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INTUITIONISTIC *I*-FUZZY SPANNING SUPERGRAPHS

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Abstract

In this paper, depth of intuitionistic *I*-fuzzyedge, height of intuitionistic *I*-fuzzyedge and intuitionistic *I*-fuzzyspanning supergraph are introduced. Some definitions, properties and theorems are given. Here intuitionistic *I*-fuzzygraph means intuitionistic interval valued fuzzy graph.

Introduction

In 1965, Zadeh [14] introduced the notation of fuzzy set as a method of presenting uncertainty. Since complete information in science and technology is not always available. Thus we need mathematical models to handle various

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types of systems containing elements of uncertainty. Intuitionistic fuzzy set was introduced by K. T. Atanassov [4]. After that Rosenfeld [8] introduced fuzzy graphs. Yeh and Bang [13] also introduced fuzzy graphs independently. Fuzzy graphs are useful to represent relationships which deal with uncertainty and it differs greatly from classical graph. It has numerous applications to problems in computer science, electrical engineering system analysis, operations research, economics, networking routing, transportation, P. V. Ramakrishnan etc. and T. Lakshmi [7] introduced depth of, height of and fuzzy spanning super graphs. Arjunan. K and Subramani. C [2, 3] introduced a new structure of fuzzy graph and *I*-Fuzzy graph. *I*-fuzzy spanning supergraphs and intuitionistic fuzzy spanning supergraphs have been defined and introduced by Vasudevan. B et al. [11, 12]. In this paper, intuitionistic *I*-fuzzy spanning subgraph is defined and introduced.

1.Preliminaries

Definition 1.1 [14]**.** Let X be any nonempty set. A mapping $M : X \to [0, 1]$ is called a fuzzy subset of X.

Definition 1.2 [14]. Let X be any nonempty set. A mapping $[M] : X \to D[0, 1]$ is called a *I*-fuzzy subset (interval valued fuzzy subset) of X, where $D[0, 1]$ denotes the family of all closed subintervals of [0, 1] and $[M](x) = [M^{-1}(x), M^{-1}(x)]$, for all x in X, where M^{-} and M^{+} are fuzzy subsets of X such that $M^{-}(x) \leq M^{-}(x)$, for all x in X. Thus $M^{-}(x)$ is an interval (a closed subset of [0, 1]) and not a number from the interval [0, 1] as in the case of fuzzy subset.

Definition 1.3 [4]**.** An intuitionistic *I*-fuzzy subset (IIFS) $[A]$ in X is defined as an object of the form $[A] = \{\langle x, \mu_{[A]}(x), \upsilon_{[A]}(x)\rangle$ $= \langle x, [\mu_{[A]^{-}}(x), \mu_{[A]^{+}}(x)], [\mu_{[A]^{-}}(x), \mu_{[A]^{+}}(x)]\rangle / x \text{ in } X\}$ where $\mu_{[A]} : X \to D[0, 1]$ and $\upsilon_{[A]} : X \to D[0, 1]$ define the degree of membership and the degree of non-membership of the element $x \in X$ respectively and for every $x \in X$ satisfying $\mu_{[A]^{+}}(x) + \upsilon_{[A]^{+}}(x) \leq 1$.

Example 1.4. $[A] = \{\langle a, [0.4, 0.7], [0.2, 0.5], [0.2, 0.3]\rangle, \langle b, [0.1, 0.5], [0.2, 0.5]\rangle,$

$\langle c, [0.5, 0.8], [0.1, 0.2]\rangle\}$ is an intuitionistic *I*-fuzzy subset of $X = \{a, b, c\}$.

Definition 1.5 [4]. Let $[A] = \{\langle x, \mu_{[A]}(x), \upsilon_{[A]}(x)\rangle / x \in X\}$, $[B] = \{\langle x, \mu_{[B]}(x)\rangle / x \in X\}$ be any two intuitionistic *I*-fuzzy subsets of *X*. We define the following relations and operations:

(i) $[A] \subseteq [B]$ if and only if $\mu_{[A]}(x) \leq \mu_{[B]}(x)$ and $\upsilon_{[A]}(x) \leq \upsilon_{[B]}(x)$ for all *x* in *X*.

(ii) $[A] = [B]$ if and only if $\mu_{[A]}(x) = \mu_{[B]}(x)$ and $\upsilon_{[A]}(x) = \upsilon_{[B]}(x)$ for all *x* in *X*.

