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COMMON FIXED POINT THEOREMS FOR RATIONALLY CONTRACTIVE MAPS IN COMPLEX VALUED METRIC SPACE

POONAM RANI, ARTI SAXENA and PRAGATI GAUTAM

Faculty of Engineering and Technology
Manav Rachna International Institute of Research and Studies
Faridabad, Haryana, 121004
Kamala Nehru College
University of Delhi, India
E-mail: poonammalhotra2692@gmail.com

1. Abstract

The focus of this article is to find a fixed point which is common for a rationally contractive pair of mappings in the setting of a complete Complex valued metric space. The result is the generalization of a variety of established theories.

2. Introduction

An unfamiliar metric space with complex codomain, which is nothing but generalization than well-known metric space, was proposed by Azam et al. [1] and known as Complex valued metric space. They have established several fixed point outcomes for a pair of contraction mappings for rational expression. Azam et al. [1] strengthened the Banach contraction principle in the framework of metric space which was Complex valued encompassing rationally contractive behaviour that could not be relevant in cone metric spaces. Ume [12] used the concept of a family of weak quasi metrics and generating spaces of quasi metric family. In the entire article *CVMS* exemplify Complex valued metric space and *CFP* for common fixed point. Numerous mathematicians have analyzed various *CFP* formulations in *CVMS* (see [14-17]).

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Banach Fixed Point Theorem has been a great source of inspiration for ancient and modern mathematicians and the various branches of science and technology. Until 1968, the contraction principle authorized by Banach [13], was the only primary tool used to examine the existence and uniqueness of fixed points. It was considered to be the root of the theory of metric fixed point, but it was not because it suffered a single disadvantage. This requires continual behavior of mapping at each and every point in the defined domain. In 1968, Kannan [18] implemented a contractivity based condition that had a single fixed point, comparable to Banach yet, unlike most of the Banachcondition, mappings have been made which have a discontinuity in their domain but have fixed points, while these mappings are continuous at a fixed point.The notion of metric space was generalized as2-metric space by Ghaler [4], which was followed by numerous papers covering this generalized space.The material having broad quantification in case of comprehensive of metric spaces likewise semi metric spaces, as well as Pseudo metric spaces have been the study of interest for so long [see, 2-11]. So we follow some kinds of definitions as well as notations which will be utilized in the discussion of the various subsequent. Here in complex valued space, metric we can learn the result of wide improvement that is involved in analysis of main theorem. In this article $\mathbb{R}^+$ and $\mathbb{C}$ are used to represent the nonnegative real number and complex numbers respectively.

3. Preliminaries

In accordance with Azam et al. [1], the described concepts and outcomes are used in relevant context.

"Let $\mathbb{C}$ be the set of all complex numbers or $z_1, z_2 \in \mathbb{C}$, define a partial order $\precsim$ on $\mathbb{C}$ as follows

$$z_1 \precsim z_2 \text{ iff } \operatorname{Re}(z_1) \leq \operatorname{Re}(z_2), \operatorname{Im}(z_1) \leq \operatorname{Im}(z_2).$$

It follows that, $z_1 \precsim z_2$ if one of the following conditions is satisfied:

(1) $\operatorname{Re}(z_1) \leq \operatorname{Re}(z_2), \operatorname{Im}(z_1) < \operatorname{Im}(z_2)$

(2) $\operatorname{Re}(z_1) < \operatorname{Re}(z_2), \operatorname{Im}(z_1) = \operatorname{Im}(z_2)$

(3) $\mathrm{Re}(z_1) < \mathrm{Re}(z_2)$, $\mathrm{Im}(z_1) < \mathrm{Im}(z_2)$

(4) $\mathrm{Re}(z_1) = \mathrm{Re}(z_2)$, $\mathrm{Im}(z_1) = \mathrm{Im}(z_2)$

In particular, we will write $z_1 \precnsim z_2$ if $z_1 \neq z_2$ and one the above conditions is not satisfied and we will write $z_1 \prec z_2$ if only iii is satisfied. Note that $0 \precsim z_1 \precnsim z_2$ implies $|z_1| < |z_2|$, $z_1 \precsim z_2$, $z_2 \prec z_3$ implies $z_2 \prec z_3$.

