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STRESSES IN A MONOCLINIC ELASTIC LAYER LYING OVER AN IRREGULAR ISOTROPIC ELASTIC HALF-SPACE

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Abstract

The closed form analytical expressions for the stresses at any point of a monoclinic elastic layer lying over an irregular isotropic elastic half-space are obtained by using Fourier Transform technique. The perfect bonding interfacing of an infinite monoclinic elastic plate of finite thickness with an irregular isotropic elastic half-space has been considered. In the present study, the isotropic half-space is considered to have rectangular shaped irregularity as a result of strip loading. Finally, we will discuss graphical representation of shearing stresses in both the medium.

1. Introduction

The study of earthquakes is the most important concept of Seismology. It explains a lot of information about how fracture occurs in the Earth and the

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complete deformation process. The concept of seismic waves helps us to make inferences about certain properties of the parts of the earth through which the waves travel. Although, from the study of earthquakes and earth structures, it has been admitted that the Earth is anisotropic in nature. An anisotropic medium of interest in Seismology has a horizontal plane of symmetry. A medium having one plane of symmetry is called Monoclinic medium. As discussed by Crampin [5], monoclinic symmetry is the symmetry of two sets of non-orthogonal parallel cracks. Monoclinic symmetry of the systems of cracks can be found near the surface of the Earth where lithostatic pressure does not have closed cracks perpendicular to the maximum compressional stress. The problems related to deformation theory have been investigated by many researchers, like Steketee [16], Chinnery [4] etc. This theory has been proved very useful to study the deformation field of Earth produced by faulting and to observe the critical region where faulting often occurs near or at the interface boundary.

The solution of the problem of the deformation of a horizontally layered elastic material under the action of surface loads has been finding wide applications in engineering, geophysics and soil mechanics. Garg et al. [6] established the representations of seismic sources causing anti plane strain deformations of orthotropic medium. After that, the same approach for the corresponding plane strain deformation of an orthotropic elastic medium has been obtained by Garg et al. [7]. Singh et al. [14] obtained the effect of anisotropy and variation of dimensionless displacements of elastic materials. Madan et al. [9] obtained the closed form analytic expressions for the stresses of monoclinic elastic medium interfacing with the base due to strip loading. The study of static deformation with irregularities present in the elastic medium has been discussed by many researchers. Acharya et al. [1], Chattopadhyay et al. [3], Madan et al. [10], Singh et al. [15], Kumar et al. [8] obtained the effect of irregularities present in the medium with different interfacing. Madan and Gaba [11] obtained the effects of rectangular and parabolic irregularities on the orthotropic elastic medium due to normal line load. Madan et al. [12] obtained a closed form analytic expression for stresses in orthotropic elastic medium lying over an irregular isotropic elastic half space.

In this paper, we will discuss the closed form analytical expression for the stresses in a horizontal monoclinic elastic layer of an infinite extent lying over an irregular isotropic base due to strip loading. In Seismology, elastic plate represents a particular type of crust of the earth. Earlier researches have discussed that the interface between elastic plate and the base may be either 'perfectly bonded', 'smooth-rigid', or 'rough-rigid'. The deformation of the monoclinic elastic plate corresponding to perfectly bonded interface with irregular isotropic elastic half-space will be considered. Finally, we will study the variation of stresses numerically and graphically.

2. Fundamental Equations

The constitutive equation in matrix form of a monoclinic material has the following form [4]

$$\begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \\ \tau_4 \\ \tau_5 \\ \tau_6 \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & c_{16} \\ c_{12} & c_{22} & c_{23} & 0 & 0 & c_{26} \\ c_{13} & c_{33} & c_{33} & 0 & 0 & c_{36} \\ 0 & 0 & 0 & c_{44} & c_{45} & 0 \\ 0 & 0 & 0 & c_{45} & c_{55} & 0 \\ c_{16} & c_{26} & c_{36} & 0 & 0 & c_{66} \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \\ e_5 \\ e_6 \end{bmatrix}. \tag{1}$$

In equation (1), we use Voigt's convention by which the tensional indices are replaced by matrix indices in the expression of the stresses and shear components τ_i and $e_i (i = 1, 2, 3, 4, 5, 6)$. The elements $c_{ij}(i, j = 1, 2, 3, 4, 5, 6)$ of the stiffness matrix from (1) represent the elasticity of the monoclinic material. The non-zero field equations for displacement, strain and stress of a monoclinic material in anti-plane strain equilibrium state are:

$$u_3 = u_3(x, y); \tag{2}$$

$$e_{31} = \partial u_3 / 2\partial x, e_{23} = \partial u_3 / 2\partial y; \tag{3}$$

$$\tau_{31} = c_{45} \partial u_3 / \partial y + c_{55} \partial u_3 / \partial x, \; \tau_{23} = c_{44} \partial u_3 / \partial y + c_{45} \partial u_3 / \partial y. \tag{4}$$

