Advances and Applications in Mathematical Sciences
Volume 21, Issue 5, March 2022, Pages 2501-2509

STUDY OF CERTAIN INTEGRAL INVOLVING GAUSS'S HYPERGEOMETRIC FUNCTION OF SERIES F_1^1 AND F_1^2

NANCY SOLANKI[1] and CHANDRAPAL SINGH CHOUHAN[2]

[1]Research Scholar
[2]Supervisor
Department of Mathematics and Statistics
Bhupal Nobles' University
Udaipur (Raj.), India

Abstract

In this present work our main aim is to obtain integral involving Hypergeometric function of the series F_1^1 and F_1^2 by employing one of the integral obtained by Mac Robert. Main interesting result of this research paper is that it comes out in the products of the ratio of the Gamma function with Special Cases. For the application point of view integral comes in terms of Gamma function is very useful in engineering applications. On specialization the parameters, we can easily obtain some new integrals by Rathie and others which are given in Book of Mathai and Saxena.

1. Introduction

The definition of the Gauss's Hypergeometric Series [6] and denoted by $F_1^2\left[\begin{matrix} a, b \\ c \end{matrix} \mid z\right]$ which can be further written as

$$F_1^2\left[\begin{matrix} a, b \\ c \end{matrix} \mid z\right] = 1 + \frac{ab}{c}\frac{z}{1!} + \frac{a(a+1)b(b+1)}{c(c+1)}\frac{z^2}{2!}$$

$$+ \frac{a(a+1)(a+2)b(b+1)(b+2)}{c(c+1)(c+2)}\frac{z^3}{z!} + \dots$$

2020 Mathematics Subject Classification: 33Dxx.
Keywords: Generalized Hypergeometric functions, Gamma function and integrals.
Received August 10, 2021; Accepted October 22, 2021

$$F_1^2\begin{bmatrix} a,\ b \\ c \end{bmatrix} \mid z \Bigg] = \sum_{n=0}^{\infty} \frac{(a)_n (b)_n}{(c)_n} \frac{z^n}{n!} \tag{1.1}$$

For $a = 1$ and $b = c$ or $b = 1$ and $a = c$, the series (1.1) reduced to the well known geometric series and for $a = 0$ and $b = 0$ or both zero, the series becomes unity. If a or b or both are negative integers, the series becomes polynomial.

The natural generalization of the above mentioned functions is the generalized Hypergeometric function with p numerator parameters and q denominator parameters denoted by F_q^p and is defined in the following manner [3].

$$F_q^p\begin{bmatrix} a_1,\ a_2,\ \ldots,\ a_p \\ b_1,\ b_2,\ \ldots,\ b_q \end{bmatrix},\ x\Bigg] = \sum_{n=0}^{\infty} \frac{\prod_{i=1}^{p} (a_i)_n x^n}{\prod_{i=1}^{q} (b_i)_n n!}$$

Also, if we take $p = q = 1$, the generalized Hypergeometric function reduces to confluent Hypergeometric function [5], given as

$$F_1^1[a;\ b \mid z] = \sum_{n=0}^{\infty} \frac{(a)_n}{(b)_n} \frac{z^n}{n!} \tag{1.2}$$

The result will be defined with the help of known and interesting result by Macrobert [1]. The aim of this paper is providing an integral involving Hypergeometric function few interested well known results have been obtained as a limiting cases of main result.