Definition 1.1. [1]. Let $\mathcal{C}$ be a nonempty set. A mapping $\partial : \mathcal{C} \times \mathcal{C} \to \mathbb{C}$ is called a complex valued metric on $\mathcal{C}$ if the following axioms are satisfied:

(c_1) $0 \precsim \partial(x, y)$ for all $x, y = \mathcal{C}$ and $\partial(x, y) = 0$ iff $x = y$,

(c_2) $\partial(x, y) = \partial(y, x)$ for all $x, y = \mathcal{C}$

(c_3) $\partial(x, z) \precsim \partial(x, z) + \partial(z, y)$ for all $x, y, z \in \mathcal{C}$.

In this case, we say that $(\mathcal{C}, \partial)$ is a complex valued metric space.

Definition 1.2 [1]. Let $(\mathcal{C}, \partial)$ be a complex valued metric space,

- We say that a sequence $\{x_n\}$ is said to be a Cauchy sequence be a sequence in $x \in \mathcal{C}$. If for every $c \in \mathbb{C}$, with $0 \prec c$ there is $n_0 \in \mathbb{N}$ such that for all $n < n_0$ such that $\partial(x_n, x_m) \prec c$.

- We say that a sequence $\{x_n\}$ converges to an element $x \in X$. If for every $c \in \mathbb{C}$, with $0 \prec c$ their exist an integer $n_0 \in \mathbb{N}$ such that for all $n > n_0$ such that $\partial(x_n, x_m) \prec c$ and we write $x_n \vec{\partial} x$.

- We say that $\partial(\mathcal{C}, \partial)$ is complete if every Cauchy sequence in $\mathcal{C}$ converges to a point in $\mathcal{C}$.

Lemma 1.1 [1]. *Any sequence $\{x_n\}$ in complex valued metric space $(\mathcal{C}, \partial)$ converges to x if and only if $|\partial(x_n, x)| \to 0$ as $n \to \infty$.*

Lemma 1.2 [1]. *Any sequence $\{x_n\}$ in complex valued metric space $(\mathcal{C}, \partial)$ is a Cauchy sequence if and only if $|\partial(x_n, x_{n+m})| \to 0$ as $n \to \infty$, where $m \in \mathbb{N}$.*"

4. Main Results

Theorem 2.1. *Consider* $(\mathcal{C}, \partial)$ *to be a complete CVMS and let the mappings* $\beta, \mathcal{S} : \mathcal{C} \to \mathcal{C}$ *satisfying*

$$\partial(\beta\chi, \mathcal{S}\omega) \precsim \delta\left(\frac{\partial(\omega, \beta\chi)(1 + \partial(\mathcal{S}\omega, \omega)) + \partial(\chi, \beta\chi)\partial(\chi, \mathcal{S}\omega)}{1 + \partial(\chi, \omega)}\right)$$

$$+ \mu\left(\frac{\partial(\mathcal{S}\omega, \beta\chi) + \partial(\mathcal{S}\chi, \chi)\partial(\beta\chi, \chi)}{1 + \partial(\chi, \omega)}\right) + \tau\left(\frac{\partial(\omega, \beta\chi) + \partial(\chi, \mathcal{S}\omega)}{2 + \partial(\beta\chi, \omega)\partial(\chi, \mathcal{S}\omega)}\right)$$

$$\partial(\beta\chi, \mathcal{S}\omega) \precsim \delta\left(\frac{\partial(\omega, \beta\chi)(1 + \partial(\mathcal{S}\omega, \omega)) + \partial(\chi, \beta\chi)\partial(\chi, \mathcal{S}\omega)}{1 + \partial(\chi, \omega)}\right)$$

$$+ \mu\left(\frac{\partial(\mathcal{S}\omega, \beta\chi) + \partial(\mathcal{S}\omega, \omega)\partial(\beta\chi, \chi)}{1 + \partial(\chi, \omega)}\right) + \tau\left(\frac{\partial(\omega, \beta\chi) + \partial(\chi, \mathcal{S}\omega)}{2 + \partial(\beta\chi, \omega)\partial(\chi, \mathcal{S}\omega)}\right)$$

$\forall \chi, \omega \in \mathcal{C}$, *where* $\delta, \mu, \rho \in \mathbb{R}^+$ *with* $(2\delta + \mu + \rho) < 1$. *Then there is an existence of CFP for* β *and* $\mathcal{S}$ *and the fixed point is unique as well.*

