Advances and Applications in Mathematical Sciences
Volume 21, Issue 8, June 2022, Pages 4427-4432

FURTHER RESULTS ON EQUITABLE POWER DOMINATION NUMBER OF GRAPHS

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Abstract

A set $S \subseteq V$ of a graph $G(V, E)$ is known as a power dominating set (PDS) if each node $x \in V - S$ is dominated by certain nodes from S by the following conditions: (1) When a node $u \in G$ is in PDS, then u dominates itself and each neighboring node of u and (2) When a dominated $u \in G$ with $r > 1$ neighboring nodes and if $r - 1$ of those neighboring nodes are already dominated, then the one non-dominated node too is dominated by u. A PDS $S \subseteq V(G)$ is known as an equitable power dominating set (EPDS), if for each $u \in V - S$ there is an adjacent $v \in S$ such that $|d(u) - d(v)| \leq 1$, where $d(u)$ denotes the degree of u. The least cardinality of an EPDS in G is known as the equitable power domination number (EPDN) of G, represented by $\gamma_{epd}(G)$. In this article, we derive EPDN of graphs constructed by performing subdivision on various classes of graphs.

1. Introduction

A dominating set (DS) is a set $S \subseteq V(G)$ so that any node in $V - S$ is adjacent to a node in S. A DS with least cardinality in G is known as the domination number of G, represented by $\gamma_d(G)$. $AD \subseteq V(G)$ is known as an

2020 Mathematics Subject Classification: 05C69.

Keywords: Power Domination, Equitable Power Domination, Equitable Power Domination Number, and Subdivision of a Graph.

Received January 10, 2022; Accepted March 2, 2022

equitable dominating set (EDS) of G [5] if for each $x \in V - D$ there is $y \in D$ such that $xy \in E(G)$ and $|d(x) - d(y)| \leq 1$. An EDS with least cardinality in G is known as the equitable domination number of G, represented by $\gamma_{ed}(G)$. $AS \subseteq V(G)$ is a PDS of G, if each $u \in V - S$ is dominated by certain nodes in S as follows: (a) when $u \in G$ is in PDS, then it dominates itself and every neighboring nodes of u, (b) when a dominated $u \in G$ with $r > 1$ neighboring nodes and if $r - 1$ of those nodes are dominated, then the one non-dominated node too is dominated by u [2]. A PDS $S \subseteq V$ is called an EPDS [3, 4] if for each $u \in V - S$, there is a neighboring $v \in S$ such that $|d(u) - d(v)| \leq 1$. An EPDS of least cardinality in G is known as the EPDN of G, denoted by $\gamma_{epd}(G)$. In this article, we have derived the EPDN of certain new classes of graphs.

2. Main results

This section is dedicated for deriving the EPDN of graphs in the context of subdivision of graphs. Let P_n, C_n, K_n, $K_{m,n}$, $W_{1,n}$ denote path on n nodes, cycle on n nodes, complete graph on n nodes, complete bipartite graph on mn nodes, and wheel on $n+1$ nodes, respectively.

Definition 1 [6]. A graph formed by subdividing every line of G is known as a subdivision of G, represented by $S(G)$.

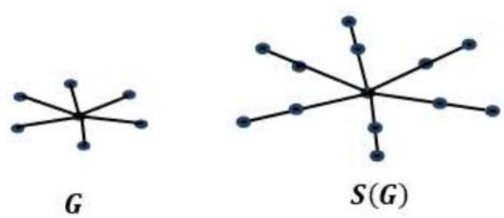

Figure 1. A graph G and $S(G)$.

Theorem 1. *If G is a graph on n nodes, then* $\gamma_{epd}(G) \leq \gamma_{epd}(S(G))$.

Proof. We take G as the given graph such that $V = \{v_1, v_2, \ldots, v_n\}$ and $E = \{e_1, e_2, \ldots, e_n\}$. Construct the subdivided graph of G with

$V(S(G)) = V(G) \cup E(G)$ and $E(S(G)) = \{(v_i e_i), (e_i v_j):$ where $1 \le i \le n$ and $i+1 \le j \le m-1\}$. There arise 2 cases in deriving an EPDS, S of $S(G)$.