(iii) $[A] \cap [B] = \{\langle x, r\min\{\mu_{[A]}(x), \mu_{[B]}(x)\}, r\max\{\upsilon_{[A]}(x), \upsilon_{[B]}(x)\}\rangle / x \in X\}$ where $r\min\{\langle\{\mu_{[A]}(x), \mu_{[B]}(x)\} = [\min\{\mu_{[A]^-}(x), \mu_{[B]^-}(x)\}, \min\{\mu_{[A]^+}(x), \mu_{[B]^+}(x)\}]$ and $r\max\{\langle\{\upsilon_{[A]}(x), \upsilon_{[B]}(x)\} = [\max\{\upsilon_{[A]^-}(x), \upsilon_{[B]^-}(x)\}, \max\{\upsilon_{[A]^+}(x), \upsilon_{[B]^+}(x)\}]$.

(iv) $[A] \cup [B] = \{\langle x, r\max\{\mu_{[A]}(x), \mu_{[B]}(x)\}, rm\min\{\upsilon_{[A]}(x), \upsilon_{[B]}(x)\}\rangle / x \in X\}$ where $r\max\{\langle\{\mu_{[A]}(x), \mu_{[B]}(x)\} = [\max\{\mu_{[A]^-}(x), \mu_{[B]^-}(x)\}, \max\{\mu_{[A]^+}(x), \mu_{[B]^+}(x)\}]$ and $r\min\{\langle\{\upsilon_{[A]}(x), \upsilon_{[B]}(x)\} = [\min\{\upsilon_{[A]^-}(x), \upsilon_{[B]^-}(x)\}, \min\{\upsilon_{[A]^+}(x), \upsilon_{[B]^+}(x)\}]$.

(v) $[A]^c = \{\langle\{x, \upsilon\mu_{[A]}(x), \mu_{[B]}(x)\}\rangle / x \in X\}$.

Definition 1.6. Let $[M] = \langle\mu_{[M]}, \upsilon_{[M]}\rangle$ be an intuitionistic I-fuzzy subset in a set *S*, the strongest intuitionistic *I*-fuzzy relation on *S*, that is an intuitionistic *I*-fuzzy relation $[V] = \langle\mu_{[V]}, \upsilon_{[V]}\rangle$ with respect to [*M*] given by $\mu_{[V]}(x, y) = r\min\{\mu_{[M]}(x), \mu_{[M]}(y)\}$ and $\upsilon_{[V]}(x, y) = r\max\{\upsilon_{[M]}(x), \upsilon_{[M]}(y)\}$ for all *x* and *y* in *S*.

Definition 1.7. Let *V* be any nonempty set, *E* be any set and $f: E \to V \times V$ be any function. Then $[A] = \langle\mu_{[A]}, \upsilon_{[A]}\rangle$ is an interval-valued intuitionistic subset of *V*, $[S] = \langle\mu_{[S]}, \upsilon_{[S]}\rangle$ is an intuitionistic *I*-fuzzy relation on *V* with respect to [*A*] and $[B] = \langle\mu_{[B]}, \upsilon_{[B]}\rangle$ is an intuitionistic *I*-

fuzzy subset of E such that $\mu_{[B]}(e) \leq \mu^{[S]}_{e \in f^{-1}(x,\, y)}(x,\, y)$ and $\upsilon_{[B]}(e) \geq \upsilon^{[S]}_{e \in f^{-1}(x,\, y)}(x,\, y)$. Then the ordered triple $[F] = ([A], [B], f)$ is called an intuitionistic I-fuzzy graph, where the elements of $[A]$ are called intuitionistic I-fuzzy points or intuitionistic I-fuzzy vertices and the elements of $[B]$ are called intuitionistic I-fuzzy lines or intuitionistic I-fuzzy edges of the Intuitionistic I-fuzzy graph $[F]$. If $f(e) = (x,\, y)$, then the Intuitionistic I-fuzzy points $(x,\, \mu_{[A]}(x),\, \upsilon_{[A]}(x)),\, (y,\, \mu_{[A]}(y),\, \upsilon_{[A]}(y))$ are called intuitionistic I-fuzzy adjacent points and intuitionistic I-fuzzy points $(x,\, \mu_{[A]}(x),\, \upsilon_{[A]}(x))$, intuitionistic I-fuzzy line $(e,\, \mu_{[B]}(e),\, \upsilon_{[B]}(e))$ are called incident with each other. If two district intuitionistic I-fuzzy lines $(e_1,\, \mu_{[B]}(e_1),\, \upsilon_{[B]}(e_1))$ and $(e_2,\, \mu_{[B]}(e_2),\, \upsilon_{[B]}(e_2))$ are incident with a common intuitionistic I-fuzzy point, then they are called intuitionistic I-fuzzy adjacent lines.

