Advances and Applications in Mathematical Sciences
Volume 21, Issue 7, May 2022, Pages 3931-3940

INVERSE DOMINATION IN INTUITIONISTIC FUZZY GRAPHS

J. JOHN STEPHAN, A. MUTHAIYAN, T. GEETHA and N. VINOTH KUMAR

Department of Mathematics
Hindustan Institute of Technology
Coimbatore, India
E-mail: johnstephan21@gmail.com

PG and Research
Department of Mathematics
Government Arts College
Ariyalur, India
E-mail: muthaiyanprof@gmail.com

PG and Research Department of Mathematics
K. N Arts College for Women
Thanjavur, India
E-mail: t.geetha@gmail.com

Department of Mathematics
Bannari Amman Institute of Technology
Erode, India
E-mail: vinoth@bitsathy.ac.in

Abstract

In this paper, we investigated the idea of an inverse dominating set (ID-Set) and inverse domination number (ID-number) of an IFG. Let $D \subset V$ be a $\gamma(G)$ set of the IFG $G(V, E)$. A dominating set D' contained in $V - D$ is called an ID set of $G(V, E)$ with respect to D. The minimum cardinality among the minimal ID set is called an ID number of $G(V, E)$ and it is denoted by $\gamma'(G)$. Further study the inverse domination number $\gamma'(G)$. of complete and

2020 Mathematics Subject Classification: 05C72.

Keywords: Intuitionistic fuzzy graphs; Inverse Dominating set, Inverse Dominating number.
Received December 21, 2021; Accepted January 14, 2022

complete bipartite IFG some results and also identify some bounds of the ID-number are investigated. The ID-number of some standard operations join of two IFGs and Cartesian product of two IFG are investigated. Some results like $G(V, E)$ is an IFG without isolated vertices, then $\gamma(G) \leq \gamma'(G)$. The IFG$_{G(V, E)}$ be a complete IFG, then $\gamma'_{if}(G) = |u|$, here u is the vertex having the second minimum cardinality in $G(V, E)$. An IFG $G(A, B)$ be a complete bipartite IFG, then $\gamma'_{if}(G) = |u| + |v|$, here u and v are the vertex having the second minimum cardinality in vertices set V_1 and V_2 in $G(V, E)$.

1. Introduction

The first Definition of fuzzy graphs was proposed by Kafmann from the fuzzy relations introduced by Zadeh. Although Rosenfeld introduced another ela borated definition, including fuzzy vertex and fuzzy edges, and several fuzzy analogs of graph theoretic concepts such as paths, cycles, connectedness and etc. The concept of domination in fuzzy graphs was investigated by Somasundaram present the concepts of independent domination, total domination, connected domination of fuzzy graphs. C. Natarajan and S. K. Ayyaswamy introduce the strong (weak) domination in fuzzy graph. The first definition of intuitionistic fuzzy graphs was proposed by Atanassov. The concept of domination in intuitionistic fuzzy graphs was investigated by R. parvathi and G. Thamizhendhi. In this paper, we presented the idea of an inverse dominating set (ID-Set) and inverse domination number (ID-number) of a IFG. Further investigate the inverse domination number of complete and complete bipartite IFG some results and also some bounds of the ID-number are investigated. The ID-number of join of two IFGs and Cartesian product of two IFG are investigated.

Let $G(V, E)$ bean IFG. A set $D \subset V$ is said to be a dominating set of G if every $v \in V - D$ there exist $u \in D$ such that u dominates v. An Intuitionistic fuzzy dominating set D of an IFG, G is called minimal dominating set of G if every node $u \in D$, $D - \{u\}$ is not a dominating set in G. An Intuitionistic fuzzy domination number $\gamma_{if}(G)$ of an IFG, G is the minimum vertex cardinality over all minimal dominating sets in G.

2. Inverse Domination

In this segment, we investigate some bounds and properties of the inverse domination number in Intuitionistic fuzzy graphs.

Definition 2.1. Let $D \subset V$ be a $\gamma(G)$ set of the IFG $G(V, E)$. A dominating set D' contained in $V - D$ is called an ID set of $G(V, E)$ with respect to D. The minimum cardinality among the minimal ID set is called an ID number of $G(V, E)$ and it is denoted by $\gamma'(G)$.

