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A COMPARATIVE STUDY ON SOLVING FUZZY TRANSPORTATION PROBLEM USING HEXAGONAL FUZZY NUMBER WITH DIFFERENT RANKING TECHNIQUES

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Abstract

In this paper fuzzy transportation problem in which the transportation cost, supply and demand are addressed as Hexagonal Fuzzy Numbers (HFN). Characterize the positioning strategies to the Hexagonal Fuzzy Number. The principle motivation behind this review is to track down the appropriate defuzzification strategy and to track down the base transportation cost by relative of different techniques.

1. Introduction

The transportation model is an extraordinary class of linear programming problem that arrangement with delivery products from sources to objections. The beginning of transportation issue was presented and created by F. L. Hitchcock in 1941. Ideal usage of the transportation framework was presented by T. C. Koopmans in 1947. Presently a-days, the transportation boundaries might be unsure by different new events uncontrolled components. So, to manage imprecision in dynamic Bellman and Zadeh [4,

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13] presented the idea of fuzziness. In 1982 O'heigeartaigh proposed a calculation to tackle Fuzzy transportations issue with triangular membership work. After crafted by Chang and Zadeh (1972), a few specialists (Chen et al. 2012. Chen and Chen 2014) all around the world have been investigating the fuzzy number in huge ways.

Positioning of ordinary fuzzy numbers was initially, presented by Jain. Chen et al. [5] used the region between the centroid dependent on distance technique to rank fuzzy numbers. Annie et al. [3] proposed the Best Candidate Method (BCM) and utilized hexagonal fuzzy numbers by centroid positioning procedures.

On contrasting the hexagonal fuzzy number and the first three-sided (triangular) (or) trapezoidal fuzzy number, we see that there are a few intricacies in line and furthermore in definition, although hexagonal fuzzy number gives additional likelihood to mean imperfect information which brings about forming replies to numerous genuine issues in an original methodology. Thus, hexagonal fuzzy numbers can be instrumental in taking care of the issue.

The point of this review is to track down the reasonable defuzzification strategies and apply with various procedures to track down the ideal answer for the hexagonal fuzzy transportation problem.

2. Preliminaries

Definition 2.1 (Fuzzy Set). The characteristic function μ_A of a crisp set $A \subset X$ doles out a worth either 0 or 1 to every part in *X*. This capacity can be summed up to a capacity to such an extent that the worth assigned to the component of the general set *X* is within a predetermined range [0, 1] i.e., $\mu_A : X \to [0, 1]$. The assigned values show the membership function and the set.

$\widetilde{A} = \{(xm, \mu_A(x)) : x \in X\}$ defined by $\mu_A(x)$ for $x \in X$ is called fuzzy set.

Definition 2.2 (Fuzzy Numbers). A fuzzy number is a speculation of a customary genuine number and which doesn't allude to a solitary worth, yet rather allude to an associated set of potential qualities, where every conceivable worth has its weight somewhere in the range of 0 and 1.

Definition 2.3 (Generalized Fuzzy Number). A fuzzy set $A = (a, b, c, d, w)$ is defined on universal set of real numbers R, is said to be generalized fuzzy number in its membership function satisfies the following properties.

(i) $\mu(x) : R \to [0, 1]$ is continuous,

(ii) $\mu_A(x) = 0$ for all $x \in A(-\alpha, a) \cup (d, \alpha)$;

(iii) $\mu_A(x)$ strictly increasing on [a, b] and strictly decreasing on $[c, d]$

(iv) $\mu_A(x) = w$ for all $x \in [b, c]$ where $0 < w \leq 1$

Definition 2.4 (Hexagonal Fuzzy Number). A fuzzy number $\widetilde{A}_H$ is a Hexagonal fuzzy Number (HFN) denoted by $\widetilde{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6)$ if it satisfies the following conditions.

(i) $\mu_{\widetilde{A}_H}(x)$ is a continuous function in $[0, 1]$.

(ii) $\mu_{\widetilde{A}_H}(x)$ is an explicitly continuously growing function in $[a_1, a_2]$ and $[a_2, a_3]$.