Definition 1.8. An intuitionistic I-fuzzy line joining an intuitionistic I-fuzzy point to itself is called an intuitionistic I-fuzzy loop.

Definition 1.9. Let $[F] = ([A], [B], f)$ be an intuitionistic I-fuzzy graph. If more than one intuitionistic I-fuzzy line joining two intuitionistic I-fuzzy vertices is allowed, then the intuitionistic I-fuzzy graph $[F]$ is called an intuitionistic I-fuzzy pseudo graph.

Definition 1.10. $[F] = ([A], [B], f)$ is called an intuitionistic I-fuzzy simple graph if it has neither intuitionistic I-fuzzy multiple lines nor intuitionistic I-fuzzy loops.

Example 1.11. $[F] = ([A], [B], f)$, where $V = \{v_1, v_2, v_3, v_4, v_5\}$, $E = \{a, b, c, d, e, h, g\}$ and $f : E \to V \times V$ is defined by $f(a) = (v_1, v_2)$, $f(b) = (v_2, v_2)$, $f(c) = (v_2, v_3)$, $f(d) = (v_3, v_4)$, $f(e) = (v_3, v_4)$, $f(h) = (v_4, v_5)$, $f(g) = (v_1, v_5)$. An intuitionistic I-fuzzy subset $[S] = \{((v_1, v_1), [0.5, 0.7], [0.2, 0.3]), (v_2, [0.4, 0.6], [0.1, 0.3]), (v_3, [0.4, 0.8], [0.2, 0.2]), (v_4, [0.3, 0.5], [0.2, 0.3]), (v_5, [0.3, 0.7], [0.2, 0.2])\}$ of V. An intuitionistic I-fuzzy relation $[S] = \{((v_1, v_1), [0.5, 0.7], [0.2, 0.3]), ((v_1, v_2), [0.4, 0.6], [0.2, 0.3]), ((v_1, v_3), [0.4, 0.7], [0.2, 0.3]), ((v_1, v_4), [0.3, 0.5], [0.2, 0.3]),$

$((v_1, v_5)$, [0., 0.7], [0.2, 0.3]) $((v_2, v_1)$, [0.4, 0.6], [0.2, 0.3]), $((v_2, v_2)$, [0.4, 0.6], [0.1, 0.3]), $((v_2, v_3)$, [0.4, 0.6], [0.2, 0.3]), $((v_2, v_4)$, [0.3, 0.5], [0.2, 0.3]), $((v_2, v_5)$, [0.3, 0.6], [0.2, 0.3]), $((v_2, v_1)$, [0.4, 0.6], [0.2, 0.3]), $((v_2, v_2)$, [0.4, 0.6], [0.1, 0.3]), $((v_2, v_3)$, [0.4, 0.6], [0.2, 0.3]), $((v_2, v_4)$, [0.3, 0.5], [0.2, 0.3]), $((v_2, v_5)$, [0.3, 0.6], [0.2, 0.3]), $((v_3, v_1)$, [0.4, 0.7], [0.2, 0.3]), $((v_3, v_2)$, [0.4, 0.6], [0.2, 0.3]), $((v_3, v_3)$, [0.4, 0.8], [0.2, 0.2]), $((v_3, v_4)$, [0.3, 0.5], [0.2, 0.3]), $((v_3, v_5)$, [0.3, 0.7], [0.2, 0.2]), $((v_4, v_1)$, [0.3, 0.5], [0.2, 0.3]), $((v_4, v_2)$, [0.3, 0.5], [0.2, 0.3]), $((v_4, v_3)$, [0.3, 0.5], [0.2, 0.3]), $((v_4, v_4)$, [0.3, 0.5], [0.2, 0.3]), $((v_4, v_5)$, [0.3, 0.5], [0.2, 0.3]), $((v_5, v_1)$, [0.3, 0.7], [0.2, 0.3]), $((v_5, v_2)$, [0.3, 0.6], [0.2, 0.3]), $((v_5, v_3)$, [0.3, 0.7], [0.2, 0.2]), $((v_5, v_4)$, [0.3, 0.5], [0.2, 0.3]), $((v_5, v_5)$, [0.3, 0.7], [0.2, 0.3])} on V with respect to $[A]$ and an intuitionistic I-fuzzy subset $[B]$ = {(a, [0.4, 0.5], [0.2, 0.4]), (b, [0.3, 0.5], [0.2, 0.3]), (c, [0.4, 0.6], [0.2, 0.4]), (d, [0.2, 0.5], [0.3, 0.4]), (e, [0.3, 0.5], [0.2, 0.3]), (h, [0.3, 0.6], [0.3, 0.4]), (g, [0.3, 0.6], [0.2, 0.4])} of E.