Theorem 2.1. *The* $G(V, E)$ *is an IFG without isolated vertices, then* $\gamma(G) \leq \gamma'(G)$.

Proof. Let $G(V, E)$ be a IFG without isolated vertices and D and D' are the $\gamma(G) \leq \gamma'(G)$ sets of $G(V, E)$ respectively. Clearly every $\gamma'(G)$ set D' is a $\gamma(G)$ set D of $G(A, B)$, but D' is not a γ dominating set of $G(V, E)$. Therefore we get

$$| D | \leq | D' | \Rightarrow \gamma(G) \leq \gamma'(G) \tag{2.1.1}$$

Theorem 2.2. *The IFG* $G(V, E)$ *be a complete IFG, then* $\gamma'_{if}(G) = | u |$, *here u is the vertex having the second minimum cardinality in* $G(V, E)$.

Proof. Let $G(V, E)$ be a complete IFG and v, u are vertex having the least two cardinality among the vertices in $G(V, E)$. In $G(V, E)$ there is a strong edge between every pair of vertices. Since $G(V, E)$ is a complete graph. Clearly $\gamma(G) = | v |$ Since the vertex $v \in V$ is adjacent to all vertices in $V - \{v\}$. The hesitancy sub graph induced by the vertices $V - \{v\}$ is also a complete IFG. Therefore there is a vertex $u \in V - \{v\}$ is adjacent to all other vertices. This implies $\{u\}$ is a minimal ID set of $G(V, E)$. Hence the ID number $\gamma'_{if}(G) = | u |$,

Example 2.1.

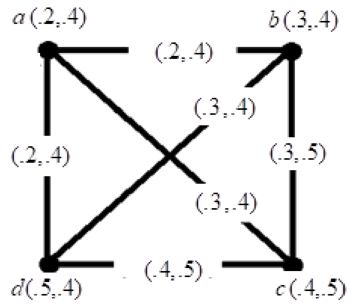

Figure 2.1. Complete IFG $G(V, E)$.

In the Complete IFG $G(V, E)$, degree of the vertices are $|a| = 0.4$, $|b| = 0.45$, $|c| = 0.45$ and $|d| = 0.55$. An inverse dominating set and inverse domination number of the complete IFG $G(V, E)$ is $D = \{b\}$ or $\{c\}$ and $\gamma'_{if}(G) = |b| = 0.45$.

Theorem 2.3. *An IFG $G(A, B)$ be a complete bipartite IFG, then $\gamma'_{if}(G) = |u| + |v|, = 0.45$. Here u and v are the vertex having the second minimum cardinality in vertices set V_1 and V_2 in $G(V, E)$.*

Proof. Let $G(V, E)$ be a complete bipartite IFG. This implies the vertices of $G(V, E)$ are partition in to V_1 and V_2. Let x and y are the vertex having the minimum cardinality in vertices set V_1 and V_2 in $G(V, E)$. Therefore x dominates vertices in V_1 and y dominates the vertices in V_2. This implies $\{x, y\}$ is a $\gamma(G)$ set of $G(V, E)$. Note that $V - \{x, y\}$ is the complete bipartite IFG. Let u and v are the vertex having the second minimum cardinality in vertices set V_1 and V_2 in $G(V, E)$. Therefore the set $\{u, v\}$ is the dominating set of the graph induced by $\langle V - \{x, y\}\rangle$, since $V - \{x, y\}$ is the complete bipartite IFG. This implies $\{u, v\}$ is the $\gamma'_{if}(G)$ set of $G(V, E)$. Hence we get $\gamma'_{if}(G) = |u| + |v|$.

Example 2.2.

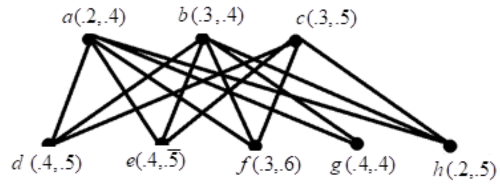

Figure 2.2. Complete Bipartite IFG $G(V, E)$.

In the Complete IFG $G(V, E)$, degree of the vertices in V_1 and V_2 are a $|a| = .4, |b| = .45, |c| = .4, |d| = .45, |e| = .45, |f| = .35, |g| = .5$ and $|h| = .35$. The total dominating set $T = \{a, f\}$ or $\{c, f\}$ or $\{a, h\}$ or $\{c, h\}$ and the total domination number of the complete IFG $G(V, E)$ is $\gamma'_{if}(G) = |c| + |h| = 0.75$.