(iii) $\mu_{\widetilde{A}_H}(x)$ will attain the value 1 in $[a_3, a_4]$

(iv) $\mu_{\widetilde{A}_H}(x)$ is an explicitly decreasing and continuous function on $[a_4, a_5], [a_5, a_6]$

Here, $\mu_{\widetilde{x}}(X)$ is the membership function for the hexagonal fuzzy number

$$\mu_{\widetilde{A}_H}(x) = \begin{cases} \frac{1}{2}\left(\frac{x - a_1}{a_2 - a_2}\right), & \text{for } a_1 \le x \le a_2 \\ \frac{1}{2} + \frac{1}{2}\left(\frac{x - a_2}{a_3 - a_2}\right), & \text{for } a_2 \le x \le a_3 \\ 1, & \text{for } a_3 \le x \le a_4 \\ 1 - \frac{1}{2}\left(\frac{x - a_4}{a_3 - a_2}\right), & \text{for } a_4 \le x \le a_5 \\ \frac{1}{2}\left(\frac{a_6 - x}{a_6 - a_5}\right), & \text{for } a_5 \le x \le a_6 \\ 0, & \text{otherwise} \end{cases}$$

2.5 Arithmetic Operations on Hexagonal Fuzzy Numbers

Suppose $\widetilde{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6)$ and $\widetilde{B}_H = (b_1, b_2, b_3, b_4, b_5, b_6)$ are two HFN's then, the usual arithmetic operations are defined as follows.

Addition. $A_H + B_H = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5, a_6 - b_6)$

Subtraction. $A_H - B_H = (a_1 - b_6, a_2 - b_5, a_3 - b_4, a_4 - b_3, a_5 - b_2, a_6 - b_1)$

Multiplication. $A_H \cdot B_H = (a_1 \cdot b_1, a_2 \cdot b_2, a_3 \cdot b_3, a_4 \cdot b_4, a_5 \cdot b_5, a_6 \cdot b_6)$

3. Ranking of Hexagonal Fuzzy Numbers

The positioning of fuzzy number is a basic component of the decision making technique in many applications. Positioning is that we can corresponds two fuzzy numbers. Defuzzification is a technique wherein the fuzzy number is converted to real number. At some point, decider can take two comparable to ideas. Here, they change the fuzzy number into fresh number and analyse them on the beginning of genuine worth.

3.1 Defuzzification Based on Alpha (α)-cut Method

The classical set $\widetilde{A}_\alpha$ called alpha cut set is the set of elements whose degree of membership is the set of elements in $\widetilde{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6)$ is no less than, it is defined as

$$\widetilde{A}_H = \{x \in X : \mu_{\widetilde{A}_H}(x) \ge \alpha\}$$

$$= \begin{cases} [D_1(\alpha),\ D_2(\alpha)] & \text{for } \alpha \in [0,\ 0.5] \\ [S_1(\alpha),\ S_2(\alpha)] & \text{for } \alpha \in [0.5,\ 1] \end{cases}$$

$$= \begin{cases} [D_1(\alpha),\ S_1(\alpha)] & \text{for } \alpha \in [0,\ 1] \\ [D_2(\alpha),\ S_2(\alpha)] & \text{for } \alpha \in [0,\ 1] \end{cases}$$

$$= a_1 + \alpha(a_3 - a_1) + a_6 + \alpha(a_4 - a_6)$$

The ranking is defined by

$$R(\widetilde{A}_H) = \int_0^1 2(0.5)\,(A^L_{h\alpha},\ A^U_{h\alpha})\,d\alpha$$

Where $(A^L_{h\alpha},\ A^U_{h\alpha})$ is the α-level cut of the fuzzy number $\widetilde{A}_H$.

$$R(\widetilde{A}_H) = \int_0^1 2(0.5)[a_1 + \alpha(a_3 - a_1) + a_6 + \alpha(a_4 - a_6)]d\alpha \text{ for } \alpha \in [0, 1]$$

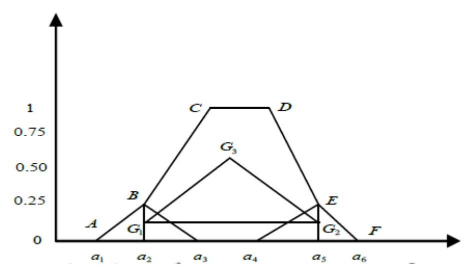
3.2 Defuzzification. using average method

In a hexagonal fuzzy number $\widetilde{A}_H = (a_1,\ a_2,\ a_3,\ a_4,\ a_5,\ a_6)$ we define the ranking function as average of $R : F(R) \to R$ by

$$R(\widetilde{A}_H) = \left(\frac{a_1,\ a_2,\ a_3,\ a_4,\ a_5,\ a_6}{6}\right)$$

