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DOMINATOR CHROMATIC NUMBER OF CENTRAL GRAPH OF SOME GRAPHS

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Abstract

Let $G = (V, E)$ be a simple, connected, undirected and finite graph. In this paper, we obtain the dominator chromatic number of central graph of Cocktail party graph, n-Antiprism graph, Musical graph. A graph G has a dominator coloring if it has a proper coloring in which each vertex of the graph dominates every vertex of some color class. The minimum number of color classes needed for the dominator coloring of a graph G is the dominator chromatic number and is denoted by $\chi_d(G)$.

1. Introduction

This concept was introduced by Ralucca Michelle Gera in 2006. The notion of central graph and dominator coloring are reviewed in the following section. Let G be a graph such that V is the vertex set and E is the edge set. A dominating set S is a subset of the vertex set V of graph G such that every vertex in the graph either belongs to S or adjacent to S.

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2. Preliminaries

Definition 2.1 [1]. A central graph $C(G)$ is obtained by subdividing each edge of G exactly once and joining all the non-adjacent vertices of G.

Definition 2.1 [1] [4]. Let G be a simple and undirected graph and let its vertex set and edge set be denoted by $V(G)$ and $E(G)$. The central graph of G, denoted by $C(G)$ is obtained by subdividing each edge of G exactly once and joining all the non-adjacent vertices of G in $C(G)$.

Definition 2.2 [1] [4]. A proper coloring of a graph G is an assignment of colors to the vertices of G in such a way that no two adjacent vertices receive the same color.

Definition 2.3 [1] [4]. The chromatic number $\chi(G)$, is the minimum number of colors required for a proper coloring of G.

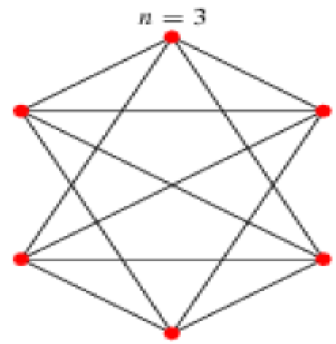
Definition 2.4 [1] [4]. A color class is the set of all vertices, having the same color. The color class corresponding to the color i is denoted by V_i.

Definition 2.5 [2]. A dominator coloring of a graph G is a proper coloring in which every vertex of G dominates every vertex of at least one color class. The convention is that if $\{v\}$ is a color class, then v dominates the color class $\{v\}$. The dominator chromatic number $\chi_d(G)$ is the minimum number of colors required for a dominator coloring of G.

3. Dominator Chromatic Number of Central Graph of Cocktail Party Graph

In this section, we obtained the dominator chromatic number of central graph of the Cocktail party graph.

Definition 3.1 [3]. The Cocktail party graph is a graph consisting of two rows of paired vertices in which all the vertices except the paired ones are joined by an edge and is denoted by CP_k, where $k = 2n$, for all $n \geq 2$. It is also called hyper octahedral graph or Roberts graph.



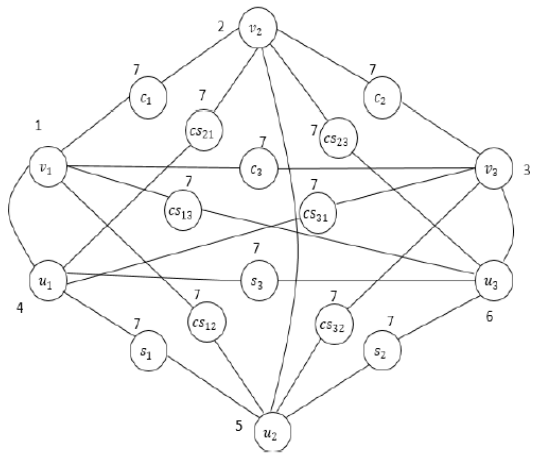
Example 3.2. Cocktail Party graph CP_k with $k = 6$ and $n = 3$.

Theorem 3.2. *For any n, the dominator chromatic number of central graph of Cocktail Party graph* $G = CP_x$ *of x-vertices, where* $x = 2n$, *then* $\chi_d[C(G)] = 2n + 1,\ n > 2$.