Theorem 2.4. *Let $G(V, E)$ be a IFG and $\gamma(G), \gamma'(G)$ are domination number and ID number of $G(V, E)$. Then we get $\gamma'_{if}(G) \leq \frac{O(G)}{2}$.*

Proof. Let $G(V, E)$ be a IFG and $\gamma(G), \gamma'_{if}(G)$ are domination number and inverse domination number of $G(V, E)$. We know that the domination number $\gamma(G)$ such that $\gamma(G) \leq \frac{O(G)}{2}$. Let D' is a $\gamma'_{if}(G)$ set of $G(V, G)$. Therefore $\gamma'_{if}(G)$ set is a dominating set in $\langle V - D \rangle$. This implies $\{V - D - D'\}$ is also an ID set of $\langle V - D \rangle$. Here $\gamma(G) \leq \frac{O(G)}{2}$ therefore we get

$$\begin{aligned} |D'| &= \min\{|D'|, |V - D|\} \\ &= \min\{|D'|, |V - D|\} \quad (2.4.1) \\ &= Min\{|D'|, |V| - |D|\} \end{aligned}$$

$\gamma'_{if}(G) \leq \frac{O(G)}{2}$.

Theorem 2.5. *Let $G(V, E)$ be a IFG and $\gamma(G), \gamma'(G)$ are domination number and ID number of $G(V, E)$. Then we get $\gamma'_{if}(G) + \gamma(G) \leq O(G)$ $-\delta_N(G)$.*

Proof. Let $G(V, E)$ be a IFG and $\gamma(G), \gamma'_{if}(G)$ are domination number and ID number of $G(V, E)$. Let u is the vertex having minimum degree in the graph $G(V, E)$, that is $\delta_N(G) = d_N(u)$. Note that the set $\{(V - D) - N(u)\}$ is an ID set not a minimal ID set of $G(A, B)$. Therefore the ID set $D' \subseteq \{(V - D) - N(u)\}$. This implies the ID number is $|D'|$. Hence we get

$$|D'| \leq |\{(V - D) - N(u)\}|$$

$$|D'| \leq |V| - |D| - |N(u)| \tag{2.5.1}$$

$$\gamma'(G) \leq O(G) - \gamma(G) - \delta_N(G)$$

$$\gamma(G) + \gamma'_{if}(G) \leq O(G) - \delta_N(G)$$

Theorem 2.6. *Let $G(V, E)$ be a IFG and D is a dominating set of $G(V, E)$. The sub graph $\langle V - D \rangle$ does not contain any strong edges, Then $\gamma'_{if}(G) + \gamma(G) = O(G)$.*

Proof. Let $G(V, E)$ be a IFG and $\gamma(G), \gamma'_{if}(G)$ are domination number and ID number of $G(V, E)$. The sub graph $\langle V - D \rangle$ does not contain any strong edges. Therefore the set $D' = \{(V - D)\}$ is an ID set of the sub graph $\langle V - D \rangle$. This implies the ID number is $|D'|$ Hence we get

$$|D'| = |\{(V - D)\}|$$

$$|D'| = |V| - |D| \tag{2.6.1}$$

$$\gamma'(G) \leq O(G) - \gamma(G)$$

$$\gamma(G) \leq \gamma'(G) - O(G)$$

Theorem 2.7. *The subsets $D_1 \subseteq V_1$ and $D_2 \subseteq V_2$ are the dominating sets of the IFG's $G(V_1, E_1)$ and $G(V_2, E_2)$ respectively. Then the ID number $\gamma'(G_1 + G_2) = \max\{|D_1|, |D_2|\}$.*

Proof. Let subsets $D_1 \subseteq V_1$ and $D_2 \subseteq V_2$ are the dominating sets of the IFG $G(V_1, E_1)$ and $G(V_2, E_2)$ respectively. In $G_1 + G_2$ there is a strong edge between every pair of vertices in V_1 and V_2. This implies