2. Result Required

In our present investigation we use the following interesting result by Macrobert [1]

$$\int_0^1 x^{\alpha-1}(1-x)^{\beta-1}[ax + b(1-x)]^{-\alpha-\beta} dx = \frac{1}{a^{\alpha} b^{\beta}} \frac{\Gamma\alpha\Gamma\beta}{\Gamma(\alpha+\beta)} \tag{2.1}$$

$\mathrm{Re}(\alpha) > 0, \mathrm{Re}(\beta) > 0$. Provided $\mathrm{Re}(\alpha) > 0, \mathrm{Re}(\beta) > 0$ and a, b are non zero constants and expression $[ax + b(1-x)]$ where $0 \le x \le 1$ is not zero and

$$(a)_{2n} = 2^{2n}\left(\frac{a}{2}\right)_n\left(\frac{a+1}{2}\right)_n \tag{2.2}$$

$$(a)_{mn} = m^{mn}\left(\frac{a}{m}\right)_n\left(\frac{a+1}{m}\right)_n\left(\frac{a+2}{m}\right)_n \cdots \left(\frac{a+m-1}{m}\right)_n \tag{2.3}$$

$$(a)_n = \frac{\Gamma(a+n)}{\Gamma a} \tag{2.4}$$

3. Main Result

In this section we evaluate integral involving confluent Hypergeometric function.

Theorem 3.1. *For* $\mathrm{Re}(\alpha) > 0, \mathrm{Re}(\beta) > 0$ *and* a, b *are non zero constants and expression* $[ax + b(1-x)]$ *where* $0 \le x \le 1$, *the following result holds true.*

$$\int_0^1 x^{\alpha-1}(1-x)^{\beta-1}[ax + b(1-x)]^{-\alpha-\beta} F_1^1\left[\begin{matrix} a \\ b \end{matrix} \mid \frac{4abx^2(1-x)^2}{[ax+b(1-x)]^4}\right] dx$$

$$= \left(\frac{\Gamma\alpha\Gamma\beta}{\Gamma(\alpha+\beta)a^{\alpha}b^{\beta}}\right) {}_5^5F\left[\begin{matrix} a, \frac{a}{2}, \frac{\beta}{2}, \frac{a+1}{2}, \frac{\beta+1}{2} \\ b, \frac{a+\beta}{4}, \frac{a+\beta+1}{4}, \frac{a+\beta+2}{4}, \frac{a+\beta+3}{4} \end{matrix} \mid \frac{1}{4ab}\right] \tag{3.1}$$

Proof. Let us consider,

$$F_1^1\left[\begin{matrix} a \\ b \end{matrix} \mid \frac{4abx^2(1-x)^2}{[ax+b(1-x)]^4}\right] = \sum_{n=0}^{\infty} \frac{(a)_n}{(b)_n} \frac{4^n a^n b^n x^{2n}(1-x)^{2n}}{[ax+b(1-x)]^{4n} n!} \tag{3.2}$$

From Left hand side of (3.1) we have

$$I = \int_0^1 x^{\alpha-1}(1-x)^{\beta-1}[ax + b(1-x)]^{-\alpha-\beta} \sum_{n=0}^{\infty} \frac{(a)_n}{(b)_n} \frac{4^n a^n b^n x^{2n}(1-x)^{2n}}{[ax+b(1-x)]^{4n} n!} dx$$

$$= \sum_{n=0}^{\infty} \frac{4^n a^n b^n (a)_n}{(b)_n n!} \int_0^1 x^{2n+\alpha-1}(1-x)^{2n+\beta-1}[ax+b(1-x)]^{-\alpha-\beta-4n} dx$$

By using (2.1) we have

$$I = \sum_{n=0}^{\infty} \frac{4^n a^n (a)_n}{(b)_n} \frac{1}{a^{2n+\alpha} b^{2n+\beta}} \frac{\Gamma(2n+\alpha)\Gamma(2n+\beta)}{\Gamma(\alpha+\beta+4n)\, n!}$$

$$I = \left(\frac{\Gamma\alpha\Gamma\beta}{\Gamma(\alpha+\beta)}\right) \sum_{n=0}^{\infty} \frac{4^n (a)_n}{(b)_n} \frac{1}{a^{n+\alpha} b^{n+\beta}} \frac{(\alpha)_{2n}(\beta)_{2n}}{(\alpha+\beta)_{4n}\, n!} \tag{3.3}$$