Proof. Let G be the Cocktail Party graph with x vertices, where $x = 2n$. So, by the definition of Cocktail Party graph, the vertex set of G is partitioned into two subsets V_1 and V_2 such that $V_1 \cup V_2 = V$ and $V_1 \cap V_2 = \emptyset$. Let $V_1 = \{v_1, v_2, v_3, \cdots, v_n\}$ and $V_2 = \{u_1, u_2, u_3, \cdots, u_n\}$ be the two subsets of $V(G)$ and all the vertices of V_1 are connected to each vertex of V_2 except the paired ones (i.e.) v_i is not adjacent to v_j for $i = j$, where $i, j = 1, 2, 3, \cdots, n$. By the definition of central graph, each edge e_{ij} for $1 \leq i \leq n, 1 \leq j \leq n$ is subdivided by a vertex c_{ij} in V_1, s_{ij} in V_2 and cs_{ij} in $V_1 \cup V_2$, $i \neq j$, $C(G)$ and join all the non-adjacent vertices of G. Let $V_1 = \{v_1, v_2, v_3, \cdots, v_n\}, V_2 = \{u_1, u_2, u_3, \cdots, u_n\}, V_3 = \{c_{ij}/1 \leq i \leq n, j \leq n, i \neq j\}$, $V_4 = \{s_{ij}/1 \leq i \leq n, 1 \leq j \leq n,, i \neq j\}$, $V_5 = \{cs_{ij}/1 \leq i \leq n, 1 \leq j \leq n,, i \neq j\}$ such that $V_1 \cup V_2 \cup V_3 \cup V_4 \cup V_5$.

The following procedure gives the dominator coloring of $C(G)$. In V_1 subset, we assign the color 1 to v_1 and v_2 vertices and color the remaining vertices v_i by $2, 3, 4, \cdots, n-1$ for $i = 3, 4, 5, \cdots, n$. Similarly, in V_2 subset, color the vertices u_j by $n, n+1, n+2, \cdots, k-1$ for $j = 1, 2, 3, \cdots, n$. Color all the remaining vertices c_{ij}, s_{ij}, cs_{ij} by k for $i = 1, 2, 3, \cdots, n$, $j = 1, 2, 3, \cdots, n$. Clearly, no two adjacent vertices have the same color and also each vertex of the graph dominates every vertex of some color class which satisfies the condition of dominator coloring. Hence, the dominator chromatic number of central graph of Cocktail Party graph is $\chi_d[C(G)] = 2n + 1$, $n > 2$.

Example 3.4. The central graph of CP_6 is depicted with a dominator coloring.



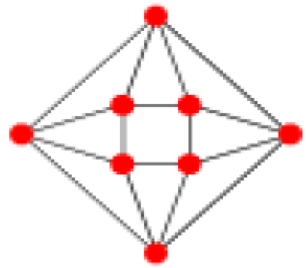
The color classes of $C(CP_6)$ are $V_1 = \{v_1\}$, $V_2 = \{v_2\}$, $V_3 = \{v_3\}$, $V_4 = \{u_1\}$, $V_5 = \{u_2\}$, $V_6 = \{u_3\}$, $V_7 = \{c_1, c_2, c_3, s_1, s_2, s_3, cs_{12}, cs_{13}, cs_{21}, cs_{23}, cs_{31}, cs_{32},\}$. The dominator chromatic number is, $\chi_d[C(CP_6)] = 7$.

4. Dominator Chromatic Number of Central Graph of *n*-Antiprism Graph

In this section, we obtained the dominator chromatic number of central graph of the n-Antiprism graph.

Definition 4.1 [3]**.** The n-Antiprism graph is a graph which can be obtained by joining two parallel copies of cycle graph C_n by an alternative band of triangles. It is denoted by AP_n.

Example 4.2. Antiprism graph AP_4 with $k = 8$ and $n = 4$.



Theorem 4.3. *For any n, the dominator chromatic number of central graph of Antiprism graph $G = APx$ of x-vertices, where $x = 2n$, then*

$$\chi_d[C(G)] = \begin{cases} 5k+1, & n = 3k \\ 5k+3, & n = 3k+1, k = 1, 2, 3, \ldots. \\ 5k+5, & n = 3k+2 \end{cases}$$

