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COMMON FIXED POINT THEOREMS FOR SEQUENCE OF MAPPINGS IN FUZZY METRIC SPACES

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Abstract

We show a few typical fixed point theorems for sequences of mappings in complete fuzzy metric spaces in this study.

1. Introduction

The concept of fuzzy metric spaces introduced by Kramosil and Michalek was modified by George and Veeramani [2]. Fang proved some fixed point

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theorems in fuzzy metric spaces which improve and generalize the results of Grabiec, also unify and extend some main results of V. M. Sehgal and A. T. Bharucha-Reid [10]. We show a few typical fixed point theorems for sequences of mappings in complete fuzzy metric spaces in this study.

Definition 2.1 [9]. A function $* : [0, 1] \times [0, 1] \to [0, 1]$ is a continuous triangular norm (t-norm) if for all $a, b, c, d \in [0, 1]$ the following condition holds

(i) $*$ is associative and commutative

(ii) $*$ is continuous

(iii) $a * 1 = a$

(iv) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$.

Definition 2.2 [2]**.** The 3-tuple $(X, M, *)$ is said to be a Fuzzy metric space where X is an arbitrary set, $*$ is a continuous t-norm and M is a Fuzzy set on $X \times X \times [0, \infty]$ satisfying the following conditions, for all $u, v, w \in X, s, t.$

(i) $M(u, v, t) > 0$

(ii) $M(u, v, t) = 1 \Leftrightarrow u = v$

(iii) $M(u, v, t) = M(v, u, t)$

(iv) $M(u, v, t) * M(v, w, s) \leq M(u, w, t + s)$

(v) $M(u, v, \cdot) : (0, \infty) \to (0, 1]$ is continuous

(vi) $\lim_{n \to \infty} M(u, v, t) = 1$

Then M is called a Fuzzy metric on X. $M(u, v, t)$ denotes the degree of nearness between u and v with respect to t.

Definition 2.3 [2]**.** If $(X, M, *)$ be a fuzzy metric space. Then

(a) $\{x_n\}$ in X converges to u in X if $\lim_{n \to \infty} M(u_n, v, t) = 1$ for all $t > 0$.

(b) $\{x_n\}$ in X is a Cauchy sequence if $\lim_{n \to \infty} M(u_{n+p}, u_n, t) = 1$ for all

$t > 0$ and $p > 0$.

(c) A fuzzy metric space is complete if every Cauchy sequence converges.

Result 2.4 [3]. $M(u, v, \cdot)$ is a non-decreasing function.

Result 2.5 [8]. Let $(X, M, *)$ be a fuzzy metric space. If there exist $q \in (0, 1)$ such that

$M(u, v, qt) \geq M(u, v, t)$ for all $u, v \in X$, $q \in (0, 1)$ and $t > 0$, then $u = v$

Result 2.6 [8]. Take a sequence $\{x_n\}$ in fuzzy metric space $(X, M, *)$. If there exist a number $q \in (0, 1)$ such that

$M(u_{n+2}, u_{n+1}, qt) \geq M(u_{n+1}, u_n, t)$ for all $t > 0$ and $n \in N$.

Then $\{u_n\}$ is a Cauchy sequence in X.

3. Main Results

Theorem 3.1. *Take a complete fuzzy metric space* $(X, M, *)$ *and* $T_n : X \to X$ *a sequence of surjective functions with* $t > 0$ *and* $q > 0$ *satisfying*

$$M(T_i u, T_j v, t) \leq \min\left\{M\left(v, T_j u, \frac{t}{2q}\right) * M\left(v, T_j v, \frac{t}{2q}\right), M\left(u, v, \frac{t}{q}\right)\right\}$$

for all $i \neq j$ *and* $u, v \in X$, *then* $\{T_n\}$ *has a unique fixed point in X which is common for all* $x \in X$.

Proof of theorem 3.1. Choose x_0 in X

Since T_n is surjective there exist point $u_1 \in T_n^{-1}(u_0)$, that is $T_n u_1 = u_0$.

In this way a sequence $\{x_n\}$ is defined in X as $u_{n-1} = T_n u_n$

If $u_{n-1} = u_n$

Then $u_{n-1} = T_n u_n = u_n$

u_n is a fixed point of T_n

Suppose $u_{n-1} \neq u_n$

$M(u_{n-1}, u_n, t) = M(T_n u_n, T_{n+1} u_n, t)$

$\leq \min\left\{M\left(u_{n+1}, T_n u_n, \frac{t}{2q}\right) * M\left(u_{n+1}, T_{n+1} u_{n+1}, \frac{t}{2q}\right), M\left(u_n, u_{n+1}, \frac{t}{q}\right)\right\}$

$= \min\left\{M\left(u_{n+1}, u_{n-1}, \frac{t}{2q}\right) * M\left(u_{n+1}, u_n, \frac{t}{2q}\right), M\left(u_n, u_{n+1}, \frac{t}{q}\right)\right\}$

$= \min\left\{M\left(u_{n-1}, u_{n+1}, \frac{t}{2q}\right) * M\left(u_{n+1}, u_n, \frac{t}{2q}\right), M\left(u_n, u_{n+1}, \frac{t}{q}\right)\right\}$