3.3 Defuzzification using centroid technique

In a hexagonal fuzzy number, the hexagon is isolated into two triangles AQB and RFE and a hexagon BQREDCB as displayed in figure. The centroid of the hexagonal fuzzy number is the middle point (adjusting point) of the hexagon ABCDFA. Using centroid of the three plane figures is



$$G_1 = \left(\frac{a_1 + a_2 + a_3}{3}\right)\left(\frac{1}{6}\right) \; G_2 = \left(\frac{a_2 + 2a_3 + 2a_4 + a_5}{6}\right)\left(\frac{1}{2}\right)$$

$$G_3 = \left(\frac{a_4 + a_5 + a_6}{3}\right)\left(\frac{1}{6}\right)$$

Plainly, they are non-colinear and they structure a triangle. The positioning capacity to the HFN, $\widetilde{A}_H = (a_1, a_2, a_3, a_4, a_5, a_6)$ are maps a bunch of all fuzzy numbers to the arrangement of genuine numbers.

$$R(\widetilde{A}_H) = \left(\frac{2a_1 + 3a_2 + 4a_3 + 4a_4 + 3a_5 + 2a_6}{18}\right)\left(\frac{5}{18}\right)$$

Sometimes, we may use appropriate weights to HFN's and the ranking function for generalized hexagonal fuzzy number is

$$R(\widetilde{A}_H) = \left(\frac{2a_1 + 3a_2 + 4a_3 + 4a_4 + 3a_5 + 2a_6}{18}\right)\left(\frac{15w}{18}\right)$$

4. Fuzzy Transportation Problem

The mathematical model to fuzzy transportation problem is to minimize the total transportation cost from m sources to n destinations as follows.

Minimize $\widetilde{Z} = \sum_{i=1}^{m} \sum_{j=1}^{n} \widetilde{c}_{ij}\widetilde{x}_{ij}$

Subject to $\sum_{j=1}^{m} \widetilde{x}_{ij} = \widetilde{a}_i, \; i = 1, 2, 3, \ldots, m \sum_{i=1}^{n} \widetilde{x}_{ij} = \widetilde{b}_i, \; j = 1, 2, 3, \ldots, n$

$\sum_{i=1}^{m} \widetilde{a}_i = \sum_{j=1}^{n} = \widetilde{b}_i, \; i = 1, 2, 3, \ldots, m, \; j = 1, 2, 3, \ldots, n$ and $\widetilde{x}_{ij} \geq 0$

In which the transportation costs are represented by generalized fuzzy numbers whereas demand and supply are crisp values. There are numerous techniques accessible to obtain initial basic feasible solution. Here we utilize. Least Cost Method, Zero Point Method, Best Candidate Method and Vogel's Approximation Method to track down the base transportation cost.

4.1 Fuzzy North-West-Corner-Rule (FNWCR)

Step (i) From the fuzzy transportation table, if the total supply and total demand are equal then, at that point, it is adjusted. In any case make it to be adjusted one.

Step (ii) Apply traditional North West Corner Method to the fresh transportation table until all the edge prerequisites are fulfilled.

4.2 Fuzzy Least cost Method (FLCM)

Step (i) Consider the crisp transportation table. Dole out a much as conceivable to the cell with the littlest unit cost in the whole scene.

Step (ii) If tie happens of any circumstance change discretionary.

Step (iii) Adjust the market interest for those lines and segments which are cross out, continue to adjust line and section complete.

4.3 Fuzzy Zero suffix Method (FZSM)

Step (i) Construct a crisp transportation table of the given FTP supplanting all the summed up hexagonal fuzzy expenses by its positioning file.

Step (ii) In each row, take away the line least from every entire's. The equivalent system should be accomplished for every column of the transportation table.

Step (iii) In the diminished crisp cost matrix, there will be at least one zero in each line and every section. Discover the list of all zero's in i^{th} line and j^{th} segment as follows.

$$Index\ (0_{ij}) = \frac{\text{Sum of non} - \text{zero costs in } i^{th} \text{ row and } j^{th} \text{ column}}{\text{Number of zero's in } i^{th} \text{ row and } j^{th} \text{ column}}$$

Step (iv) Choose the maximum of S, assuming it has one most extreme worth, first stock to that request relating to the cell. On the off chance that it has more equivalent qualities in this way select $\{a_i, b_j\}$ furthermore, supply to that request most extreme conceivable.

Step (v) After the above advances, the depleted requests or supplies to be managed. The resultant lattice should has at least one zero in each line and section. In any case rehash step (ii).

Step (vi) Repeat steps (iii) to (v) until the ideal arrangement is acquire.