Proof. Let $G = AP_x$ be the Antiprism graph with x-vertices, where $x = 2n$. So, by the definition of Antiprism graph, is constructed by joining two parallel copies of the cycle C_n by an alternative band of triangles. Let $V_1 = \{v_1, v_2, v_3, \cdots, v_n\}$ be the vertex set of the outer cycle C_n and $V_1 = \{u_1, u_2, u_3, \cdots, u_n\}$ be the vertex set of the inner cycle C_n. By the definition of central graph $C(G)$, each edge e_{ij} for $1 \leq i \leq n, 1 \leq j \leq n$ is subdivided by a vertex c_{ij} in V_1, s_{ij} in V_2, cs_{ij} in $V_1 \cup V_2, i \neq j$, and join all the non-adjacent vertices of. Let $V_1 = \{v_1, v_2, v_3, \cdots, v_n\}, V_2 = \{u_1, u_2, u_3, \cdots, u_n\}$, $V_3 = \{c_{ij} / 1 \leq i \leq n, 1 \leq j \leq n, i \neq j\}$, $V_4 = \{s_{ij} / 1 \leq i \leq n, 1 \leq j \leq n, , i \neq j\}$, $V_5 = \{cs_{ij} / 1 \leq i \leq n, 1 \leq j \leq n, i \neq j\}$ such that $V_1 \cup V_2 \cup V_3 \cup V_4 \cup V_5$. The following procedure gives the dominator coloring of $C(G)$.

Case (i). For $n = 3k, k = 1, 2, 3, \cdots n > 3$. Let $S = \{c_i, s_j, cs_{ij}$, for $i = 1, 2, 3, \ldots, n, j = 1, 2, 3, \ldots, n\}$ be the dominating set of G. Now, assign the dominator coloring to the graph G. Let us assign the color c_1 to the vertices c_j, s_j, cs_{ij}, for $i = 1, 2, 3, \cdots, n, j = 1, 2, 3, \cdots, n$ of the dominating set S and assign the minimum color classes $c_2, c_3, c_4, \cdots c_n$ to the remaining non-adjacent vertices of the graph. By the definition of dominator coloring all the vertices in the set S dominates the color class of S itself, also the dominating set S dominates all the color classes. Therefore, the dominator chromatic number of central graph of Antiprism graph is $5k+1, k=1, 2, 3, \cdots$

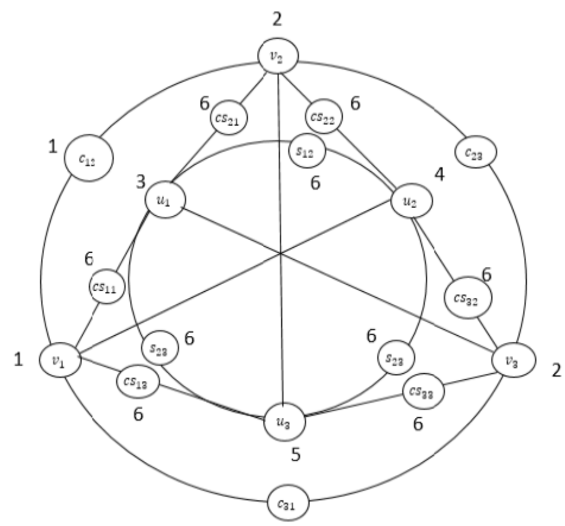
Case (ii). For $n = 3k+1$, $k = 1, 2, 3, \cdots n > 3$. Let $S = \{c_i, s_j, cs_{ij}$, for $i = 1, 2, 3, \cdots, n$, $j = 1, 2, 3, \cdots n\}$ be the dominating set of G. Now, assign the dominator coloring to the graph G. Let us assign the color c_1 to the vertices c_j, s_j, cs_{ij}, for $i = 1, 2, 3, \cdots, n$, $j = 1, 2, 3, \cdots n$ of the dominating set S and assign the minimum color classes $c_2, c_3, c_4, \cdots c_n$ to the remaining non-adjacent vertices of the graph. By the definition of dominator coloring all the vertices in the set S dominates the color class of S itself, also the dominating set S dominates all the color classes. Therefore, the dominator chromatic number of central graph of Antiprism graph is $5k+3$, $k = 1, 2, 3, \ldots$.

Case (iii). For $n = 3k+2$, $k = 1, 2, 3, \cdots n > 3$. Let $S = \{c_i, s_j, cs_{ij}$, for $i = 1, 2, 3, \cdots, n$, $j = 1, 2, 3, \cdots n\}$ be the dominating set of G. Now, assign the dominator coloring to the graph G. Let us assign the color c_1 to the vertices c_j, s_j, cs_{ij}, for $i = 1, 2, 3, \cdots, n$, $j = 1, 2, 3, \cdots n$ of the dominating set S and assign the minimum color classes $c_2, c_3, c_4, \cdots c_n$ to the remaining non-adjacent vertices of the graph. By the definition of dominator coloring all the vertices in the set S dominates the color class of S itself, also the dominating set S dominates all the color classes. Therefore, the dominator chromatic number of central graph of Antiprism graph is $5k+5$, $k = 1, 2, 3, \ldots$

By proceeding this way, we get the successive sequence of dominator chromatic number of central graph of Antiprism graph is

$$\chi_d[C(G)] = \begin{cases} 5k+1, & n = 3k \\ 5k+3, & n = 3k+1,\ k = 1, 2, 3, \ldots \\ 5k+5, & n = 3k+2 \end{cases}$$

Example 4.4. The central graph of AP_6 is depicted with a dominator coloring.