By using (2.2), (2.3), (2.4) in (3.3) we have

$$I = \left(\frac{\Gamma\alpha\Gamma\beta}{\Gamma(\alpha+\beta)}\right) \sum_{n=0}^{\infty} \frac{(a)_n \left(\frac{a}{2}\right)_n \left(\frac{\alpha+1}{2}\right)_n \left(\frac{\beta}{2}\right)_n \left(\frac{\beta+1}{2}\right)_n}{(b)_n n!\, a^{n+\alpha} b^{n+\beta} 2^{2n} \left(\frac{\alpha+\beta}{4}\right)_n \left(\frac{\alpha+\beta+2}{4}\right)_n \left(\frac{\alpha+\beta+3}{4}\right)_n}$$

$$I = \left(\frac{\Gamma\alpha\Gamma\beta}{\Gamma(\alpha+\beta) a^{\alpha} b^{\beta}}\right) \sum_{n=0}^{\infty} \frac{(a)_n \left(\frac{a}{2}\right)_n \left(\frac{\alpha+1}{2}\right)_n \left(\frac{\beta}{2}\right)_n \left(\frac{\beta+1}{2}\right)_n}{(b)_n \left(\frac{\alpha+\beta}{4}\right)_n \left(\frac{\alpha+\beta+2}{4}\right)_n \left(\frac{\alpha+\beta+3}{4}\right)_n} \frac{1}{n!\, a^n b^n 2^{2n}}$$

$$I = \left(\frac{\Gamma\alpha\Gamma\beta}{\Gamma(\alpha+\beta) a^{\alpha} b^{\beta}}\right) {}_5^5F \left[\begin{matrix} a, \frac{a}{2}, \frac{\beta}{2}, \frac{a+1}{2}, \frac{\beta+1}{2} \\ b, \frac{a+\beta}{4}, \frac{a+\beta+1}{4}, \frac{a+\beta+2}{4}, \frac{a+\beta+3}{4} \end{matrix} \mid \frac{1}{4ab} \right]$$

$$I = \int_0^1 x^{\alpha-1}(1-x)^{\beta-1}[ax+b(1-x)]^{-\alpha-\beta} F_1^1 \left[\begin{matrix} a \\ b \end{matrix} \mid \frac{4abx^2(1-x)^2}{[ax+b(1-x)]^4} \right] dx$$

$$I = \left(\frac{\Gamma\alpha\Gamma\beta}{\Gamma(\alpha+\beta) a^{\alpha} b^{\beta}}\right) {}_5^5F \left[\begin{matrix} a, \frac{a}{2}, \frac{\beta}{2}, \frac{a+1}{2}, \frac{\beta+1}{2} \\ b, \frac{a+\beta}{4}, \frac{a+\beta+1}{4}, \frac{a+\beta+2}{4}, \frac{a+\beta+3}{4} \end{matrix} \mid \frac{1}{4ab} \right]$$

This completes the proof of Theorem 3.1.

Theorem 3.2. *For* $\mathrm{Re}(\alpha) > 0$, $\mathrm{Re}(\beta) > 0$ *and* a, b *are non zero constants and expression* $[ax + b(1-x)]$ *where* $0 \le x \le 1$, *the following result holds true.*

$$\int_0^1 x^{\alpha-1}(1-x)^{\beta-1}[ax+b(1-x)]^{-\alpha-\beta} F_1^1\left[\begin{matrix} a+b \\ ab \end{matrix} \mid \frac{4abx^2(1-x)^2}{[ax+b(1-x)]^4}\right] dx =$$

$$\left(\frac{\Gamma\alpha\Gamma\beta}{\Gamma(\alpha+\beta)a^\alpha b^\beta}\right) F_5^5\left[\begin{matrix} a, \frac{a}{2}, \frac{\beta}{2}, \frac{a+1}{2}, \frac{\beta+1}{2} \\ b, \frac{a+\beta}{4}, \frac{a+\beta+1}{4}, \frac{a+\beta+2}{4}, \frac{a+\beta+3}{4} \end{matrix} \mid \frac{1}{4ab}\right] \tag{3.4}$$