4.4 Fuzzy Best Candidate Method (FBCM)

Step (i) Check the fuzzy transportation table is balanced. If not make it to adjusted one by adding faker row (or) column.

Step (ii) The best contender for limiting issues to the base expense and expanding benefit to the greatest expense is chosen. Consequently, this progression is done by choosing best up-and-comer in each column.

Step (iii) Identify the line with the littlest expense competitor from the pick blend. Then, at that point, apportion the interest and the stockpile however much as could reasonably be expected to the variable with least unit cost in the chose line or segment. Likewise, the stock and request is changed by intersection out the line/section and appoint to nothing.

Step (iv) The cycle is rehashed until an ideal fundamental attainable arrangement is acquired.

4.5 Fuzzy Vogel's Approximation method

Step (i) In a fuzzy transportation table utilizing suitable defuzzification procedure to make it as crisp one.

Step (ii) To confirm the reasonable one or make it.

Step (iii) Processed a conventional VAM Procedure.

5. Numerical Example

In the part, fuzzy transportation issue is settled utilizing different defuzzification procedures represented in section-3. Every defuzzification procedures we utilize the techniques in section-4 to track down the base transportation cost.

Consider the following Fuzzy Transportation Problem with HFN.

	d_1	d_2	d_3	d_4	Supply
0_1	(3,7,11,15, 19,24)	(13,18,23,28, 33,40)	(6,13,20,28, 36,45)	(15,20,25,31, 38,45)	(6,8,11,14, 19,25)
0_2	(16,19,24,29, 34,39)	(3,5,7,910, 12)	(5,7,10,13, 17,21)	(20,23,26,30, 35,40)	(9,11,13,15, 18,20)

0_3	(11,14,17,21, 25,30)	(7,9,11,14, 18,22)	(2,3,4,6, 7,9)	(5,7,8,11, 14,17)	(7,9,11,13, 16,20)
Demand	(3,4,5,6, 8,10)	(3,5,7,9, 12,15)	(6,7,9,11, 13,16)	(10,12,14,16, 20,24)	Balanced

Solution.

Method I. (Defuzzification based as α-method

$$R(\widetilde{A}_H) = \int_0^1 2(0.5)\,[a_1 + \alpha(a_3 - a_1) + a_6 + (a_4 - a_6)]\,d\alpha$$

$$R(3, 7, 11, 15, 19, 24) = \int_0^1 2(0.5)(27 - \alpha)\,d\alpha = \left[27\alpha - \frac{\alpha^2}{2}\right]_0^1 = 26.5$$

Similarly, $R(13, 18, 23, 28, 33, 40) = 52$ $R(6\ 13, 20, 28, 36, 45) = 49.5$

$R(15, 20, 25, 31, 38, 45) = 58$ $R(6, 19, 24, 29, 34, 39) = 54$

$R(3, 5, 7, 9, 10, 12) = 15.5$ $R(5, 7, 10, 13, 17, 21) = 24.5$

$R(20, 23, 26, 30, 35, 40) = 58$ $R(11, 14, 17, 21, 25, 30) = 39.5$

$R(7, 9, 11, 14, 18, 22) = 27$ $R(2, 3, 4, 6, 7, 9) = 10.5$

$R(5, 7, 8, 11, 14, 17) = 20.5$ $R(3, 4, 5, 6, 8, 10) = 12$

$R(3, 5, 7, 9, 12, 15) = 17$ $R(6, 7, 9, 11, 13, 16) = 21$

$R(10, 12, 14, 16, 20, 24) = 32$ $R(6, 8, 11, 14, 19, 25) = 28$

$R(9, 11, 13, 15, 18, 20) = 28.5$ $R(7, 9, 11, 13, 16, 20) = 25.5$

By applying these ranking, the following crisp transportation table is,

	d_1	d_2	d_3	d_4	Supply
0_1	26.5	52	49.5	58	28
0_2	54	15.5	24.5	58	28.5
0_3	39.5	27	10.5	20.5	25.5

Demand	12	17	21	32	82

Solving the crisp TP using the methods in sec-4, the solution is obtained.

Method II. (Defuzzification using average method)

$R(\widetilde{A}_H) = \left(\frac{a_1, a_2, a_3, a_4, a_5, a_6}{6}\right)$ Using this ranking technique to each cells in HFN's, the crisp one is obtained.