Consequently, Cauchy's first two equations are identically satisfied and the third equation becomes

$$\partial \tau_{13} / \partial x + \partial \tau_{23} / \partial y = 0. \tag{5}$$

Using equations (4) and (5), the equilibrium equation satisfied by u_3 can be written in the following form:

$$\partial^2 u_3/\partial x^2 + c_{45}/c_{55}\,\partial^2 u_3/\partial x\partial y + c_{44}/c_{55}\,\partial^2 u_3/\partial y^2 = 0. \tag{6}$$

3. Formulation and Solution of the Problem

Here, we consider a horizontal infinite monoclinic elastic plate of thickness *'T'* lying over an infinite isotropic elastic medium with x-axis vertically downwards. The origin of the cartesian coordinate system (x, y, z) is taken at the upper boundary of the plate. The monoclinic elastic plate occupying the region $0 \le x \le T$ is described as Medium I whereas $x > T$ is the region of isotropic elastic half space over which the plate is lying and is described as Medium II (Figure 1).

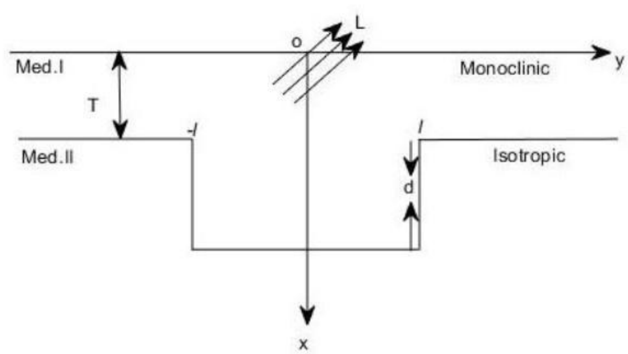

Figure 1.

Suppose a shear load L per unit area is acting over the strip $\| y \| \le t$ of the surface $x = 0$ in the positive x-direction. The boundary condition at the surface $x = 0$ is

$$\tau_{31} = \begin{cases} -L; & | y | \le t \\ 0; & | y | > t, \end{cases} \tag{7}$$

The irregularity is assumed to be rectangular with length $2l$ and depth d. The equation of the rectangular irregularity is represented as:

$$x = \varepsilon f(y) = \begin{cases} d; & |y| \le l \\ 0; & |y| > l \end{cases} \tag{8}$$

where $\varepsilon = d/2l \ll 1$ is the perturbation factor.

At the interface $(y,\, x = \varepsilon f(y))$, the boundary conditions are

$$u_3^I = u_3^{II}$$

$$\tau_{31}^I - i\varepsilon f'(y)\tau_{32}^I = \tau_{31}^{II} - i\varepsilon f(y)\tau_{32}^{II}. \tag{9}$$

Taking the Fourier Transformation of equation (6), we get

$$\frac{d^2\overline{u}_3^I}{dx^2} - 2(is\frac{c_{45}}{c_{55}})\frac{d\overline{u}_3^I}{dx} - \frac{c_{44}}{c_{55}}s^2\overline{u}_3^I = 0. \tag{10}$$

The solution of the above ordinary differential equation is

$$u_3^I(Ae^{m_1|s|x} + Be^{-m_1|s|x})e^{im_2sx} \tag{11}$$

where $m_1 = \sqrt{m_3 - m_2^2}$, $m_2 = c_{45}/c_{55}$, $m_3 = c_{44}/c_{55}$ and A and B may be functions of s.

Taking inverse Fourier Transformation of equation (11), we get

$$u_3^I = \frac{1}{2\pi}\int_{-\infty}^{\infty}(A_e^{m_1|s|x} + Be^{-m_1|s|x})e^{-i(y-m_2x)s}ds. \tag{12}$$

By using equations (12) and (4), we get

$$\tau_{31}^I = \frac{T_1}{2\pi}\int_{-\infty}^{\infty}(A_e^{m_1|s|x} - Be^{-m_1|s|x})e^{-i(y-m_2x)s}|s|ds \tag{13}$$

$$\tau_{32}^I = \frac{T_1}{2\pi}[m_2\int_{-\infty}^{\infty}(A_e^{m_1|s|x} + Be^{-m_1|s|x})e^{-i(y-m_2x)s}|s|ds - im_1$$

$$\int_{-\infty}^{\infty}(A_e^{m_1|s|x} + Be^{-m_1|s|x})e^{-i(y-m_2x)s}ds] \tag{14}$$

where $T_1 = m_1c_{55}$.