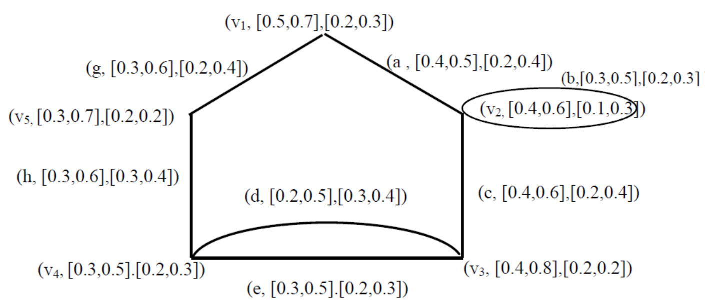

Figure 1.1.

In figure 1.1, (i) (v_1, [0.5, 0.7], [0.2, 0.3]) is an intuitionistic I-fuzzy point. (ii) (a, [0.4, 0.5], [0.2, 0.4])) is an intuitionistic I-fuzzy edge. (iii) (v_1, [0.5, 0.7], [0.2, 0.3]) and (v_2, [0.4, 0.6], [0.1, 0.3]) are intuitionistic I-fuzzy adjacent points. (iv) (a, [0.4, 0.5], [0.2, 0.4]) join with (v_1, [0.5, 0.7], [0.2, 0.3]) and (v_2, [0.4, 0.6], [0.1, 0.3]) and therefore it is incident with (v_1, [0.5, 0.7], [0.2, 0.3]) and (v_2, [0.4, 0.6], [0.1, 0.3]). (v) (a, [0.4, 0.5,], [0.2, 0.4]) and (g, [0.3, 0.6], [0.2, 0.4]) are intuitionistic I-fuzzy adjacent lines. (vi) (b, [0.3, 0.5], [0.2, 0.3]) is an intuitionistic I-fuzzy loop. (vii) (d, [0.2, 0.5], [0.3, 0.4]) and (e, [0.3, 0.5], [0.2, 0.3]) are intuitionistic I-fuzzy multiple edges. (viii) It is not an intuitionistic I-fuzzy simple graph. (ix) It is an intuitionistic I-fuzzy pseudo graph.

Definition 1.12. The fuzzy graph $[F] = ([A], [B], f)$ where $[C] = \langle \mu_{[C]}, \upsilon_{[C]} \rangle$ and $[D] = \langle \mu_{[D]}, \upsilon_{[D]} \rangle$ is called an intuitionistic I-fuzzy subgraph of $[F] = ([A], [B], f)$ if $[C] \subseteq [A]$ and $[D] \subseteq [B]$.

Definition 1.13. The intuitionistic I-fuzzy subgraph $[H] = ([C], [D], f)$ is said to be an intuitionistic I-fuzzy spanning subgraph of $[F] = ([A], [B], f)$ if $[C] = [A]$.

Definition 1.14. Let $[F] = ([A], [B], f)$ be an intuitionistic I-fuzzy graph. Then the degree of an intuitionistic I-fuzzy vertex is defined by $d(\upsilon) = (d_\mu(\upsilon), d_\upsilon(\upsilon))$ where $d_\mu(\upsilon) = \sum_{e \in f^{-1}(u, y)} \mu_{[B]}(e) + 2\sum_{e \in f^{-1}(\upsilon, \upsilon)} \mu_{[B]}(e)$ and $d_\upsilon(\upsilon) = \sum_{e \in f^{-1}(u, \upsilon)} \mu_{[B]}(e) + 2\sum_{e \in f^{-1}(\upsilon, \upsilon)} \upsilon_{[B]}(e)$.