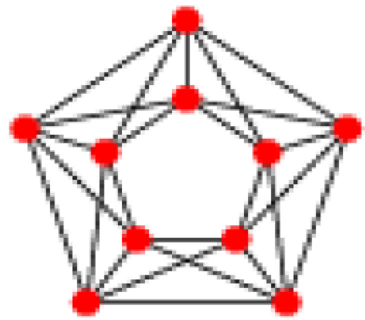
The color classes of $C(AP_6)$ are $V_1 = \{v_1, v_2\}$, $V_2 = \{v_3\}$, $V_3 = \{u_1\}$, $V_4 = \{u_2\}$, $V_5 = \{u_3\}$, $V_6 = \{c_1, c_2, c_3, s_1, s_2, s_3, cs_{11}, cs_{12}, cs_{21}, cs_{22}, cs_{32}, cs_{33}\}$. The dominator chromatic number is, $\chi_d[C(CP_6)] = 6$.

5. Dominator Chromatic Number of Central Graph of Musical Graph

In this section, we obtained the dominator chromatic number of central graph of the Musical graph.

Definition 5.1 [3]**.** The Musical graph $n \geq 3$ of order n consists of two parallel copies of cycle graph C_n in which all the paired vertices and the neighbouring vertices are connected and is denoted by M_{2n}, $\forall n \geq 3$.

Example 5.2. Musical graph M_8 with $k = 8$ and $n = 4$.



Theorem 5.3. *For any n, the dominator chromatic number of central graph of Musical graph $G = M_x$ of x-vertices, where $x = 2n$.* then

$$\chi_d[C(G)] = \begin{cases} 5k+1, & n = 3k \\ 5k+3, & n = 3k+1, \; k = 1, 2, 3, \ldots, n > 3. \\ 5k+5, & n = 3k+2 \end{cases}$$

Proof. Let $G = AP_x$ be the Musical graph with x-vertices, where $x = 2n$. So, by the definition of Musical graph, is constructed by joining two parallel copies of the cycle C_n by an alternative band of triangles. Let $V_1\{v_1, v_2, v_3, \cdots v_n\}$ be the vertex set of the outer cycle C_n and $V_2 = \{u_1, u_2, u_3, \ldots, u_n\}$ be the vertex set of the inner cycle C_n. By the definition of central graph $C(G)$, each edge e_{ij} for $1 \le i \le n, 1 \le j \le n$ is subdivided by a vertex c_{ij} in V_1, s_{ij} in V_2 cs_{ij} in $V_1 \cup V_2, i \ne j$, and join all the non-adjacent vertices of . Let $V_1 = \{v_1, v_2, v_3, \cdots v_n\}, V_2\{u_1, u_2, u_3, \cdots u_n\}$, $V_3\{c_{ij} / 1 \le i \le n, 1 \le j \le n, i \ne j\}, V_4 = \{s_{ij} / 1 \le i \le n, 1 \le j \le n, i \ne j\}, V_5 = \{cs_{ij} / 1 \le i \le n,$ $1 \le j \le n, i \ne j\}$ such that $V_1 \cup V_2 \cup V_3 \cup V_4 \cup V_5$. The following procedure gives the dominator coloring of $C(G)$.

Case (i). For $n = 3k, k = 1, 2, 3, \cdots n > 3$. Let $\{c_i, s_j, cs_{ij},$ for $i = 1, 2, 3, \cdots n, j = 1, 2, 3, \cdots n\}$ be the dominating set of G. Now, assign the dominator coloring to the graph G. Let us assign the color c_1 to the vertices c_i, s_j, cs_{ij}, for $i = 1, 2, 3, \ldots, n, j = 1, 2, 3, \ldots, n$ of the dominating set S and assign the minimum color classes $c_2, c_3, c_4, \cdots c_n$ to the remaining non-adjacent vertices of the graph. By the definition of dominator coloring all the vertices in the set S dominates the color class of S itself, also the dominating set S dominates all the color classes. Therefore, the dominator chromatic number of central graph of Musical graph is $5k + 1,\ k = 1, 2, 3, \ldots$.