Taking Fourier Transform of equation (7), we get

$$\bar{\tau}_{31}^{I} = -2L/s \sin st. \tag{15}$$

Therefore,

$$\bar{\tau}_{31}^{I} = \frac{-L}{\pi}\int_{-\infty}^{\infty}(\sin st/s)e^{-isy}ds. \tag{16}$$

From equations (13) and (15), we get

$$A - B = -2L/T_1(\sin st/s|\, s\,|). \tag{17}$$

The displacement component in the isotropic elastic half space $x > T$ is obtained as

$$u_{31}^{II} = \frac{1}{2\pi}\int_{-\infty}^{\infty}Ce^{-|\,s\,|x}e^{-iys}ds. \tag{18}$$

From equations (4) and (18), we get

$$\tau_{31}^{II} = \frac{-\mu}{2\pi}\int_{-\infty}^{\infty}Ce^{-|\,s\,|x}e^{-iys}|\,s\,|ds \tag{19}$$

$$\tau_{32}^{II} = \frac{-i\mu}{2\pi}\int_{-\infty}^{\infty}Ce^{-|\,s\,|x}e^{-iys}sds. \tag{20}$$

Equations (9), (12), (13), (14), (18), (19), (20) give the relations

$$(A_e^{m_1|\,s\,|x} + Be^{-m_1|\,s\,|x})e^{-im_{2,s,x}} - Ce^{-|\,s\,|x} = 0 \tag{21}$$

$$T'A(s'-i\varepsilon f'(y)m_2s' - \varepsilon f'(y)m_1)e^{m_1|\,s\,|x}e^{im_2sx} - T'B(s' - i\varepsilon f'(y)m_2s' + \varepsilon f'(y)m_1)$$

$$e^{-m_1|\,s\,|x}e^{im_2\,skx} + Ce^{-|\,s\,|x}(s' + \varepsilon f'(y)) = 0 \tag{22}$$

where $T' = T_1/\mu$ and $S' = |\,s\,|/s.$ Solving (17), (21) and (22), we get

$$A = \frac{2L\sin st}{T_1 s|s|}\left(\frac{(s' + \varepsilon f'(y)V' - i\varepsilon f'(y)m_2 s'\left(\frac{1+V}{2}\right)e^{-2m_1|s|\varepsilon f(y)}}{\left(s'V - \varepsilon f'(y)\left(V' + im_2 s'\left(\frac{1+V}{2}\right)\right)\right) - \left(s' + \varepsilon f'(y)\left(V' - im_2 s'\left(\frac{1+V}{2}\right)\right)\right)e^{-2m_1|s|\varepsilon f(y)}}\right) \tag{23}$$

$$B = \frac{2L\sin st}{T_1 s|s|}\left(1 + \frac{\left(s + \varepsilon f'(y)V' - i\varepsilon f'(y)m_2 s\left(\frac{1+V}{2}\right)\right)e^{-2m_1|s|\varepsilon f(y)}}{\left(sV - \varepsilon f'(y)\left(V' + im_2 s'\left(\frac{1+V}{2}\right)\right)\right) - \left(s' + \varepsilon f'(y)\left(V - im_2 s'\left(\frac{1+V}{2}\right)\right)\right)e^{-2m_1|s|\varepsilon f(y)}}\right) \tag{24}$$

$$C = \frac{2L\sin st}{T_1 s|s|}\left(\frac{(1+V)(s' - i\varepsilon f'(y)m_2 s')e^{-\varepsilon f(y)(|k|(m_1-1)-im_2 s)}}{\left(s'V - \varepsilon f'(y)\left(V' + im_2 s'\left(\frac{1+V}{2}\right)\right)\right) - \left(s + \varepsilon f'(y)\left(V' - im_2 s'\left(\frac{1+V}{2}\right)\right)\right)e^{-2m_1|s|\varepsilon f(y)}}\right) \tag{25}$$

where $V = (T'-1)/(T'+1)$ and $V' = (T'm_1 - 1)/(T'-1)$.

By applying Fourier Transformation technique on equation (8), we get

$$\bar{f}(s) = (4l/s)\sin(sl). \tag{26}$$

Therefore,

$$f(y) = sigh(l-y) + sigh(l+y), \tag{27}$$

where sign represents the signum function.

On substituting the values of constants A, B and C from equations (23), (24), (25) in equations (12), (13), (14) for Medium I and in (18), (19), (20) for Medium II and also substituting the value of $f(y)$ for rectangular irregularity from equation (27), we will obtain the following expression for the displacement and stress component.