Proof. Let us consider,

$$F_1^1\left[\begin{matrix} a+b \\ ab \end{matrix} \mid \frac{4abx^2(1-x)^2}{[ax+b(1-x)]^4}\right] = \sum_{n=0}^{\infty} \frac{(a+b)_n}{(ab)_n} \frac{4^n a^n b^n x^{2n}(1-x)^{2n}}{[ax+b(1-x)]^{4n} n!}$$

From Left hand side of (3.4) we have

$$I = \int_0^1 x^{\alpha-1}(1-x)^{\beta-1}[ax+b(1-x)]^{-\alpha-\beta} \sum_{n=0}^{\infty} \frac{(a+b)_n}{(ab)_n} \frac{4^n x^{2n}(1-x)^{2n}}{[ax+b(1-x)]^{4n} n!} dx$$

$$I = \sum_{n=0}^{\infty} \frac{4^n a^n b^n (a+b)_n}{(ab)_n n!} \int_0^1 x^{2n+\alpha-1}(1-x)^{2n+\beta-1}[ax+b(1-x)]^{-\alpha-\beta-4n} dx$$

$$I = \sum_{n=0}^{\infty} \frac{4^n (a+b)_n}{(ab)_n} \frac{1}{a^{n+\alpha} b^{n+\beta}} \frac{\Gamma(2n+\alpha)\Gamma(2n+\beta)}{\Gamma(\alpha+\beta+4n)\, n!} \tag{3.6}$$

By using (2.2), (2.3), (2.4) in (3.6) we have

$$I = \left(\frac{\Gamma\alpha\Gamma\beta}{\Gamma(\alpha+\beta)}\right) \sum_{n=0}^{\infty} \frac{4^n (a)_n}{(b)_n} \frac{1}{a^{n+\alpha} b^{n+\beta}} \frac{(\alpha)_{2n}(b)_{2n}}{(\alpha+\beta)_{4n} n!}$$

$$I = \left(\frac{\Gamma\alpha\Gamma\beta}{\Gamma(\alpha+\beta)}\right)$$

$$\times \sum_{n=0}^{\infty} \frac{(a+b)_n \left(\frac{a}{2}\right)_n \left(\frac{\alpha+1}{2}\right)_n \left(\frac{\beta}{2}\right)_n \left(\frac{\beta+1}{2}\right)_n}{(ab)_n n!\, a^{n+\alpha} b^{n+\beta} 2^{2n} \left(\frac{\alpha+\beta}{4}\right)_n \left(\frac{\alpha+\beta+1}{4}\right)_n \left(\frac{\alpha+\beta+2}{4}\right)_n \left(\frac{\alpha+\beta+3}{4}\right)_n}$$

$$I=\left(\frac{\Gamma\alpha\Gamma\beta}{\Gamma(\alpha+\beta)a^{\alpha}b^{\beta}}\right)\sum_{n=0}^{\infty}\frac{(a+b)_n\left(\frac{a}{2}\right)_n\left(\frac{\alpha+1}{2}\right)_n\left(\frac{\beta}{2}\right)_n\left(\frac{\beta+1}{2}\right)_n}{(ab)_n\left(\frac{\alpha+\beta}{4}\right)_n\left(\frac{\alpha+\beta+2}{4}\right)_n\left(\frac{\alpha+\beta+3}{4}\right)_n}\frac{1}{n!\,a^n b^n 2^{2n}}$$

$$I=\left(\frac{\Gamma\alpha\Gamma\beta}{\Gamma(\alpha+\beta)a^{\alpha}b^{\beta}}\right){}_5^5F\left[\begin{matrix}(a+b),\ \frac{a}{2},\ \frac{\beta}{2},\ \frac{a+1}{2},\ \frac{\beta+1}{2}\\ ab,\ \frac{a+\beta}{4},\ \frac{a+\beta+1}{4},\ \frac{a+\beta+2}{4},\ \frac{a+\beta+3}{4}\end{matrix}\ \Big|\ \frac{1}{4ab}\right]$$