Proof. If we assume χ_0 as an arbitrarily chosen point of $\mathcal{C}$, by define

$$\begin{cases} \chi_{2s+1} = \beta\chi_{2s} \\ \chi_{2s+2} = \mathcal{S}\chi_{2s}, \; s = 0, 1, 2, \ldots \end{cases} \tag{1}$$

then the main equation of Theorem 2.1 results,

$$\partial(\chi_{2s+1}, \chi_{2s+2}) = \partial(\beta\chi_{2s}, \mathcal{S}\chi_{2s+1})$$

$$\precsim \delta\left(\frac{\partial(\chi_{2s+1}, \beta\chi_{2s})(1 + \partial(\mathcal{S}\chi_{2s+1}, \chi_{2s+1})) + \partial(\chi_{2s}, \beta\chi_{2s})\partial(\chi_{2s}, \mathcal{S}\chi_{2s+1})}{1 + \partial(\chi_{2s}, \chi_{2s+1})}\right)$$

$$+ \mu\left(\frac{\partial(\mathcal{S}\chi_{2s+1}, \beta\chi_{2s}) + \partial(\mathcal{S}\chi_{2s+1}, \chi_{2s+1})\partial(\beta\chi_{2s}, \chi_{2s})}{1 + \partial(\chi_{2s}, \chi_{2s+1})}\right)$$

$$+ \rho\left(\frac{\partial(\chi_{2s+1}, \beta\chi_{2s}) + \partial(\chi_{2s}, \mathcal{S}\chi_{2s+1})}{2 + \partial(\beta\chi_{2s}, \chi_{2s+1})\partial(\chi_{2s}, \mathcal{S}\chi_{2s+1})}\right)$$

using the defining criteria of sequence by equation (1), we have result as

$$\precsim \delta\left(\frac{\partial(\chi_{2s+1}, \chi_{2s+1})(1 + \partial(\chi_{2s+2}, \chi_{2s+1})) + \partial(\chi_{2s}, \chi_{2s+1})\partial(\chi_{2s}, \chi_{2s+2})}{1 + \partial(\chi_{2s}, \chi_{2s+1})}\right)$$

$$+\mu\left(\frac{\partial(\chi_{2s+2}, \chi_{2s+1})+\partial(\chi_{2s+2}, \chi_{2s+1})\partial(\chi_{2s+1}, \chi_{2s})}{1+\partial(\chi_{2s}, \chi_{2s+1})}\right)$$

$$+\rho\left(\frac{\partial(\chi_{2s+1}, \chi_{2s+1})+\partial(\chi_{2s}, \chi_{2s+2})}{2+\partial(\chi_{2s+1}, \chi_{2s+1})\partial(\chi_{2s}, \chi_{2s+2})}\right)$$

using the axioms (c_1) and (c_3), then the resultant becomes

$$\partial(\chi_{2s+1}, \chi_{2s+2}) \precsim (\partial(\chi_{2s}, \chi_{2s+1})+\partial(\chi_{2s+1}, \chi_{2s+2}))+\mu(\partial(\chi_{2s+2}, \chi_{2s+1}))$$

$$+\frac{\rho}{2}(\partial(\chi_{2s}, \chi_{2s+1})+\partial(\chi_{2s+1}, \chi_{2s+2}))$$

since it is obvious that $\frac{\rho\,\partial(\chi_{2s}, \chi_{2s+1})}{1+\partial(\chi_{2s}, \chi_{2s+1})}<1$. Also,

$$\left(1-\delta-\mu-\frac{\rho}{2}\right)\partial(\chi_{2s+1}, \chi_{2s+2}) \precsim \left(\delta+\frac{\rho}{2}\right)\partial(\chi_{2s}, \chi_{2s+1})$$

$$\partial(\chi_{2s+1}, \chi_{2s+2}) \precsim \frac{\left(\delta+\frac{\rho}{2}\right)}{\left(1-\delta-\mu-\frac{\rho}{2}\right)}\partial(\chi_{2s}, \chi_{2s+1})$$

using the condition defined on δ, μ and γ, we have

$$\partial(\chi_{2s+1}, \chi_{2s+2}) \precsim h\partial(\chi_{2s}, \chi_{2s+1}), \text{ where } h=\frac{(\delta+\rho)}{(1-\delta-\mu-\rho)}.$$