4. Displacement and Stresses for Medium I

$$u_3^I = \frac{-L}{\pi T_1}\int_{-\infty}^{\infty}\frac{\sin st}{s|s|}\left(1+\sum_{n=1}^{\infty}V^n e^{m_1|s|(2n\varepsilon(sign(l-y)+sign(l+y)))}\right)$$

$$\left(e^{m_1|s|x-i(y-m_2x)s}+Ve^{-m_1|s|x-i(y-m_2x)s}\right)ds \tag{28}$$

$$\tau_{31}^I = \frac{L}{\pi}\Bigg[(1+V)\tan^{-1}\left(\frac{2tm_1x}{(y-m_2x)^2+m_1^2x^2-t^2}\right)+$$

$$\sum_{n=1}^{\infty}V^n\Bigg(V\tan^{-1}\frac{2tm_1(x-2n\varepsilon(sign(l-y)+sign(l+y)))}{(m_1^2(x-2n\varepsilon(sign(l-y)+sign(l+y)))^2)+(y-m_2x)^2-t^2}$$

$$-\tan^{-1}\frac{2tm_1(x+2n\varepsilon(sign(l-y)+sign(l+y)))}{(m_1^2(x+2n\varepsilon(sign(l-y)+sign(l+y)))^2)+(y-m_2x)^2-t^2}\Bigg)\Bigg] \tag{29}$$

$$\tau_{32}^I = \frac{L}{\pi}\Bigg[m_2\Bigg[(1+V)\tan^{-1}\left(\frac{2tm_1x}{(y-m_2x)^2+m_1^2x^2-t^2}\right)$$

$$+\sum_{n=1}^{\infty}V^n\Bigg(V\tan^{-1}\frac{2tm_1(x-2n\varepsilon(sign(l-y)+sign(l+y)))}{(m_1^2(x-2n\varepsilon(sign(l-y)+sign(l+y)))^2)+(y-m_2x)^2-t^2}$$

$$-\tan^{-1}\frac{2tm_1(x-2n\varepsilon(sign(l-y)+sign(l+y)))}{(m_1^2(x+2n\varepsilon(sign(l-y)+sign(l+y)))^2)+(y-m_2x)^2-t^2}\Bigg)\Bigg]$$

$$+\frac{m_1}{2}\Bigg[(1-V)\log\frac{m_1^2x^2+(y-m_2x+t)^2}{m_1^2x^2+(y-m_2x-t)^2}$$

$$+\sum_{n=1}^{\infty}V^n\Bigg(\log\frac{(y-m_2x+t)^2+m_1^2(x+2n\varepsilon(sign(l-y)+sign(l+y)))^2}{(y-m_2x-t)^2+m_1^2(x+2n\varepsilon(sign(l-y)+sign(l+y)))^2}\Bigg)$$

$$+V\log\frac{(y-m_2x+t)^2+m_1^2(x-2n\varepsilon(sign(l-y)+sign(l+y)))^2}{(y-m_2x-t)^2+m_1^2(x-2n\varepsilon(sign(l-y)+sign(l+y)))^2}\Bigg]\Bigg]. \tag{30}$$

5. Displacement and Stresses for Medium II

$$u_3^{II} = \frac{-L}{\pi}\int_{-\infty}^{\infty} \frac{\sin st}{s|s|}(1+V)(1+\sum_{n=1}^{\infty} V^n e^{2m_1 n\varepsilon|s|f(y)})$$

$$e^{|s|\varepsilon(m_1+1)sign(l-y)sign(l-y)+sign(l+y)-x)} e^{is(m_2 x - y)} \quad (31)$$

$$\tau_{31}^{II} = \frac{L\mu}{\pi T_1}(1+V)[\tan^{-1}\frac{2h(x-(m_1+1)\varepsilon(sign(l-y)+sign(l+y)))}{(x-\varepsilon(sign(l-y)+sign(l+y))((2n+1)m_1+1))^2 + (m_2 x - y)^2 - t^2}$$

$$+\sum_{n=1}^{\infty} V^n \{\tan^{-1}\frac{2t(x-\varepsilon(sign(l-y)+sign(l+y))((2n+1)m_1+1))}{(x-\varepsilon(sign(l-y)+sign(l+y))((2n+1)m_1+1))^2 + (m_2 x - y)^2 - t^2})\}] \quad (32)$$

$$\tau_{32}^{II} = \frac{L\mu}{2\pi T_1}(1+V)[\log\frac{(m_2 x - y - t)^2 + (x-(m_1+1)\varepsilon(sign(l-y)+sign(l+y)))^2}{(m_2 x - y + t)^2 (x-(m_1+1)\varepsilon(sign(l-y)+sign(l+y)))^2}$$

$$+\sum_{n=1}^{\infty} V^n \log\frac{(x-\varepsilon(sign(l-y)+sign(l+y))((2n+1)m_1+1))^2 (m_2 x - y + t)^2}{(x-\varepsilon(sign(l-y)+sign(l+y))((2n+1)m_1+1))^2 + (m_2 x - y + t)^2}] \quad (33)$$