The crisp transportation table is (correct to two decimal places).

	d_1	d_2	d_3	d_4	Supply
0_1	13.17	25.83	24.67	29	13.83
0_2	26.83	7.67	12.17	29	14.33
0_3	19.67	13.5	5.17	10.33	12.67
Demand	6	8.5	10.33	16	40.83

Upon solving the reduced crisp transportation problem using the methods in section-4 to the minimum transportation cost is obtained.

Method III. (Defuzzzification using centroid technique)

$$R(\widetilde{A}_H) = \left(\frac{2a_1, 3a_2, 4a_3, 4a_4, 3a_5, 2a_6}{18}\right)\left(\frac{5}{18}\right)$$

(Sometimes we may appropriate weights to HFN 's)

By defuzzification to the given HFT is reduced to crisp transportation table as follows.

	d_1	d_2	d_3	d_4	Supply
0_1	3.64	7.15	6.81	7.99	3.75
0_2	7.42	2.15	3.33	7.99	3.97
0_3	5.42	3.69	1.42	2.82	3.47
Demand	1.64	2.33	2.84	4.38	11.19

To solve this crisp transportation problem using North-West-Corner-Method, Least-Cost-Method, Zero Suffix Method, Best Candidate Method and Vogel's Approximation Method.

The solution table using all the defuzzificaiton techniques and using discussed method of finding minimum cost is given below.

Defuzzificaiton Technique	Methods to find minimum	α-cut technique	Average method	Centroid method
NWCR		2579.75	639.58	48.04
LCM		2489.75	617.93	46.75
ZSM		2219.00	549.90	41.40
BCM		2489.25	617.93	46.76
VAM		2233.25	553.63	41.67

From the arrangement table of given mathematical model. We can presume that defuzzifying by centroid strategy is giving the base transportation cost. Anyway we utilize Zero Suffix Method yields much more least expense equivalent to other four techniques.

Henceforth defuzzification by centroid strategy and through Zero Suffix Method gives the base expense for hexagonal transportation issue. .

Conclusion

In this paper, we dealt three diverse defuzzificaiton methods to make the HFN into crisp one and track down the initial basic feasible solution and optimum solution to this by utilizing five different techniques. At last, we contrast every one of the arrangements with close the decision of best defuzzficiation for reasonable strategy to yield least transportation cost.

References

[1] Abbasbandys and T. Hajjari An improvement an centroid point method for ranking of fuzzy numbers J. Sci IAU78: (2011) 109-119.

[2] Amarpreet Kaur and Amit Kumar, A new method for solving fuzzy transportation

problem using ranking functions, Applied Mathematicas 35(12) (2011), 5622-5661.

[3] M. S. Annie Christi and D. Malini, Solving transportation problems with hexagonal fuzzy numbers using best candidate method and different ranking techniques, International Journal of Engineering Research and Applications 6 (2016), 76-81.

[4] R. E. Bellmann and L. A. Zadeh, Decision making in fuzzy environment, Management Science 17 (1970), 141-164.

[5] T. Chu and C. Tsao, Ranking fuzzy number with an area between the centroid point and original point Computer math Appl 43 (2002), 111-117.

[6] K. Dhurai and A. Karpagam, A new Ranking Technique of Hexagonal fuzzy Numbers, International Journal of Science and Research (2013), 533-535.

[7] P. Jayaraman and R. Jahirhussian, Fuzzy Optimal Transportation Problem by improved zero suffix method via Robust rank techniques, International Journal of Fuzzy Mathematics and Systems 3(4) (2013), 303-311.

[8] P. K. Kantiswarup, Gupta and Mammohan, Operations Research- I, 12th edition (2004).

[9] A. Kaur and A. Kumar, A method for solving fuzzy transportation problems using ranking function, Applied Mathematical Modeling 35(12) (2011), 5652-5661.

[10] P. Rajeswari and A. Sahaya Sudha, A New Operations on Hexagonal Fuzzy Number, International Journal of Fuzzy Logic systems 3(3) (2013), 15-26.

[11] A. Thamaraiselvi and R. Santhi, Optimal Solution of Fuzzy Transportation Problem using Hexagonal Fuzzy Numbers, International Journal of scientific and Engineering Research 6(3) (2015), 40-45.

[12] Y. J. Wang and S. M. Lee, The revised method of ranking fuzzy numbers with an area between the centroid and original points, Computers and Mathematics with Applications 55 (2008), 2033-2042.

[13] L. A. Zadeh, Fuzzy sets, Information and control 8 (1965), 338-353.

[14] H. J. Zimmerman, Fuzzy set theory and its application third edition, Kluwer Academic Publisher, Boston Massachusetts, (1996).