vertices and edges of the path P_n respectively. Let $\{u_i\}_{i=1}^{n}$ and $\{f_i\}_{i=1}^{n}$ denote the pendant vertices and pendant edges respectively.

Let us consider, $e_i = v_i v_{i+1}; i = 1, 2, \ldots, n-1$

$$f_i = v_i u_i; i = 1, 2, \ldots, n$$

Define $h : E(G) \to \{1, 2, \ldots, |E|\}$ as follows.

$h(e_i) = 2i; i = 1, 2, \ldots, n-1$

$h(f_i) = 2i - 1; i = 1, 2, \ldots, n$

We get, $|V_h(0)| = |V_h(1)| = n$.

Hence, $P_n \circ K_1$ admits Product B_{LCL}.

Example 1.

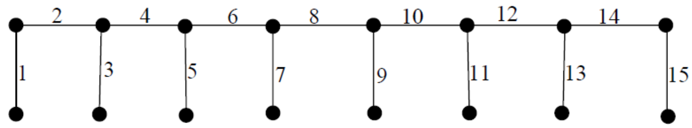

Figure 4.1. $P_8 \circ K_1$.

Theorem 4.5. *Dumbell graph* Db_n *admits Product* B_{LCL}.

Proof. Let $C_1 = (v_1, v_2, \ldots, v_n, v_1)$ denote cycle with edges $e_i = (v_i v_{i+1})$; $i = 1, 2, \ldots, n-1$ and $e_n = v_n v_1$ and $C_2 = (w_1, w_2, \ldots, w_n, w_1)$ be the cycle with edges $g_i = (w_i w_{i+1})$; $i = 1, 2, \ldots, n-1$ and $g_n = w_n w_1$.

And let $k = v_1 w_1$. Now label the edges as below,

$h(e_i) = 2i - 1; i = 1, 2, \ldots, n$

$h(g_i) = 2i; i = 1, 2, \ldots, n$

$h(k) = 2n + 1$

We get, $| V_h(0) | = | V_h(1) |$

Hence Db_n admits Product B_{LCL}.

Example 2.

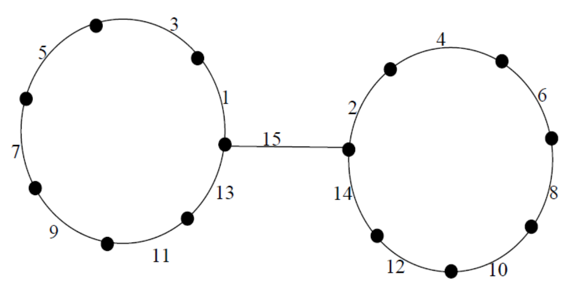

Figure 4.2. Db_7.

Theorem 4.6. *Bull graph Bull* (C_n) *on odd vertices admits Product* B_{LCL}.

Proof. Let $\{v_i / i = 1, 2, \ldots, n\}$ be the vertex set of the cycle C_n and $\{w_1, w_2\}$ be the set of pendant vertices. Let $\{e_i / i = 1, 2, \ldots, n\}$ be the edge set of the cycle C_n and $g_1 = (v_n, w_1)$ and $g_2 = (v_1, w_2)$ be the set of pendant edges.

Label the edges of the cycle as follows.

$$h(e_i) = 2i - 1; i = 1, 2, \ldots, \left\lceil \frac{n}{2} \right\rceil$$

$$h\left(e_{\left\lceil \frac{n}{2} \right\rceil + i}\right) = 2i; i = 1, 2, \ldots, \left\lfloor \frac{n}{2} \right\rfloor$$

Now, label the pendant edges as $h(g_1) = n + 1$ and $h(g_2) = n + 2$.

We get, $|V_h(1)| = |V_h(0)| + 1$.

Hence, Bull graph Bull (C_n) on odd vertices admits Product B_{LCL}.

Example 3.

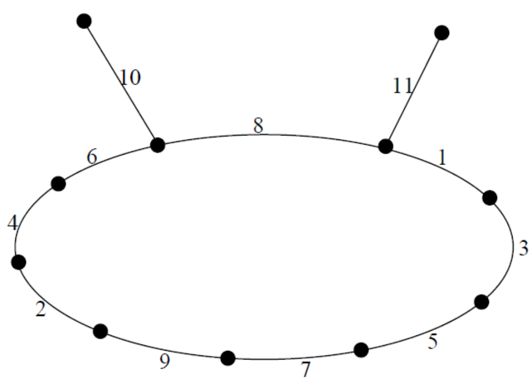

Figure 4.3. $Bull(C_9)$.

Theorem 4.7. *Flag graph* $Fl(C_n)$ *admits Product* B_{LCL}.

Proof. Let $\{v_i\}_{i=1}^{n}$ denote the vertex set and $\{e_i\}_{i=1}^{n}$ denote the edge set of the cycle C_n. Let w and e denote the pendant vertex and pendant edge respectively.

Case 1. When 'n' is odd.

$$h(v_i v_{i+1}) = 2i; i = 1, 2, \dots, \left\lceil \frac{n}{2} \right\rceil$$

$$h(v_{\left\lceil \frac{n}{2} \right\rceil + i} v_{\left\lceil \frac{n}{2} \right\rceil + i + 1}) = 2i + 1; i = 1, 2, \dots, \left\lfloor \frac{n}{2} \right\rfloor \text{ where } v_{n+1} = v_1$$

Let us label the pendant edge as $h(e) = 1$

Hence, $| V_h(0) | = | V_h(1) |$.

Case 2. When 'n' is even.

$$h(v_i v_{i+1}) = 2i - 1; i = 1, 2, \dots, \frac{n}{2} + 1$$

$$h(v_{\frac{n}{2}+1+i} v_{\frac{n}{2}+i+2}) = 2i; i = 1, 2, \dots, \frac{n}{2} - 1$$

Label the pendant edges as $h(e) = n$.

We get, $| V_h(0) | = | V_h(1) | + 1$

Hence the theorem.

Theorem 4.8. *Crown graph admits Product* B_{LCL}.

Proof. Let the cycle C_n be defined as $(v_1, c_1, v_2, c_2, \dots, v_{n-1}, c_n, v_n)$.

Let $\{w_i\}_{i=1}^{n}$ denote the pendant vertices and $\{e_i / i = 1, 2, \dots, n\}$ denote the pendant edges.

Label the pendant edges as follows.

$$h(e_i) = 2i - 1; i = 1, 2, \dots, n$$

Now, label the edges of the cycle as below.

$$h(c_i) = 2i; i = 1, 2, 3, \dots, n$$

Hence, we get $|V_h(0)| = |V_h(1)|$

Hence proved.

5. Conclusion

In the above assessment we have defined B_{LCL} and have shown some graphs admit Product B_{LCL} with needed examples.

References

[1] Mukund V. Bapat, Some vertex prime graphs and a new type of graph labeling, International Journal of Mathematics Trends and Technology (IJMTT) 47(1) (2017), 49-55.

[2] Mukund V. Bapat, New results on L-cordial labeling of graphs, International Journal of Statistics and Applied Mathematics 3(2) (2018), 641-644.

[3] Mukund V. Bapat, A note on L-cordial labeling of graphs, International Journal of Mathematics and It's Applications 5(4-D) (2017), 457-460.

[4] J. Arthy, Cycle and path related graphs on L-cordial labeling 4(6) (2019).

[5] J. A. Gallian, A dynamic survey of graph labeling, The Electronics Journal of Combinatories (2018)#DS6.

[6] S. Nila Steffi and J. Subhashini, Sum Binary L-Cordial labeling, AIP Conference Proceedings, (Communicated).