6. Numerical Results

In this section, we want to analyze the effect of rectangular irregularity on the stresses due to shear line load acting at any point of the monoclinic elastic layer lying over an irregular isotropic half space. For numerical calculations we take the values of elastic constants for Dolomite in monoclinic medium given by Rasolofosaon and Zinszner [13] and for Glass in Isotropic medium given by love [8]. Here we will calculate all stresses τ_{31}^{I}, τ_{32}^{I}, τ_{31}^{II}, τ_{32}^{II} for $T' = -1$ for different strip length 'indicated by $|y| \le t$' on upper boundary of monoclinic layer. Figures (2)-(5) represent the variation of shearing stresses τ_{31}^{I} with horizontal distance for different values of $t = 1, 2.3, 1.6, 2$ and for different depth levels $x = 0.25, 0.5, 0.75, 1$. It is observed that shearing stresses for $t = 1$ and $t = 1.3$ overlap over entire depth. For $t = 1.6$ and $t = 2$, the difference between shearing stresses in magnitude significantly decreases as depth increases. Similarly figures (6)-(9) represent the variation of shearing stresses τ_{31}^{I} with horizontal distance for different values of $t = 1, 1.3, 1.6, 2$ and for different depth levels $x = 0.25, 0.5, 0.75, 1$ respectively. The shearing stresses for $t = 1$ and $t = 2$, overlap over entire depth. Clearly the stresses coincide at zero on horizontal distance for all different depth levels. Figures (10)-(13) represent the variation of shearing stresses τ_{31}^{II} with horizontal distance for different values of $t = 1, 1.3, 1.6, 2$ and for different depth levels $x = 0.25, 0.5, 0.75, 1$ respectively. It is observed from these curves that stresses for different values of t decrease for negative horizontal distance and further increase for positive horizontal distance. Figures (14)-(17) represent the variation of shearing stresses τ_{32}^{II} with horizontal distance for different values of $t = 1, 1.3, 1.6, 2$ and for different depth levels $x = 0.25, 0.5, 0.75, 1$ respectively. Stresses for different values of t coincide in neighborhood of zero horizontal distance.

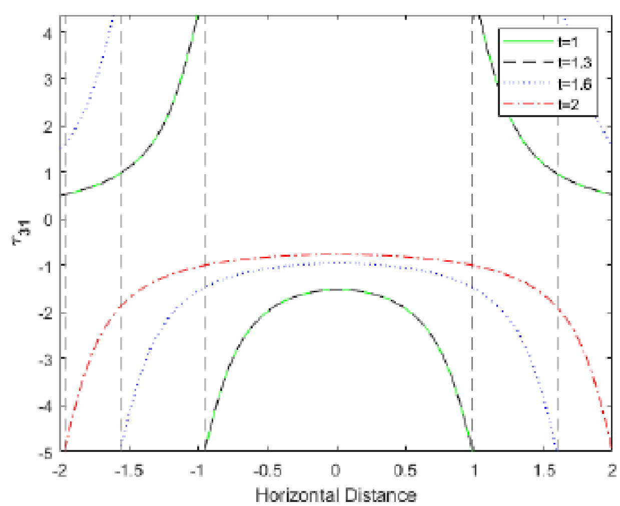

Figure 2. Variation of the shearing stress τ_{31} in Med. I with horizontal distance y at $x = 0.25$.

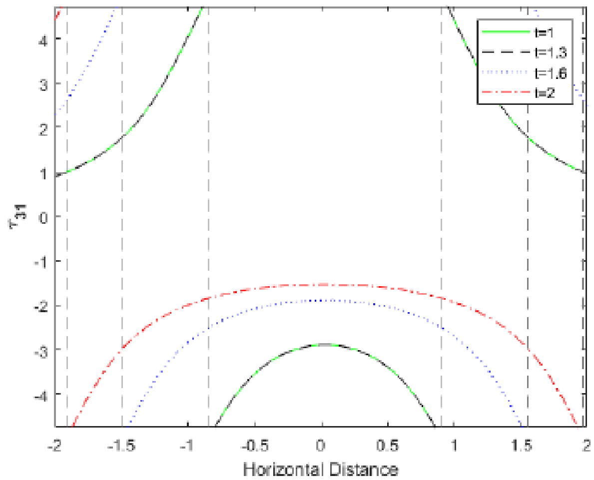

Figure 3. Variation of the shearing stress τ_{31} in Med. I with horizontal distance y at $x = 0.5$.