Definition 1.15. Let $[F] = ([A], [B], f)$ be an intuitionistic I-fuzzy graph. The total degree of intuitionistic I-fuzzy vertex υ is defined by $d_T(\upsilon) = (d_{T_\mu}(\upsilon), d_{T_\upsilon}(\upsilon))$

where

$$d_{T_\mu}(\upsilon) = \sum_{e \in f^{-1}(u, \upsilon)} \mu_{[B]}(e) + 2\sum_{e \in f^{-1}(\upsilon, \upsilon)} \mu_{[B]}(e) + \mu_{[A]}(\upsilon) = d_\mu(\upsilon) + \mu_{[A]}(\upsilon)$$

and

$$d_{T_\mu}(\upsilon) = \sum_{e \in f^{-1}(u, \upsilon)} \upsilon_{[B]}(e) + 2\sum_{e \in f^{-1}(\upsilon, \upsilon)} \upsilon_{[B]}(e) + \upsilon_{[A]}(\upsilon) = d_\upsilon(\upsilon) + \upsilon_{[A]}(\upsilon)$$

for all υ in V.

Definition 1.16. The minimum degree of the intuitionistic I-fuzzy graph $[F] = ([A], [B], f)$ is $\delta[F] = (\delta_\mu[F], \delta_\upsilon[F])$ where $\delta_\mu[F] = r \min \{d_\mu(\upsilon)/\upsilon \in V\}$ and $\delta_\upsilon[F] = r \max \{d_\upsilon(\upsilon)/\upsilon \in V\}$ and the maximum degree of $[F]$ is $\Delta[F] = (\Delta_\mu[F], \Delta_\upsilon[F])$ where $\Delta_\mu[F] = r\max \{d_\mu(\upsilon)/\upsilon \in V\}$ and $\Delta_\mu[F] = r \min \{d_\upsilon(\upsilon)/\upsilon \in V\}$.

Definition 1.17. Let $[F] = ([A], [B], f)$ be an intuitionistic I-fuzzy graph. Then the order of intuitionistic I-fuzzy graph $[F]$ is defined to be $O[F] = (O_\mu[F], O_\upsilon[F])$ where $O_\mu[F] = \sum_{\upsilon \in V} \mu_{[A]}(\upsilon)$ and $O_\mu[F] = \sum_{\upsilon \in V} \upsilon_{[A]}(\upsilon)$.

Definition 1.18. Let $[F] = ([A], [B], f)$ be an intuitionistic I-fuzzy graph. Then the size of the intuitionistic I-fuzzy graph $[F]$ is defined to be $S[F] = (S_\mu[F], S_\upsilon[F])$ where $S_\mu[F] = \sum_{e \in f^{-1}(x, y)} \mu_{[B]}(e)$ and $S_\upsilon[F] = \sum_{e \in f^{-1}(x, y)} \upsilon_{[B]}(e)$.

Definition 1.19. Let $[F] = ([A], [B], f)$ be an intuitionistic I-fuzzy graph. Then the depth of intuitionistic I-fuzzy edge $[B]$ is defined by $D([B]) = (\mu_{[d]}([B]), \upsilon_{[d]}([B])) = (r \min \{\mu_{[B]}(e)/e \in E)\}, r \max \{\upsilon_{[B]}(e)/e \in E)\})$.

Definition 1.20. Let $[F] = ([A], [B], f)$ be an intuitionistic I-fuzzy graph. Then the height of intuitionistic I-fuzzy edge $[B]$ is defined by $H([B]) = (\mu_{[d]}([B]), \upsilon_{[d]}([B])) = (r \max \{\mu_{[B]}(e)/e \in E)\}, r \min \{\upsilon_{[B]}(e)/e \in E)\})$.

Example 1.21.