Similarly, it is obvious that $\partial(\chi_{2s+2}, \chi_{2s+3})=\partial(\beta\chi_{2s+2}, \mathcal{S}\chi_{2s+1})$

$$\precsim \delta\left(\frac{\partial(\chi_{2s+1}, \beta\chi_{2s+2})(1+\partial(\chi_{2s+1}, \chi_{2s+1}))+\partial(\chi_{2s+2}, \beta\chi_{2s+2})\partial(\chi_{2s+2}, \mathcal{S}\chi_{2s+1})}{1+\partial(\chi_{2s+2}, \chi_{2s+1})}\right)$$

$$+\mu\left(\frac{\partial(\mathcal{S}\chi_{2s+1}, \beta\chi_{2s+2})+\partial(\mathcal{S}\chi_{2s+1}, \chi_{2s+1})\partial(\beta\chi_{2s+2}, \chi_{2s+2})}{1+\partial(\chi_{2s+2}, \chi_{2s+1})}\right)$$

$$+\rho\left(\frac{\partial(\chi_{2s+1}, \beta\chi_{2s+2})+\partial(\chi_{2s+2}, \mathcal{S}\chi_{2s+1})}{2+\partial(\beta\chi_{2s+2}, \chi_{2s+1})\partial(\chi_{2s+2}, \mathcal{S}\chi_{2s+1})}\right)$$

using the defining criteria of sequence by equation (1), we have result as

$$\precsim \delta\left(\frac{\partial(\chi_{2s+1}, \chi_{2s+3})(1+\partial(\chi_{2s+2}, \chi_{2s+1})) + \partial(\chi_{2s+2}, \chi_{2s+3})\partial(\chi_{2s+2}, \chi_{2s+2})}{1+\partial(\chi_{2s+2}, \chi_{2s+1})}\right)$$

$$+ \mu\left(\frac{\partial(\chi_{2s+2}, \chi_{2s+3}) + \partial(\chi_{2s+2}, \chi_{2s+1})\partial(\chi_{2s+3}, \chi_{2s+2})}{1+\partial(\chi_{2s+2}, \chi_{2s+1})}\right)$$

$$+ \rho\left(\frac{\partial(\chi_{2s+1}, \chi_{2s+3}) + \partial(\chi_{2s+2}, \chi_{2s+2})}{2+\partial(\chi_{2s+3}, \chi_{2s+1})\partial(\chi_{2s+2}, \chi_{2s+2})}\right)$$

using the axioms (c_1) and (c_3), it can easily be concluded that

$$\partial(\chi_{2s+2}, \chi_{2s+3}) \precsim \delta(\partial(\chi_{2s+1}, \chi_{2s+2}) + \partial(\chi_{2s+2}, \chi_{2s+3})) + \mu(\partial(\chi_{2s+2}, \chi_{2s+3}))$$

$$+ \frac{\rho}{2}(\partial(\chi_{2s+1}, \chi_{2s+2}) + \partial(\chi_{2s+2}, \chi_{2s+3}))$$

$$(1-\delta-\mu-\rho)\partial(\chi_{2s+2}, \chi_{2s+2}) \precsim \left(\delta+\frac{\rho}{2}\right)\partial(\chi_{2s}, \chi_{2s+1})$$

$$\partial(\chi_{2s+1}, \chi_{2s+2}) \precsim \frac{\left(\delta+\frac{\rho}{2}\right)}{(1-\delta-\mu-\rho)}\partial(\chi_{2s}, \chi_{2s+1})$$

using the condition defined on δ, μ and γ, we have

$$\partial(\chi_{s+1}, \chi_{s+2}) \precsim h\partial(\chi_s, \chi_{s+1})$$

$$\precsim h^2\partial(\chi_{s-1}, \chi_s)$$

$$\precsim h^3\partial(\chi_{s-2}, \chi_{s-1}), \ldots, \precsim h^{s-1}\partial(\chi_0, \chi_1).$$