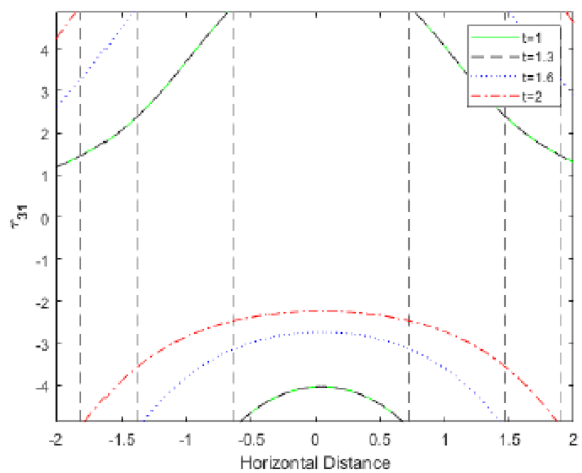

Figure 4. Variation of the shearing stress τ_{31} in Med. I with horizontal distance y at $x = 0.75$.

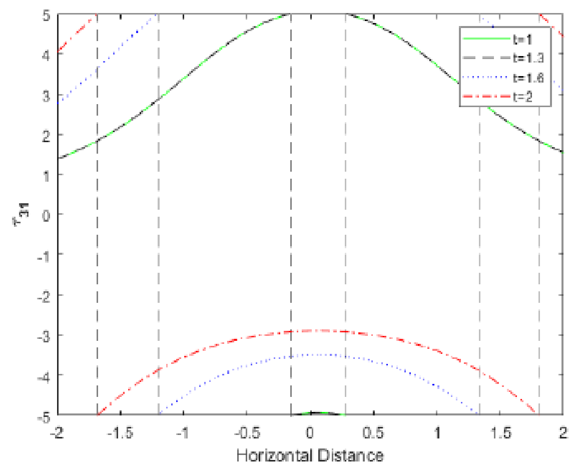

Figure 5. Variation of the shearing stress τ_{31} in Med. I with horizontal distance y at $x = 1$.

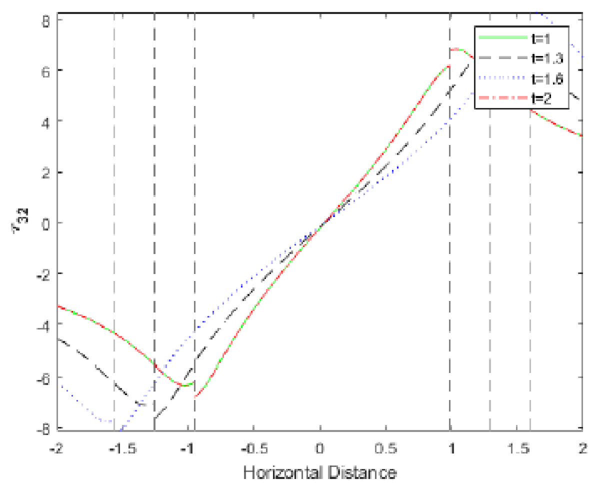

Figure 6. Variation of the shearing stress τ_{32} in Med. I with horizontal distance y at $x = 0.25$.

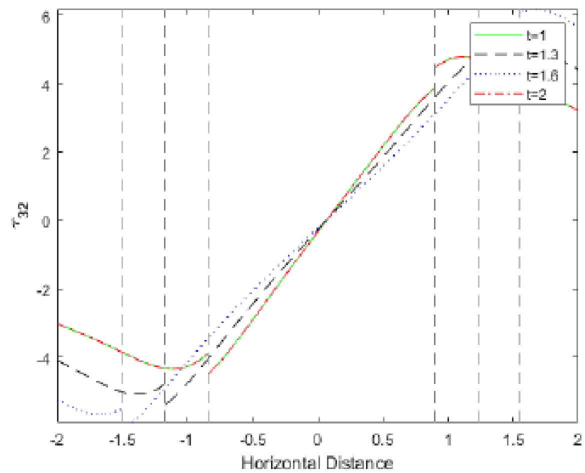

Figure 7. Variation of the shearing stress τ_{32} in Med. I with horizontal distance y at $x = 0.5$.

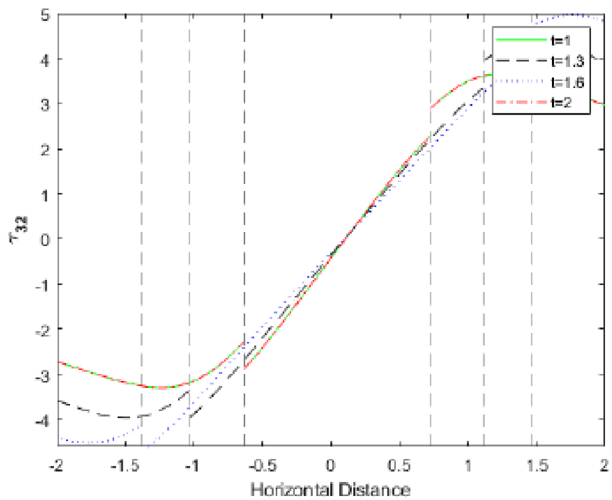

Figure 8. Variation of the shearing stress τ_{32} in Med. I with horizontal distance y at $x = 0.75$.