$$I=\int_0^1 x^{\alpha-1}(1-x)^{\beta-1}[ax+b(1-x)]^{-\alpha-\beta}F_1^1\left[\begin{matrix}(a+b)\\ ab\end{matrix}\ \Big|\ \frac{4abx^2(1-x)^2}{[ax+b(1-x)]^4}\right]dx$$

$$I=\left(\frac{\Gamma\alpha\Gamma\beta}{\Gamma(\alpha+\beta)a^{\alpha}b^{\beta}}\right)F_5^5\left[\begin{matrix}(a+b),\ \frac{a}{2},\ \frac{\beta}{2},\ \frac{a+1}{2},\ \frac{\beta+1}{2}\\ ab,\ \frac{a+\beta}{4},\ \frac{a+\beta+1}{4},\ \frac{a+\beta+2}{4},\ \frac{a+\beta+3}{4}\end{matrix}\ \Big|\ \frac{1}{4ab}\right]$$

This completes the proof of Theorem 3.2.

Theorem 3.3. *For* $\operatorname{Re}(\alpha)>0$, $\operatorname{Re}(\beta)>0$ *and* a, b *are non zero constants and expression* $[ax+b(1-x)]$ *where* $0\le x\le 1$, *the following result holds true.*

$$\int_0^1 x^{\alpha-1}(1-x)^{\beta-1}[ax+b(1-x)]^{-\alpha-\beta}F_1^2\left[\begin{matrix}a,\ b\\ ab\end{matrix}\ \Big|\ \frac{4abx^2(1-x)^2}{[ax+b(1-x)]^4}\right]dx$$

$$=\left(\frac{\Gamma\alpha\Gamma\beta}{\Gamma(\alpha+\beta)a^{\alpha}b^{\beta}}\right)F_5^6\left[\begin{matrix}a,\ b,\ \frac{a}{2},\ \frac{\beta}{2},\ \frac{a+1}{2},\ \frac{\beta+1}{2}\\ ab,\ \frac{a+\beta}{4},\ \frac{a+\beta+1}{4},\ \frac{a+\beta+2}{4},\ \frac{a+\beta+3}{4}\end{matrix}\ \Big|\ \frac{1}{4ab}\right]\quad(3.7)$$

Proof. Let us consider,

$$F_1^2\left[\begin{matrix}a,\ b\\ b\end{matrix}\ \Big|\ \frac{4abx^2(1-x)^2}{[ax+b(1-x)]^4}\right]=\sum_{n=0}^{\infty}\frac{(a+b)_n}{(ab)_n}\frac{4^n a^n b^n x^{2n}(1-x)^{2n}}{[ax+b(1-x)]^{4n}n!}\quad(3.8)$$

From Left hand side of (3.7) we have

$$I=\int_0^1 x^{\alpha-1}(1-x)^{\beta-1}[ax+b(1-x)]^{-\alpha-\beta}\sum_{n=0}^{\infty}\frac{(a)_n(b)_n}{(ab)_n}\frac{4^n a^n b^n x^{2n}(1-x)^{2n}}{[ax+b(1-x)]^{4n}n!}dx$$

$$I = \sum_{n=0}^{\infty} \frac{4^n a^n b^n (a)_n (b)_n}{(ab)_n n!} \int_0^1 x^{2n+\alpha-1} (1-x)^{2n+\beta-1} [ax + b(1-x)]^{-\alpha-\beta-4n} dx$$

$$I = \sum_{n=0}^{\infty} \frac{4^n (a)_n (b)_n}{(ab)_n} \frac{1}{a^{2n+\alpha} b^{2n+\beta}} \frac{\Gamma(2n+\alpha)\Gamma(2n+\beta)}{\Gamma(\alpha+\beta+4n) n!} \tag{3.9}$$