$\gamma(G_1, G_2) = \min\{| D_1 |, | D_2 |\}$. Suppose D_1 is γ set of $G_1 + G_2$. Now we prove D_2 is an ID set of $G_1 + G_2$. That is to prove D_2 is a dominating set of $\langle V - D_1 \rangle$. If $u \in V - D_1$ there is a vertex $v \in D_2$. Suppose $u \in V_2$ the set D_2 dominates u since D_2 is a minimal dominating set of $G(V_2, E_2)$. Suppose $u \in V_1$, by the definition of $G_1 + G_2$ we get

$$(\mu_{12} + \mu_{22})(uv) = \mu_{11}(u) \wedge \mu_{21}(v)$$
$$(\eta_{12} + \eta_{22})(uv) = \eta_{11}(u) \wedge \eta_{21}(v) \qquad (2.7.1)$$
$$(\beta_{12} + \beta_{22})(uv) = \beta_{11}(u) \wedge \beta_{21}(v)$$

This implies there is a strong edge between $u \in V_1$ and $v \in D_2$. Clearly D_2 dominates every vertices in $\langle V - D_1 \rangle$. Here D_2 is a minimal dominating set of $\langle V - D_1 \rangle$. Hence D_2 is a minimal ID set of $\langle V - D_1 \rangle$. Similarly, we prove if D_2 is γ set of $G_1 + G_2$. D_1 is an ID set of $G_1 + G_2$. Hence the ID number of $G_1 + G_2$ is $\gamma'(G_1 + G_2) = \max\{| D_1 |, | D_2 |\}$.

Example 2.4.

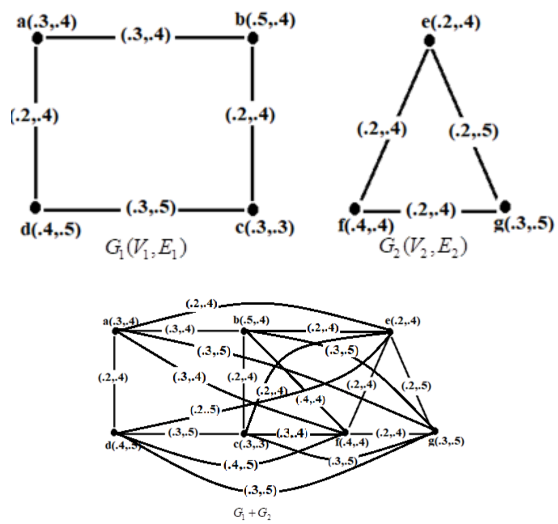

Figure 2.4. Join Graph $G_1 + G_2$ of $G(V_1, E_1)$ and $G(V_2, E_2)$.

In Figure 3.4, the dominating sets of the IFG's $G_1(A_1, B_1)$ and

$G_2(A_2, B_2)$ are $D_1 = \{a, c\}$ and $D_2 = \{e\}$. The domination number of $G_1(A_1, B_1)$ and $G_2(A_2, B_2)$ are $\gamma(G_1) = 0.66$, $\gamma_T(G_2) = 0.4$. An ID set of $G_1 + G_2$ is $D = D_1$ and ID number $\gamma'_{if}(G_1 + G_2) = 0.66$.

Theorem 2.8. *The subsets $D_1 \subseteq V_1$ and $D_2 \subseteq V_2$ are the dominating sets of the IFG's $G(V_1, E_1)$ and $G(V_2, E_2)$ respectively. Then*

(i) If $(D_1 \times V_2)$ is a domination set of $(G_1 \times G_2)$. Then the ID number $\gamma'_{iF}(G_1 \times G_2) = | D_1 \times (V_2 \times D_2) |$.

(ii) *If $(V_1 \times D_2)$ is a domination set of $(G_1 \times G_2)$. Then the ID number $\gamma'_{iF}(G_1 \times G_2) = | (V_1 \times D_1) \times D_2 |$.*

Proof. The subsets $D_1 \subseteq V_1$ and $D_2 \subseteq V_2$ are the minimal dominating sets of the IFG $G(V_1, E_1)$ and $G(V_2, E_2)$ respectively. Therefore every vertex $v_1 \in V_1$ and $v_2 \in V_2$ are adjacent to a vertex in D_1 and D_2 respectively. By the known result $(D_1 \times V_2)$ or $(V_1 \times D_2)$ is the minimal dominating set of $(G_1 \times G_2)$.