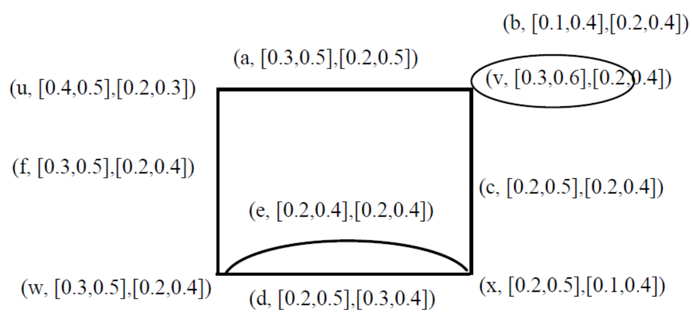

Figure 1.2. intuitionistic I-fuzzy graph $[F]$.

Here $D(B) = ([0.1, 0.4], [0.3, 0.5])$ and $H(B) = ([0.3, 0.5], 0.2, 0.4)$.

Remark 1.22. Clearly $D([B]) \leq [B](e) \leq H([B])$.

2. Intuitionistic *I*-fuzzy Spanning Supergraphs

Definition 2.1. An intutionistic *I*-fuzzy graph $[F] = ([A], [B], f)$ is called a intuitionistic *I*-fuzzy $([k_1], [k_2])$- regular graph if $d(v) = ([k_1], [k_2])$ for all v in *V*.

Definition 2.2. An intuitionistic *I*-fuzzy graph $[F] = ([A], [B], f)$ is called a intuitionistic *I*-fuzzy complete graph if every pair of distinct intuitionistic *I*-fuzzy vertices are intuitionistic *I*-fuzzy adjacent and $\mu_{[B]}(e) = \mu_{[S]_{e \in f^{-1}(x, y)}}(x, y)$ and $\gamma_{[B]}(e) = \gamma_{[S]_{e \in f^{-1}(x, y)}}(x, y)$ for all (x, y) in *V*.

Definition 2.3. An intuitionistic *I*-fuzzy graph $[F] = ([A], [B], f)$ is a intuitionistic *I*-fuzzy strong graph if $\mu_{[B]}(e) = \mu_{[S]_{e \in f^{-1}(x, y)}}(x, y)$ and $\gamma_{[B]}(e) = \gamma_{[S]_{e \in f^{-1}(x, y)}}(x, y)$ for all x, y in *V*.

Definition 2.4. An intuitionistic *I*-fuzzy graph [*F*] is intuitionistic *I*-fuzzy $[k] = ([k_1], [k_2])$- totally regular graph if each vertex of [*F*] has the same total degree $[k] = ([k_1], [k_2])$.

Definition 2.5. Let $[F] = ([A], [B], f)$ be an intuitionistic *I*-fuzzy simple graph, E^1 be an extension set of E and $g : E^1 \to V \times V$ be an extension function of *f*. An intuitionistic fuzzy spanning supergraph $[F'] = ([A], [B'], g)$ of [*F*] is defined as

$$\mu_{[B']}(e) = \begin{cases} \mu_{[B]}(e) & \text{if } e \in f^{-1}(u, v) \\ \mu_{[A]}(u) \cap \mu_{[A]}(v) & \text{if } e \in (g^{-1}(u, v) - f^{-1}(u, v)) \end{cases}$$

$$\gamma_{[B']}(e) = \begin{cases} \gamma_{[B]}(e) & \text{if } e \in f^{-1}(u, v) \\ \gamma_{[A]}(u) \cup \gamma_{[A]}(v) & \text{if } e \in (g^{-1}(u, v) - f^{-1}(u, v)) \end{cases}$$

for all *e* in E^1.

Example 2.6.

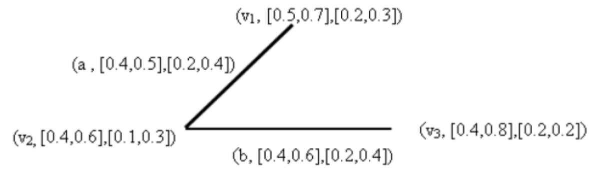

Figure 2.1. A Intuitionistic fuzzy graph.

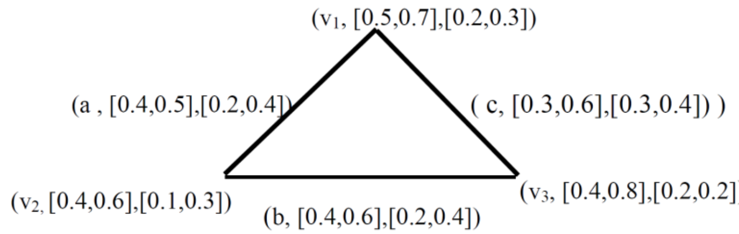

Figure 2.2. A Intuitionistic fuzzy spanning supergraph.