For the case $r > s$,

$$\partial(\chi_s, \chi_r) \precsim \partial(\chi_s, \chi_{s+1}) + \partial(\chi_{s+1}, \chi_{s+2}) + \ldots + \partial(\chi_{r-1}, \chi_r)$$

$$\precsim [k^2 + h^{s+1} + h^{s+2} + \ldots + h^{r-1}]\partial(\chi_0, \chi_1)$$

$$\precsim \left[\frac{h^2}{1-h}\right]\partial(\chi_0, \chi_1)$$

and hence we have, $|\,\partial(\chi_s, \chi_r)\,| \precsim \left[\frac{h^s}{1-h}\right]|\,\partial(\chi_0, \chi_1)$ which converges to 0 as n

approaches to ∞ and hence $\{\chi_s\}$ is a sequence which satisfies Cauchy's behavior and since it is also given that the space $\mathcal{C}$ is complete, so the limiting value, let it be η and $\eta \in \mathcal{C}$ such that $\{\chi_s\}$ converges to η, which follows that $\eta = \beta\eta$, if not then $\partial(\beta\eta, \eta) = p > 0$ and in such a case

$$p \precsim \partial(\eta, \chi_{2s+2}) + \partial(\chi_{2s+2}, \beta\eta)$$

$$\precsim \partial(\eta, \chi_{2s+2}) + \partial(\mathcal{S}\chi_{2s+1}, \beta\eta)$$

$$\precsim \partial(\eta, \chi_{2s+2}) + \partial(\beta\eta, \mathcal{S}\chi_{2s+1})$$

$$\precsim \partial(\eta, \chi_{2s+2}) + \delta\left(\frac{\partial(\chi_{2s+1}, \beta\eta)(1 + \partial(\chi_{2s+2}, \chi_{2s+1})) + \partial(\eta, \beta\eta)\partial(\eta, \chi_{2s+2})}{1 + \partial(\eta, \chi_{2s+1})}\right)$$

$$+ \mu\left(\frac{\partial(\chi_{2s+2}, \beta\eta) + \partial(\chi_{2s+2}, \chi_{2s+1})p}{1 + \partial(\chi_{2s}, \chi_{2s+1})}\right)$$

$$+ \rho\left(\frac{\partial(\chi_{2s+1}, \beta\eta) + \partial(\eta, \chi_{2s+2})}{2 + \partial(\beta\eta, \chi_{2s+1})\partial(\eta, \chi_{2s+2})}\right)$$

which results, $| p | \precsim | \partial(\eta, \chi_{2s+2}) |$

$$+ \delta\frac{| \partial(\chi_{2s+1}, \beta\eta)(1 + \partial(\chi_{2s+2}, \chi_{2s+1})) + \partial(\eta, \beta\eta)\partial(\eta, \chi_{2s+2}) |}{| 1 + \partial(\eta, \chi_{2s+1}) |}$$

$$+ \mu\frac{| \partial(\chi_{2s+2}, \eta\beta) + \partial(\chi_{2s+2}, \chi_{2s+1}) || p |}{| 1 + \partial(\chi_{2s}, \chi_{2s+1}) |}$$

$$+ \rho\frac{| \partial(\chi_{2s+1}, \beta\eta) + \partial(\eta, \chi_{2s+2}) |}{| 2 + \partial(\beta\eta, \chi_{2s+1})\partial(\eta, \chi_{2s+2}) |},$$

results $| p | = 0$, which is a contradiction and hence $\beta\eta = \eta$. It tends to follow the same way as $\mathcal{S}\eta = \eta$. Now, it is to prove that β and $\mathcal{S}$ have a *CFP* with uniqueness. Make the assumption, for this reason, that in $\eta^* \in \mathcal{C}$ is a distinct *CFP* point of β and $\mathcal{S}$, $\partial(\eta, \eta^*) = \partial(\beta\eta, \mathcal{S}\eta^*)$

$$\precsim \delta\left(\frac{\partial(\eta^*, \beta\eta)(1 + \partial(\mathcal{S}\eta^*, \eta^*)) + \partial(\eta, \beta\eta)\partial(\eta, \mathcal{S}\eta^*)}{1 + \partial(\eta^*, \eta^*)}\right)$$