By using (2.2), (2.3), (2.4) in (3.9) we have

$$I = \left(\frac{\Gamma\alpha\Gamma\beta}{\Gamma(\alpha+\beta)}\right) \sum_{n=0}^{\infty} \frac{4^n (a)_n (b)_n}{(b)_n} \frac{1}{a^{2n+\alpha} b^{2n+\beta}} \frac{(\alpha)_{2n} (b)_{2n}}{(\alpha+\beta)_{4n}\, n!}$$

$$I = \left(\frac{\Gamma\alpha\Gamma\beta}{\Gamma(\alpha+\beta)}\right)$$

$$\times \sum_{n=0}^{\infty} \frac{(a)_n (b)_n \left(\frac{a}{2}\right)_n \left(\frac{\alpha+1}{2}\right)_n \left(\frac{\beta}{2}\right)_n \left(\frac{\beta+1}{2}\right)_n}{(ab)_n n! a^{n+\alpha} b^{n+\beta} 2^{2n} \left(\frac{\alpha+\beta}{4}\right)_n \left(\frac{\alpha+\beta+1}{4}\right)_n \left(\frac{\alpha+\beta+2}{4}\right)_n \left(\frac{\alpha+\beta+3}{4}\right)_n}$$

$$I = \left(\frac{\Gamma\alpha\Gamma\beta}{\Gamma(\alpha+\beta) a^{\alpha} b^{\beta}}\right) \sum_{n=0}^{\infty} \frac{(a)_n (b)_n \left(\frac{a}{2}\right)_n \left(\frac{\alpha+1}{2}\right)_n \left(\frac{\beta}{2}\right)_n \left(\frac{\beta+1}{2}\right)_n}{(ab)_n \left(\frac{\alpha+\beta}{4}\right)_n \left(\frac{\alpha+\beta+1}{4}\right)_n \left(\frac{\alpha+\beta+2}{4}\right)_n \left(\frac{\alpha+\beta+3}{4}\right)_n}$$

$$\times \frac{1}{n! a^n b^n 2^{2n}}$$

$$I = \left(\frac{\Gamma\alpha\Gamma\beta}{\Gamma(\alpha+\beta) a^{\alpha} b^{\beta}}\right) F_5^6 \left[\begin{matrix} a, b, \frac{a}{2}, \frac{\beta}{2}, \frac{a+1}{2}, \frac{\beta+1}{2} \\ ab, \frac{a+\beta}{4}, \frac{a+\beta+1}{4}, \frac{a+\beta+2}{4}, \frac{a+\beta+3}{4} \end{matrix} \mid \frac{1}{4ab}\right]$$

$$I = \int_0^1 x^{\alpha-1} (1-x)^{\beta-1} [ax + b(1-x)]^{-\alpha-\beta} F_1^2 \left[\begin{matrix} a, b \\ ab \end{matrix} \mid \frac{4abx^2 (1-x)^2}{[ax + b(1-x)]^4}\right] dx$$

$$I = \left(\frac{\Gamma\alpha\Gamma\beta}{\Gamma(\alpha+\beta) a^{\alpha} b^{\beta}}\right) F_5^6 \left[\begin{matrix} a, b, \frac{a}{2}, \frac{\beta}{2}, \frac{a+1}{2}, \frac{\beta+1}{2} \\ ab, \frac{a+\beta}{4}, \frac{a+\beta+1}{4}, \frac{a+\beta+2}{4}, \frac{a+\beta+3}{4} \end{matrix} \mid \frac{1}{4ab}\right]$$

This completes the proof of Theorem 3.3.