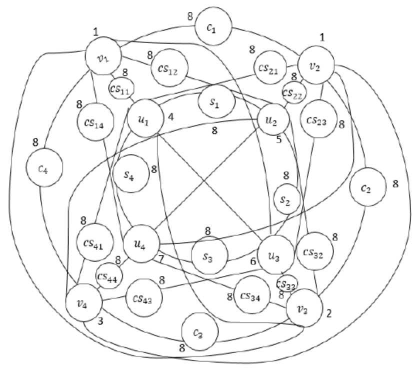
Case (ii). For $n = 3k + 1,\ k = 1, 2, 3, \ldots, n > 3$. Let $S = \{c_i, s_j, cs_{ij}$, for $i = 1, 2, 3, \ldots, n,\ j = 1, 2, 3, \ldots, n\}$ be the dominating set of G. Now, assign the dominator coloring to the graph G. Let us assign the color c_1 to the vertices c_i, s_j, cs_{ij}, for $i = 1, 2, 3, \cdots, n,\ j = 1, 2, 3, \cdots n$ of the dominating set S and assign the minimum color classes $c_2, c_3, c_4, \cdots c_n$ to the remaining non-adjacent vertices of the graph. By the definition of dominator coloring all the vertices in the set S dominates the color class of S itself, also the dominating set S dominates all the color classes. Therefore, the dominator chromatic number of central graph of Musical graph is $5k + 3,\ k = 1, 2, 3, \ldots$.

Case (iii). For $n = 3k + 2,\ k = 1, 2, 3, \ldots$. Let $S = \{c_i, s_j, cs_{ij}$, for $i = 1, 2, 3, \ldots, n,\ j = 1, 2, 3, \ldots, n\}$ be the dominating set of G. Now, assign the dominator coloring to the graph G. Let us assign the color c_1 to the vertices c_i, s_j, cs_{ij}, for $i = 1, 2, 3, \ldots, n,\ j = 1, 2, 3, \ldots, n$ of the dominating set S and assign the minimum color classes $c_2, c_3, c_4, \cdots c_n$ to the remaining non-adjacent vertices of the graph. By the definition of dominator coloring all the vertices in the set S dominates the color class of S itself, also the dominating set S dominates all the color classes. Therefore, the dominator chromatic number of central graph of Musical graph is $5k + 5,\ k = 1, 2, 3, \ldots$.

By proceeding this way, we get the successive sequence of dominator chromatic number of central graph of Musical graph is

$$\chi_d[C(G)] = \begin{cases} 5k + 1, & n = 3k \\ 5k + 3, & n = 3k + 1,\ k = 1, 2, 3, \ldots, n > 3. \\ 5k + 5, & n = 3k + 2 \end{cases}$$

Example 5.4. The central graph of M_8 is depicted with a dominator coloring.



The color classes of $C(M_6)$ are $V_1 = \{v_1, v_2\}$, $V_2 = \{v_3\}$, $V_3 = \{v_4\}$, $V_4 = \{u_1\}$, $V_5 = \{u_2\}$, $V_6 = \{u_3\}$, $V_7 = \{u_4\}$, $V_8 = \{c_1, c_2, c_3, c_4, s_1, s_2, s_3, s_4, cs_{11}, cs_{12}, cs_{14}, cs_{21}, cs_{22}, cs_{23}, cs_{32}, cs_{33}, cs_{34}, cs_{41}, cs_{43}, cs_{44}\}$. The dominator chromatic number is, $\chi_d[C(CP_6)] = 8$.

6. Results

- $\chi_d[C(CP_x)] > \chi_d[CP_x]$, $x = 2n$
- $\chi_d[C(AP_x)] = \chi_d[C(M_x)]$, $x = 2n$, $\forall n \geq 4$
- $\chi_d[C(CP_x)] > \chi_d[C(AP_x)]$, $x = 2n$, $\forall n \geq 3$
- $\chi_d[C(CP_x)] > \chi_d[C(M_x)]$, $x = 2n$, $\forall n \geq 4$.

7. Conclusion

In this paper, we obtained the dominator chromatic number of central graph of some graphs. This paper can further be extended to find the dominator chromatic number of various graph families.

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