Theorem 2.7. *Let* $[F] = ([A], [B], f)$ *be an intuitionistic I-fuzzy strong graph with p-intuitionistic fuzzy vertices. Then* $[F'] = ([A], [B'], g)$ *is an intuitionistic I-fuzzy complete graph.*

Proof. Assume that $[F] = ([A], [B], f)$ be an intuitionistic *I*-fuzzy strong graph with *p*-intuitionistic fuzzy vertices. That is $\mu_{[B]}(e) = \mu_{[S]_{s \in f^{-1}(x,\, y)}}(x, y)$ and $\gamma_{[B]}(e) = \gamma_{[S]_{s \in f^{-1}(x,\, y)}}(x, y)$ for all x, y in V, implies that $\mu_{[B']}(e) = \mu_{[A]}(x) \cap \mu_{[A]}(y)$ and $\gamma_{[B']}(e) = \gamma_{[A]}(x) \cup \gamma_{[A]}(y)$ for all x and y in $V(F')$. Then $\mu_{[B']}(e)$ is $\mu_{[B']}(e) = \mu_{[S]_{s \in f^{-1}(x,\, y)}}(x, y)$ and $\gamma_{[B]}(e) = \gamma_{[S]_{s \in f^{-1}(x,\, y)}}(x, y)$ for all x, y in $V([F'])$.

Therefore $[F] = ([A], [B], f)$ is an intuitionistic *I*-fuzzy complete graph.

Theorem 2.8. *Let* $[F] = ([A], [B], f)$ *be an intuitionistic I-fuzzy connected graph.* $[F]$ *is intuitionistic I-fuzzy strong graph if and only if* $[F']$ *is intuitionistic I-fuzzy complete graph.*

Proof. The proof follows from the definitions of intuitionistic *I*-fuzzy strong graph and intuitionistic *I*-fuzzy complete graph.

Proposition 2.9. *If [F] is an intuitionistic I-fuzzy strong graph*. Then $[F']$ *need not be intuitionistic I-fuzzy regular graph*.

Proof. The proof follows from the definitions of *I*-intuitionistic fuzzy strong graph and intuitionistic *I*-fuzzy spanning super graph.

Theorem 2.10. *Let* $[F] = ([A], [B], f)$ *be an intuitionistic I-fuzzy strong graph with p-intuitionistic I-fuzzy vertices and* $[A]$ *be a* $[k]$*- constant function*. *Then* $[F'] = ([A], [B'], g)$ *is a intuitionistic I-fuzzy* $(p-1)[k]$ *regular graph*.

Proof. Assume that $[F] = ([A], [B], f)$ is an intuitionistic *I*-fuzzy strong graph with *p*-intuitionistic *I*-fuzzy vertices. By Theorem 2.7, is intuitionistic *I*-fuzzy complete graph and also $[A](v) = [k]$ for all v in $V([F'])$ That is $\mu_{[s']}(e)$ is $\mu_{[B]}(e) = \mu_{[S]_{e \in f^{-1}(x,\, y)}}(x,\, y)$ and $\gamma_{[B]}(e) = \gamma_{[S]_{e \in f^{-1}(x,\, y)}}(x,\, y)$ for all x and y in V. Then $\mu_{[B']}(e) = \mu_{[A]}(x) \cap \mu_{[A]}(y)$ and $\gamma_{[B']}(e) = \gamma_{[A]}(x) \cap \mu_{[A]}(y) = [k]$ for all x and y in V. Therefore $d(v) = (p-1)[k]$ for all v in $V([F'])$. Hence $[F']$ is an intuitionistic *I*-fuzzy $(p-1)[k]$- regular graph.

Theorem 2.11. *Let* $[F] = ([A], [B], f)$ *be an intuitionistic I-fuzzy strong graph with p-intuitionistic I-fuzzy vertices and* $[A]$ *be a* $[k]$*- constant function*. *Then* $S([F']) = \dfrac{p(p-1)}{2}[k]$.