4. Special Cases

In Theorem 3.1, if we take $a = 1, b = 1$ then we get following result,

Corollary 4.1. *For* $\text{Re}(\alpha) > 0, \text{Re}(\beta) > 0$ *and* $0 \leq x \leq 1,$ *the following result holds true.*

$$\int_0^1 x^{\alpha-1}(1-x)^{\beta-1} F_1^1\left[\begin{matrix}1\\1\end{matrix} \mid 4x^2(1-x)^2\right] dx$$

$$= \left(\frac{\Gamma\alpha\Gamma\beta}{\Gamma(\alpha+\beta)}\right) F_4^4\left[\begin{matrix}\frac{a}{2}, \frac{\beta}{2}, \frac{a+1}{2}, \frac{\beta+1}{2}\\ \frac{a+\beta}{4}, \frac{a+\beta+1}{4}, \frac{a+\beta+2}{4}, \frac{a+\beta+3}{4}\end{matrix} \mid \frac{1}{4}\right] \tag{4.1}$$

In Theorem 3.1, if we take $a = b$ then we get following result,

Corollary 4.2. *For* $\text{Re}(\alpha) > 0, \text{Re}(\beta) > 0$ *and b is non zero constants and where* $0 \leq x \leq 1,$ *the following result holds true.*

$$\int_0^1 x^{\alpha-1}(1-x)^{\beta-1} b^{-\alpha-\beta} F_1^1\left[\begin{matrix}b\\b\end{matrix} \mid 4x^2b^2(1-x)^2\right] dx$$

$$= \left(\frac{\Gamma\alpha\Gamma\beta}{\Gamma(\alpha+\beta)b^{\alpha+\beta}}\right) F_4^4\left[\begin{matrix}\frac{a}{2}, \frac{\beta}{2}, \frac{a+1}{2}, \frac{\beta+1}{2}\\ \frac{a+\beta}{4}, \frac{a+\beta+1}{4}, \frac{a+\beta+2}{4}, \frac{a+\beta+3}{4}\end{matrix} \mid \frac{1}{4b^2}\right]$$

References

[1] T. M. MacRobert, Beta functions formula and integral involving *E*-function, Math. Annalen. 142 (1961), 450-452.

[2] S. Ali, On an interesting integral involving gauss's Hypergeometric function, Advances in Computational Mathematics and its Applications 1(4) (2012), 244-246.

[3] S. Ali, On a reducibility of the kampe de feriet Function, Advances in Computational Mathematics and its Applications 1(1) (2012), 24-27.

[4] Pandey and Ujjwal, On an interesting double integral involving Hypergeometric function, Advances in Computational Mathematics and its Applications 1(4) (2012), 188-190.

[5] S. Ali and B. Usha, An integral containing Hypergeometric function, Advances in Computational Mathematics and its Applications 2(2) (2012), 263-266.

[6] E. D. Rainville, Special functions, The Macmillan Company, Yew York, (1961).

[7] Bateman Harry, Tables of integral transforms I and II, McGraw-Hill (1954).

[8] M. A. Sharma, A double integral using Dixon theorem, Advances in Computational Mathematics and its Applications 1(2) (2012), 136-138.

[9] S. A. Ali, A single integral involving kampe deferiet function, Advances in Mechanical Engineering and its Applications 1(1) (2012), 9-11.

[10] Sunil Joshi and Rupakshi Pandey Mishra, On a Finite integral involving generalized Hypergeometric function, Advances in Computational Mathematics and its Applications 1(3) (2012), 322-323.

[11] W. N. Bailey, Generalized hyper geometric series, Cambridge University Press, Cambridge (1932).

[12] I. N. Sneddon, The use of integral transforms, Tata Mcgraw-Hill, New Delhi, (1979).

[13] H. M. Srivastava and P. W. Karlson, Multiple gaussian hyper geometric series, Halsted Press (Ellis Horwood Limited, Chichester), John Wiley and Sons, New York, Chichester, Brisbane and Toronto, (1985).

[14] A. K. Rathie, A new generalization of generalized Hypergeometric function, Le Mathematic he 52 (1997), 297-310.