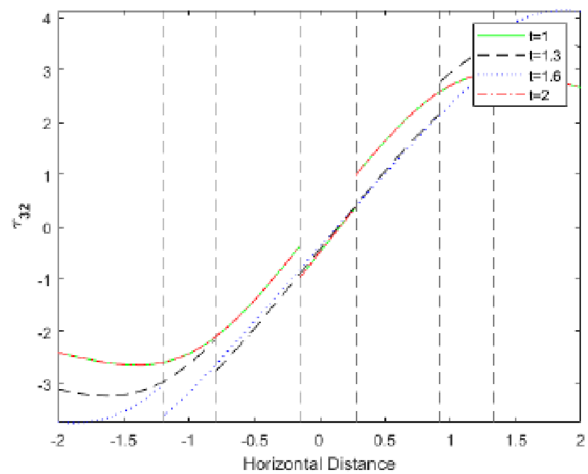

Figure 9. Variation of the shearing stress τ_{32} in Med. I with horizontal distance y at $x = 1$.

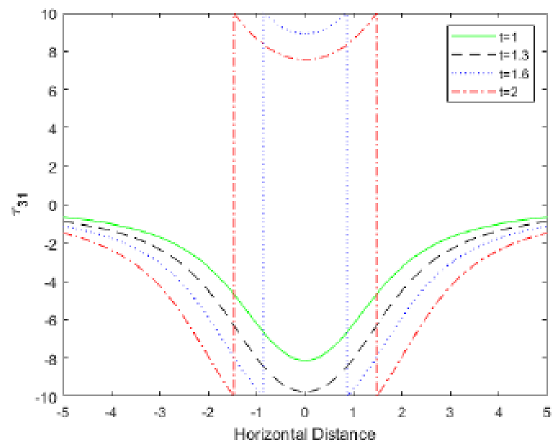

Figure 10. Variation of the shearing stress τ_{31} in Med. II with horizontal distance y at $x = 0.25$.

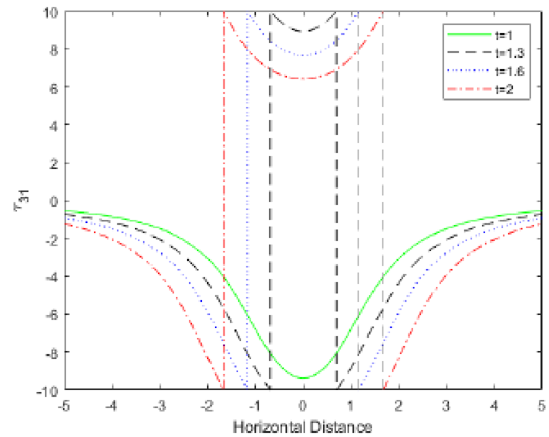

Figure 11. Variation of the shearing stress τ_{31} in Med. II with horizontal distance y at $x = 0.5$.

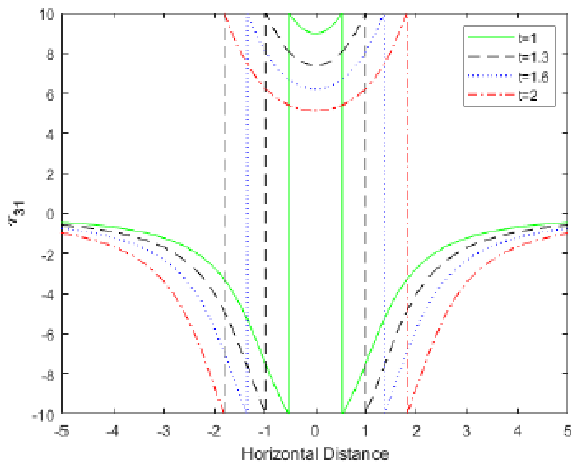

Figure 12. Variation of the shearing stress τ_{31} in Med. II with horizontal distance y at $x = 0.75$.

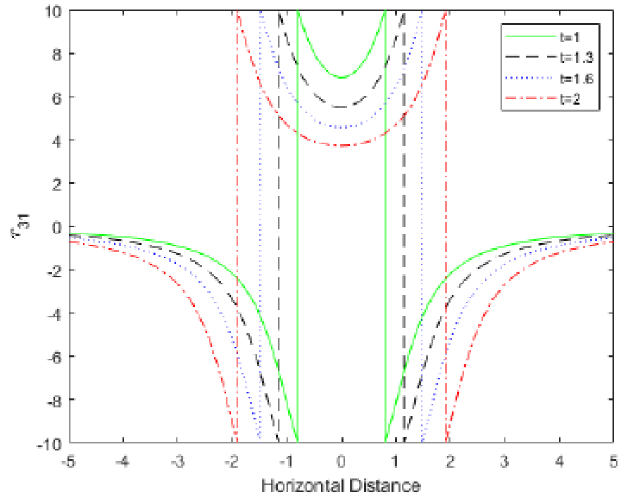

Figure 13. Variation of the shearing stress τ_{31} in Med. II with horizontal distance y at $x = 1$.