Proof. By Theorem 2.10, $d(v) = (p-1)[k]$ for all v in $V([F'])$. Then $\sum_{v \in V} d(v) = \sum_{v \in V} (p-1)[k]$ which implies that $2S([F']) = p(p-1)[k] \Rightarrow$ $S'([F']) = \dfrac{p(p-1)}{2}[k]$.

Proposition 2.12. *If* $[F] = ([A], [B], f)$ *is an intuitionistic I-fuzzy complete graph*. *Then* $[F']$ *need not be intuitionistic I-fuzzy regular graph*.

Theorem 2.13. *Let* $[F] = ([A], [B], f)$ *be an intuitionistic I-fuzzy complete graph with p-intuitionistic I-fuzzy vertices and* $[A]$ *be a* $[k]$*- constant function*.

Then $[F] = ([A], [B'], g)$ is an intuitionistic *I*-fuzzy $(p-1)[k]$ regular graph.

Theorem 2.14. *Let* $[F] = ([A], [B], f)$ *be an intuitionistic I-fuzzy complete graph with p-intuitionistic I-fuzzy vertices and* $[A]$ *be a* $[k]$*- constant function.*

Then $S([F']) = \frac{p(p-1)}{2}[k].$

Remark 2.15. If $[F]$ is an intuitionistic *I*-fuzzy complete graph. Then $[F'] = [F]$. But converse need not be true.

Theorem 2.16. *Let* $[F] = ([A], [B], f)$ *be an intuitionistic I-fuzzy strong graph with p-intuitionistic I-fuzzy vertices and* $[A]$ *be a* $[k]$*- constant function. Then* $[F']$ *is an intuitionistic I-fuzzy* $[k]$*- totally regular graph.*

Proof. By Theorem 2.10, $d(v) = (p-1)[k]$ for all v in $V([F'])$. Then $d_T(v) = dv + [A](v) = (p-1)[k] + [k] = p[k] - [k] + [k] = p[k]$ for all v in $V([F'])$. Hence $[F']$ is an intuitionistic *I*-fuzzy $p[k]$- totally regular graph.

Theorem 2.17. *If* $[F]$ *is an intuitionistic I-fuzzy strong graph with p-intuitionistic I-fuzzy vertices. Then* $S([F']) = \frac{p^2[k] - o([F'])}{2}$.

Proof. By Theorem 2.16, then $d_T(v) = p[k]$ for all in $V([F']) \Rightarrow d(v) + [A](v) = p\,[k] \Rightarrow \sum_{v \in V} d(v) + \sum_{v \in V}[A](v) = p[k] \Rightarrow 2S([F']) = p^2[k].$

Hence $S([F']) = \frac{p^2[k] - o([p'])}{2}$.

Theorem 2.18. *If* $[F] = ([A], [B], f)$ *is an intuitionistic I-fuzzy strong graph and* $[A]$ *be a* $[k]$*- constant function. Then* $o([F']) = p[k]$.

Proof. By Theorems 2.10 and 2.16, is both intuitionistic *I*-fuzzy $(p-1)[k]$ regular graph and intuitionistic *I*-fuzzy $[k]$- totally regular graph.

By Theorems 2.11 and 2.17, $S([F']) = p(p-1)[k]$ and $2S([F']) = p^2[k] - o([F'])$.

Then $p(p-1)[k] = p^2[k] - o([F'])$ which implies that $o([F']) = p^2[k] - p(p-1)[k] = p^2[k] - p^2[k] + p[k] = p[k]$.

Theorem 2.19. *Let* $[F] = ([A], [B], f)$ *be an intuitionistic I-fuzzy strong graph with p-intuitionistic I-fuzzy vertices and* $[A]$ *be a* $[k]$*- constant function. Then sum of the total degree of all intuitionistic I*-fuzzy *vertices in an intuitionistic I-fuzzy spanning super graph* $[F']$ *is* $p^2[k]$.

Proof. By Theorem 2.10, $d(v) = (p-1)[k]$ for all v in $V([F'])$. We have $[F] = ([A], [B], f)$ for all v in $\sum_{v \in V} d_T(v) = \sum_{v \in V} d(v) + \sum_{v \in V} [A](v) = p(p-1)[k]$ $+ o([F']) = p(p-1)[k] + p[k] = p^2[k]$.

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