Case 1. If v_i incidents on e_i for whom $|d(v_i) - d(e_i)| \le 1$ to minimum one 'i', then we have to select e_i to be in S. Therefore, $\gamma_{epd}(G) \le \gamma_{epd}(S(G))$.

Case 2. If v_i incidents on e_i for whom $|d(v_i) - d(e_i)| \le 1$, $1 \le i \le n$, then S is unchanged. Hence $\gamma_{epd}(G) \le \gamma_{epd}(S(G))$.

Theorem 2 [3]. *For* P_n, $\gamma_{epd}(P_n) = 1$, *for* $n \ge 3$.

Corollary 1. *For* P_n, $n \ge 3$, $\gamma_{epd}(S(P_n)) = 1$.

Proof. Take P_n with $V(P_n) = \{x_1, x_2, \ldots, x_n\}$. When we do the subdivision on P_n, again we get the path on $2n-1$ nodes and by using Theorem 2, we get $\gamma_{epd}(S(P_n)) = 1$.

Theorem 3 [3]. *If* C_n, $n \ge 3$, *is the cycle*, *then* $\gamma_{epd}(C_n) = 1$.

Theorem 4. *If* C_n, $n \ge 3$, *is the cycle*, *then* $\gamma_{epd}(S(C_n)) = 1$.

Proof. Take C_n with $V(C_n) = \{c_1, c_2, \ldots, c_n\}$. Then when we perform subdivision on C_n again we get a cycle of $2n$ vertices and so by Theorem 3, one can deduce that $\gamma_{epd}(s(C_n)) = 1$.

For an example, one can see from the figure 2 that subdivision of a circle is again a circle and hence $\gamma_{epd}(s(C_n)) = 1$.

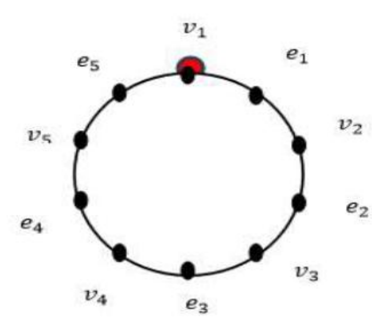

Figure 2. Subdivision of C_5, $S(C_5)$.

Theorem 5. *If* $S(K_n)$ *is the subdivision of* K_n, *then* $\gamma_{epd}(s(K_n)) = nC_2 + n, \forall\, n \geq 5.$

Proof. Take K_n with $V(K_n) = \{x_1, x_2, \ldots, x_n\}$ and $E(K_n) = \{y_1, y_2, \ldots, y_m\}$. From the nature of K_n, one can note that $d(v_i) = n-1, \forall\, 1 \leq i \leq n$. Construct subdivision of K_n, $s(K_n)$ as $V(s(K_n)) = V_1 \cup V_2$, with $V_1 = \{x_1, x_2, \ldots, x_n\}$ and $V_2 = E(K_n)$. We make an interesting observation here that the subdivided graph of K_n produces a graph which has no 2 adjacent nodes satisfying $|d(u) - d(v)| \leq 1$ and thereby violating equitable property. Therefore in order to form an EPDS, we must select all the vertices of $S(K_n)$ to be presented in EPDS. Hence, $|S| = nC_2 + n.$

Theorem 6. *For* $K_{m,n}$, $m, n \geq 4$,

$$\gamma_{epd}(s(K_n)) = \begin{cases} mn + m + n & if\,|m - n| \geq 2; \\ 1 & otherwise. \end{cases}$$

Proof. If $K_{m,n}$ is with $V(K_{m,n}) = V_1 \cup V_2$, then $V_1 = \{x_1, x_2, \ldots, x_m\}$ and $V_2 = \{y_1, y_2, \ldots, y_n\}$ are the 2 partitions of $V(K_{m,n})$. Obtain the subdivision of $K_{m,n}$, then $|V(S(K_{m,n}))|$ is $m + n + mn$. There arise 2 cases.