(i). Suppose $(D_1 \times V_2)$ is γ set of $G_1 \times G_2$. Now we find the dominating set of the sub graph $\langle (V_1 - D_1) \times V_2 \rangle$. Assume $(V_1 - D_1) \times D_2$ is not an minimal dominating set of $G_1 \times G_2$ there exist a vertex $uv \in (V_1 - D_1) \times (V_2 - D_2)$ is not dominated by the vertex in the set $(V_1 - D_1) \times D_2$. There is no strong edge between vertex $uv \in (V_1 - D_1) \times (V_2 - D_2)$ and every vertex $xy \in (V_1 - D_1) \times D_2$. Therefore we get there is no strong edge between $v, y \in V_2$. This implies D_2 is not a minimal dominating set of $G(V_2, E_2)$. This is contradict to our assumption D_2 is a minimal dominating set $G(V_2, E_2)$. Therefore we get $(V_1, E_1) \times D_2$ is a dominating set of $\langle (V_1 - D_1) \times V_2 \rangle$.

(ii) Suppose $(V_1 \times D_2)$ is γ set of $G_1 \times G_2$. Now we find the dominating set of the sub graph $\langle (V_1 - D_1) \times V_2 \rangle$. Assume $D_1 \times (V_2 - D_2)$ is not an minimal dominating set of $G_1 \times G_2$ there exist a vertex $uv \in (V_1 - D_1) \times (V_2 - D_2)$ is not dominated by the vertex in the set $D_1 \times (V_2 - D_2)$. There is no strong edge between vertex and every vertex $xy \in D_1 \times (V_2 - D_2)$. Therefore we get there is no strong edge between $u, x \in V_2$. This implies D_1 is not a minimal dominating set of $G(V_2, E_2)$. This is contradict to our assumption D_2 is a minimal dominating set $G(V_1, E_1)$. Therefore we get $D_1 \times (V_2 - D_2)$ is a dominating set of $\langle (V_1 - D_1) \times V_2 \rangle$. Hence ID number of the graph $(G_1 \times G_2)$ is $\gamma'(G_1 \times G_2) = \min\{| D_1(V_2 \times D_2) |, | (V_1 \times D_1) \times D_2 |\}$.

Example 2.5.

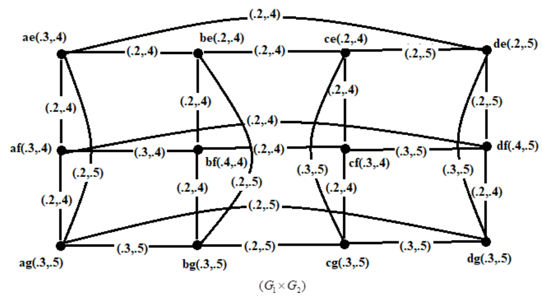

Figure 2.5. Cartesian product $(G_1 \times G_2)$ of $G(V_1 \times E_1)$ and $G(V_2 \times E_2)$.

In the IFG's $G(V_1 \times E_1)$ and $G(V_2 \times E_2)$. The dominating sets are $D_1 = \{a, c\}$ and $D_2 = \{e\}$. The ID set of $G_1 \times G_2$ are $D = \{af, ag, cf, cg\}$ and ID number $\gamma'(G_1 \times G_2) = 1.7$.

3. Conclusion

In this paper, we investigated the idea of an inverse dominating set (ID Set) and inverse domination number (ID-number) of an IFG. Further study the inverse domination number of complete and complete bipartite IFG some results and also some bounds of the ID-number are investigated. The ID-number of join of two IFGs and Cartesian product of two IFG are investigated.

References

[1] K. Atanasson, Intuitionistic Fuzzy Sets, Theory and Applications, Physicaverlag, New York, (1999).

[2] J. N. Mordeson and P. S. Nair, Fuzzy graphs and Fuzzy Hyper graphs, Physica-Verlag, Heidelberg, 1998, second edition, 2001.

[3] F. Harary, Graph Theory, Addition Wesley, Third Printing, October 1972.

[4] A. Somasundaram and S. Somasundaram, Domination in Fuzzy Graphs-I, Pattern Recognition Letters 19 (1998), 787-791.

[5] T. Haynes, S. T. Hedetniemi and P. J. Slater, Fundamentals of Domination in Graph, Marcel Decker, New York, (1998).

[6] A. Rosenfeld, Fuzzy Sets and their Applications to Cognitive and Decision Processes, Academic Press, New York (1975), pp. 77-95.