$\leq \min\left\{M\left(u_{n-1}, u_n, \frac{t}{2q}\right), M\left(u_n, u_{n+1}, \frac{t}{q}\right)\right\}$

$= M\left(u_n, u_{n+1}, \frac{t}{q}\right)$

$= M(u_n, u_{n+1}, kt)$ where $k = \frac{1}{q} < 1$

Hence, $M(u_{n-1}, u_n, t) \leq M(u_n, u_{n+1}, kt)$ for all $t > 0$

Hence by Result (2.6) $\{u_n\}$ is a Cauchy sequence in X

Since X is complete, $\{u_n\}$ converges to u in X

Now

$M(T_n u, u, t) = \lim_{n\to\infty} M(T_n u, u, t)$

$= \lim_{n\to\infty} M(T_n u, T_{n+1} u_{n+1}, t)$

$\leq \lim_{n\to\infty} \min$

$\left\{M\left(u_{n+1}, T_n u, \frac{t}{2q}\right) * M\left(u_{n+1}, T_{n+1} u_{n+1}, \frac{t}{2q}\right), M\left(u, u_{n+1}, \frac{t}{q}\right)\right\}$

$= \lim_{n\to\infty} \min\left\{M\left(u_{n+1}, T_n u, \frac{t}{2q}\right) * M\left(u_{n+1}, u_n, \frac{t}{2q}\right), M\left(u, u_{n+1}, \frac{t}{q}\right)\right\}$

$= \lim_{n\to\infty} \min\left\{M\left(T_n u, u_{n+1}, \frac{t}{2q}\right) * M\left(u_{n+1}, u_n, \frac{t}{2q}\right), M\left(u, u_{n+1}, \frac{t}{q}\right)\right\}$

$$\leq \lim_{n\to\infty} \min\left\{M\left(T_n u, u_n, \frac{t}{q}\right), M\left(u, u_{n+1}, \frac{t}{q}\right)\right\}$$

$$= \min\left\{M\left(T_n u, u, \frac{t}{q}\right), M\left(u, u, \frac{t}{q}\right)\right\}$$

$$= \min\left\{M\left(T_n u, u_n, \frac{t}{q}\right), 1\right\}$$

$$= M\left(T_n u, u, \frac{t}{q}\right)$$

$$= M(T_n u, u, kt) \text{ where } k = \frac{1}{q} < 1$$

$$M(T_n u, u, t) \leq M(T_n u, u, kt)$$

Therefore by Result (2.5) $T_n u = u$ for all n

Hence u is a common fixed point of $\{T_n\}$ for all n.

Uniqueness:

Let $v \neq u$ be another common fixed point of $\{T_n\}$

Then

$$M(u, v, t) = M(T_n u, T_n v, t)$$

$$\leq \min\left\{M\left(v, T_n u, \frac{t}{2q}\right) * M\left(v, T_n v, \frac{t}{2q}\right), M\left(u, v, \frac{t}{q}\right)\right\}$$

$$= \min\left\{M\left(v, u, \frac{t}{2q}\right) * M\left(v, v, \frac{t}{2q}\right), M\left(u, v, \frac{t}{q}\right)\right\}$$

$$= \min\left\{M\left(u, v, \frac{t}{2q}\right) * M\left(v, v, \frac{t}{2q}\right), M\left(u, v, \frac{t}{q}\right)\right\}$$

$$= \min\left\{M\left(u, v, \frac{t}{q}\right) * M\left(u, v, \frac{t}{q}\right)\right\}$$

$$= M\left(u, v, \frac{t}{q}\right)$$

$= M(u, v, kt)$ where $k = \frac{1}{q} < 1$

$M(u, v, t) \leq M(u, v, kt)$

Therefore by Remark (2.5) $u = v$.

Theorem 3.2. *Take a complete fuzzy metric space* $(X, M, *)$ *and* $T_n : X \to X$ *a sequence of surjective functions with* $t > 0$ *and* $q > 0$ *satisfying*

$$M(T_i u, T_j v, t) \leq \min$$

$$\left\{M\left(v, T_i u, \frac{t}{2q}\right) * M\left(v, T_j v, \frac{t}{2q}\right), M\left(v, T_i u, \frac{t}{2q}\right), M\left(v, T_j v, \frac{t}{2q}\right) M\left(u, v, \frac{t}{q}\right)\right\}$$

for all $i \neq j$ *and* $u, v \in X$, *then* $\{T_n\}$ *has a unique fixed point in X which is common for all* $x \in X$.

Proof of theorem 3.2. Proof is similar

Theorem 3.3. *Take a complete fuzzy metric space* $(X, M, *)$ *and* $T_n : X \to X$ *a sequence of surjective functions with* $t > 0$ *and* $q > 0$ *satisfying*

$$M(T_i u, T_j v, t) \leq \min$$

$$\left\{M\left(v, T_i u, \frac{t}{2q}\right) * M\left(v, T_j v, \frac{t}{2q}\right), M\left(v, T_i u, \frac{t}{2q}\right), M\left(v, T_j v, \frac{t}{2q}\right) M\left(u, v, \frac{t}{q}\right)\right\}$$

for all $i \neq j$ *and* $u, v \in X$, *then* $\{T_n\}$ *has a unique fixed point in X which is common for all* $x \in X$.

Proof of theorem 3.3. Proof is similar

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