$$+\mu\left(\frac{\partial(\mathcal{S}\eta^*, \beta\eta)+\partial(\mathcal{S}\eta^*, \eta^*)\partial(\beta\eta, \eta)}{1+\partial(\eta, \eta^*)}\right)+\rho\left(\frac{\partial(\eta^*, \beta\eta)+\partial(\eta, \mathcal{S}\eta^*)}{2+\partial(\beta\eta, \eta^*)\partial(\eta, \mathcal{S}\eta^*)}\right)$$

using the defining criteria of sequence by equation (1), we have result as

$$\precsim \delta\left(\frac{\partial(\eta^*, \eta)(1+\partial(\eta^*, \eta^*))+\partial(\eta, \eta)\partial(\eta, \eta^*)}{1+\partial(\eta, \eta^*)}\right)$$

$$+\mu\left(\frac{\partial(\eta^*, \eta)+\partial(\eta^*, \eta^*)\partial(\eta, \eta)}{1+\partial(\eta, \eta^*)}\right)+\rho\left(\frac{\partial(\eta^*, \eta)+\partial(\eta, \eta^*)}{2+\partial(\eta, \eta^*)\partial(\eta, \eta^*)}\right)$$

using the axioms (c_1) and (c_3), we have $\partial(\eta^*, \eta)\precsim[\delta+\mu+\rho]\partial(\eta^*, \eta)$ since it is obvious that $\frac{1}{1+\partial(\eta, \eta^*)}<1$ and $\frac{2}{2+\partial(\eta, \eta^*)\partial(\eta, \eta^*)}<1,$ which confirms the uniqueness and the completion of the confirmation of this theorem.

Corollary 2.1. *Let* $(\mathcal{C}, \partial)$ *be a complete complex valued metric space and let the mappings* $\mathcal{S}:\mathcal{C}\to\mathcal{C}$ *satisfying*

$$\partial(\mathcal{S}\chi, \mathcal{S}\omega)\precsim\delta\left(\frac{\partial(\omega, \mathcal{S}\chi)(1+\partial(\mathcal{S}\omega, \omega))+\partial(\chi, \mathcal{S}\chi)\partial(\chi, \mathcal{S}\omega)}{1+\partial(\chi, \omega)}\right)$$

$$+\mu\left(\frac{\partial(\mathcal{S}\omega, \mathcal{S}\chi)\,\partial(\mathcal{S}\omega, \omega))+\partial(\mathcal{S}\chi, \chi)}{1+\partial(\chi, \omega)}\right)$$

$$+\rho\left(\frac{\partial(\omega, \mathcal{S}\chi)+\partial(\chi, \mathcal{S}\omega,)}{2+\partial(\mathcal{S}\chi, \omega)\partial(\chi, \mathcal{S}\omega)}\right)$$

for all $\chi, \omega\in\mathcal{C}$, where δ, μ and ρ are nonnegative reals with $(2\delta+\mu+\rho)<1.$ Then $\mathcal{S}$ has a unique fixed point.

Corollary 2.2. *Let* $(\mathcal{C}, \partial)$ *be a complete complex valued metric space and let the mappings* $\mathcal{S}:\mathcal{C}\to\mathcal{C}$ *satisfying*

$$\partial\left(\mathcal{S}^s\chi, \mathcal{S}^s\omega\right)\precsim\delta\left(\frac{\partial(\omega, \mathcal{S}^s\chi)(1+\partial(\mathcal{S}^s\omega, \omega))+\partial(\chi, \mathcal{S}^s\omega)\partial(\chi, \mathcal{S}^s\omega)}{1+\partial(\chi, \omega)}\right)$$

$$+\mu\left(\frac{\partial(\mathcal{S}^s\omega, \mathcal{S}^s\chi)\,\partial(\mathcal{S}^s\omega, \omega))+\partial(\mathcal{S}^s\chi, \chi)}{1+\partial(\chi, \omega)}\right)$$

$$+ \rho\left(\frac{\partial(\omega, \mathcal{S}^s\chi) + \partial(\chi, \mathcal{S}^s\omega)}{2 + \partial(\mathcal{S}^s\chi, \omega)\partial(\chi, \mathcal{S}^s\omega)}\right)$$

for all $\chi, \omega \in \mathcal{C}$, *where* $\delta, \mu, \rho \in \mathbb{R}^+$ *with* $(2\delta + \mu + \rho) < 1$. *Then* $\mathcal{S}$ *has a unique fixed point.*

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