Case (i). $|m - n| \geq 2$

Note that $d(x_i) = n$ for each $x_i \in V_1$ and $d(y_i) = m$ for each $y_2 \in V_2$. Also the degree of newly introduced nodes is two. The equitable property is clearly violating for any 2 adjacent nodes as $|m - n| \geq 2$. So the entire node set of $S(K_{m,n})$ needs to be selected to make the required EPDS.

Case (ii). $|m - n| < 2$

Take $S = \{x_1\}$. That is, selecting only 1 node from any of the 2 partitions whose degree is greater than 1 is sufficient to derive an EPDS, S. Hence, $\gamma_{epd}(K_{m,n}) = 1$ whenever $|m - n| \geq 2$.

Remark 1. One can easily verify that $\gamma_{epd}(S(K_{m,n})) = 1$ for $n = 2$ or 3.

Theorem 7. *For* $W_{1,n}$, $n \geq 3$, $\gamma_{epd}(S(W_{1,n})) = n+1$.

Proof. Take $W_{1,n}$ on '$n+1$' nodes with $V(W_{1,n}) = \{v_1, v_2, \ldots, v_n\}$, where V_0 and $v_i : 1 \leq i \leq n$ denote the hub and rim nodes, respectively. Let $u_i : 1 \leq i \leq n$ denote the spokes in $W_{1,n}$. Form the subdivision of $W_{1,n}$, $S(W_{1,n})$. Clearly, $|V(S(K_{m,n}))| = 3n+1$. Next in order to construct an EPDS, S, we have to select v_0 as it's of highest degree and no neighboring nodes equitably power dominate (EPD) with it. Moreover, out of the remaining nodes, select v_1 to be in S as v_1 EPDs v_1', v_n' and u_1. For v_1' and v_n'. the only non-dominated nodes are v_2 and v_n, respectively and so they are dominated. For v_2 and v_n, there are 2 non-dominated nodes (one on the rim and the other on the spoke). So one has to select any one of these nodes, so to get the least cardinality, let us select v_2 and v_n to be in S. Similarly, continuing the same, we have to select every node on the spokes of $W_{1,n}$. Therefore, $S = \{v_0, u_i : 1 \leq i \leq n\}$ and $|S| = n+1$.

Corollary 2. *For the gear graph* G_n, $n \geq 5$, $\gamma_{epd}(S(G_n)) = 2$.

Proof. The proof is in similar lines with Theorem 7.

Definition 3 [1]. The n-barbell is formed by joining 2 copies of K_n by a cut edge.

Figure 3. -barbell graph.

Theorem 8 [3]**.** *For* K_n, $\gamma_{epd}(K_n) = 1$.

Theorem 9. *If G is the n-barbell, then* $\gamma_{epd}(S(G)) = 2(m+n)+1$, $\forall\, n > 6$.

Proof. Let G be the n-barbell with $V(G) = \{u_1, u_2, \ldots, u_n, u_1', u_2', \ldots, u_n'\}$. By Theorem 8, $\gamma_{epd}(K_n) = 1$. Note that when subdivision is performed on n barbell graph, the resultant graph has no neighboring nodes EPD in g any of its neighboring nodes. Hence, we have to select every node to be in S. Therefore, $\gamma_{epd}(S(G)) = 2(m+n)+1$.

Definition 5 [3]**.** A Windmill graph, $W_d(K, n)$, has n copies of K_k joined to a common node of degree $n(k-1)$.

Theorem 10. *For* $W_d(K, n)$, $\gamma_{epd}(S(W_d(K, n))) = n+1$, *for* n, $K \geq 2$.

Proof. Note by the Definition 5 that, $W_d(K, n)$ has n-copies of K_k connecting to the node of degree $n(K-1)$ say, v_0. We know that $\gamma_{epd}(K_n) = 1$. Therefore, we have to select minimum one node from each copy of K_k. Also these 'n' nodes do not EPD v_0 as $| d(v_0) - d(v); v \in V - v_0 | \geq 2$. Thus one must choose v_0 to be in the EPDS, S. So, $\gamma_{epd}(W_d(K, n)) = n+1$.

Conclusion

In this paper, the subdivision of certain special classes of graphs has been taken into consideration and the EPDN of those graphs are derived. Determining the EPDN of other families of graphs for different graph operations is the further area of research.

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