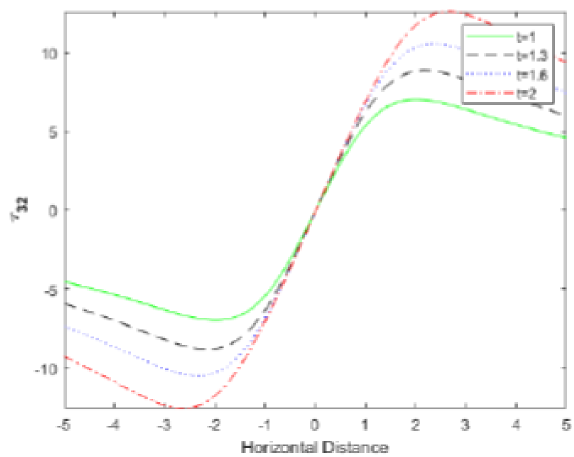

Figure 14. Variation of the shearing stress τ_{32} in Med. II with horizontal distance y at $x = 0.25$.

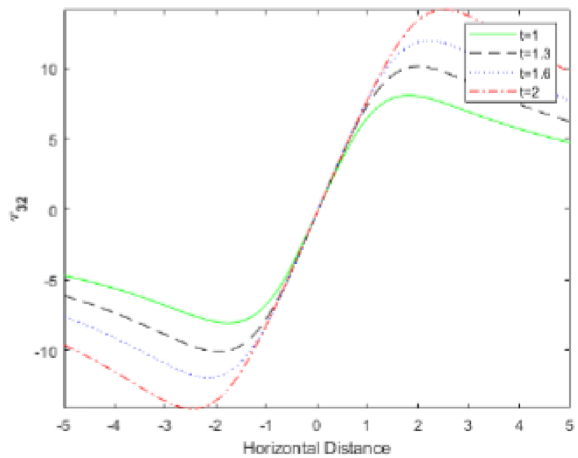

Figure 15. Variation of the shearing stress τ_{32} in Med. II with horizontal distance y at $x = 0.5$.

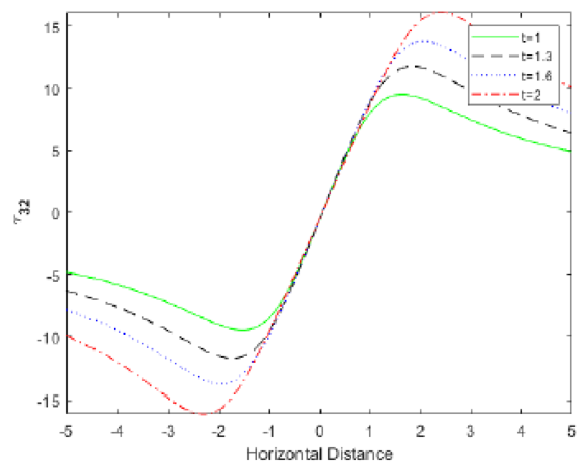

Figure 16. Variation of the shearing stress τ_{32} in Med. II with horizontal distance y at $x = 0.75$.

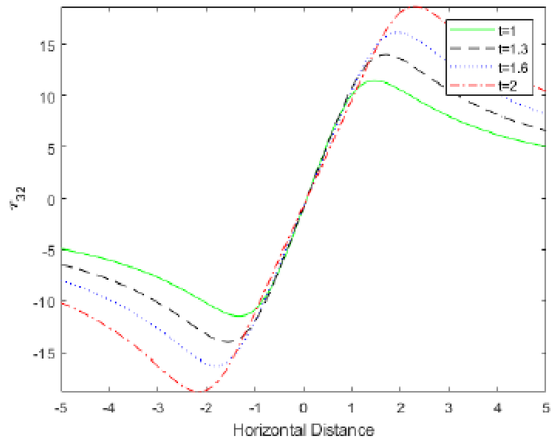

Figure 17. Variation of the shearing stress τ_{32} in Med. II with horizontal distance y at $x = 1$.

7. Conclusions

The closed form expressions for the stresses in an elastic medium consisting of Monoclinic elastic layer lying over an irregular isotropic half space due to shearing load has been concluded. The results are useful to study the effect of irregularity lying between two or more mediums. Graphically, it has been concluded that the stress is distributed in an infinite layer with irregularity present at the interface with a half space and is affected by not only the presence of irregularity but also by anisotropy of the elastic medium as a result of shear load acting over the strip of a monoclinic elastic medium. Also, the obtained results are useful to study the static deformation around different crust of the earth.

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