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VERTEX ODD MEAN LABELING OF SOME GRAPHS WITH PENDANT EDGES

A. SASIKALA and P. OVIYA

Department of Mathematics
Periyar Maniammai Institute of Science and Technology
Vallam, Thanjavur, Tamilnadu, India
E-Mail: sasikala@pmu.edu
oviyaprakash97@gmail.com

Abstract

While considering the graph G with vertices are denoted as P and edges are denoted as Q it plotted as a vertex odd mean graph, in this there is an injective Function f deriving from the vertices G to $\{1, 3, 5, \ldots, 2q-1\}$ on such way while each edges uv is labeled with $(f(u)+f(v))/2$ if $f(u)+f(v)$ is even and $(f(u)+f(v)+1)/2$ if $f(u)+f(v)$ is odd then the resulting edges are distinct in nature. Similar way of plotting and labeling methodology is called a "Vertex Odd Mean Labeling". Here $P_m + \overline{K}_n$ is a graph which we used for plotting in "Vertex Odd Mean Labeling" method.

1. Introduction

Graph plotted in this paper which is a simple finite graph without traces of isolated vertices. In this paper the terminology and notations used are in the precedent of Harary-Ref [1]. A graph labelling is as simple that to mapping a set of elements which usually vertices or edges or both together in to a set of numbers. There are lot of methods of labelling have been studied and explained in "The Excellent Survey of Graph Labelling" found in Ref. [2]. Origin of Graph labelling and its different techniques were introduced in work "Graceful Labelling" by Alexander Rosa in his paper at 1967. Let consider $G(V, E)$ be a graph with P is a vertices and Q is an Edges. The Concept of Mean Labelling was expelled and introduced in the work by

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Somasundaram and Ponraj [4]. Similarly the concept of odd mean labelling of graph was introduced by Manickam and Marudai in their studies Ref. [4]. Even mean labelling of some graph work was presented by the author Revathi N in her work [5]. With the above references and analysis we tried and investigated the possibilities of Odd Mean Labelling of Graph with $P_{(m+n)}$ with pendant edges.

Definition 1. Mean labeling of graph

A graph G with (p, q) is a mean graph if there is one to one function f from the vertices of G to $\{0, 1, 2, \ldots, q\}$ such that when each edge uv is labeled with $(f(u) + f(v))/2$ if $f(u) + f(v)$ is even and $(f(u) + f(v) + 1)/2$ if $f(u) + f(v)$ is odd then the ensuring edges are different.

Definition 2. Vertex odd mean labeling

A graph G with (p, q) is said to be an vertex odd mean graph if there is one to one function: $f : v(G) \to \{1, 3, 5, \ldots, 2q - 1\}$ such that when each edge uv is labelled with $(f(u) + f(v))/2$ if $f(u) + f(v)$ is even and $(f(u) + f(v) + 1)/2$ if $f(u) + f(v)$ is odd, after that the terminating edges are different. Such a function is called a vertex odd mean labeling.

Definition 3. Join of graph

The join of two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ denoted by $G_1 + G_2$, consists of $G_1 \cup G_2$ and all edges joining V_1 with V_2.

2. Main Result

In this section the vertex odd mean labeling of graph $P_m + \overline{K}_n$ and $P_7 + \overline{K}_2$ is presented.

Theorem 2.1. *The graph obtained by adding* 3*-pendant edges to each vertex of* $\overline{K}_n$ *in the graph* $P_m + \overline{K}_n$ *admits vertex odd mean labeling*.

Proof. The order and size of the graph G obtained by adding 3-pendant edges to each vertex of $\overline{K}_n$ in the graph $P_m + \overline{K}_n$ are respectively $4n + m$

and $(3n-1)+(n+1)m$. Let v_1 and v_2 be the vertices of P_m, $u_i(1 \le i \le n)$ be the vertices of $\overline{K}_n$. Obviously it $u_{it}(1 \le t \le 3n)$ will be the pendant vertices corresponding to u_i.

Define a vertex labeling $f : (P_m + \overline{K}_n) \to \{1, 3, 5, \ldots, 2q-1\}$ by as follows:

$$f(v_1) = 1$$

$$f(v_2) = 2$$

$$f(v_3) = 23$$

$$f(v_4) = 31$$

$$f(v_5) = 35$$

$$f(v_n) = 6i + 9, \text{ if } i = 6, 7, \ldots, m$$

$$f(u_i) = 10i + 1, 1 \le i \le n.$$

$$f(u_{it}) = \begin{cases} 10i - 5t, \ t = 1, 1 \le i \le 3n. \\ 10i - 3t + 3, \ t = 2, 1 \le i \le 3n \\ 10i - t + 2, \ t = 3, 1 \le i \le 3n \end{cases}$$

Clearly f is injective.

Clearly labels of the edges received by the mean of the labels on end vertices are all distinct. Hence the graph $P_m + \overline{K}_n$ has vertex odd mean labeling.

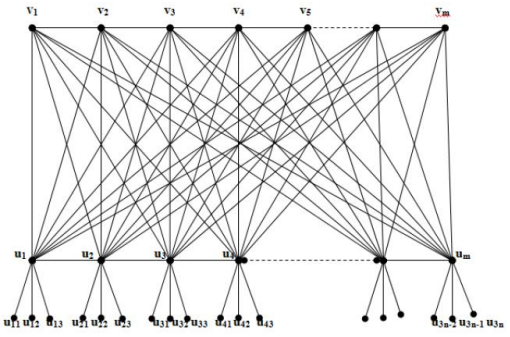

Figure 1: Vertex Odd Mean Labeling Graph $P_m+\overline{K}_n$

Example 2.2.

A graph obtained by adding 3-pendant edges to each vertex of the graph $P_7 + \overline{K}_2$ and its vertex odd mean labeling are shown in figure 2 respectively.

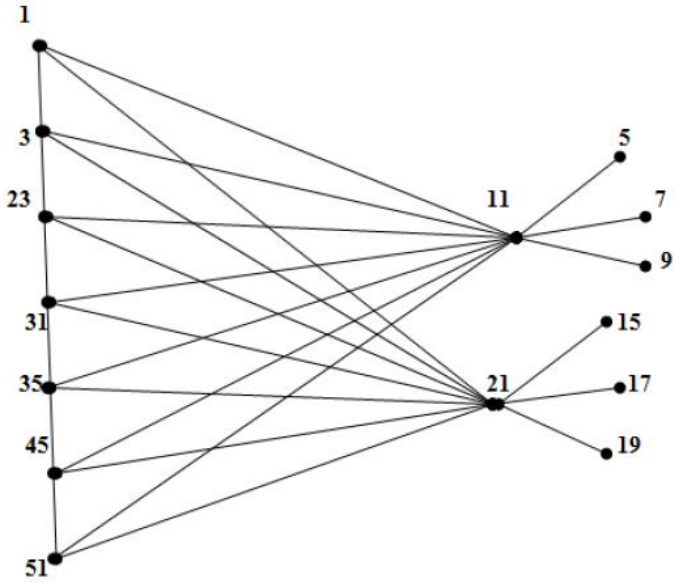

Figure 2. Vertex odd mean labeling of the graph $P_7 + \overline{K}_2$.

3. Conclusion

Graph theory has some interesting applications in system analysis, operations research and economics. The notion of labeling of graphs is an active research area and it has been broadly studied by several researchers. In wide area network (WAN), some systems are attached to the main server; the labeling system plays a very important role to label the cables. The labeling of graphs has been applied in the fields such as circuit design, communication network, coding theory, and crystallography.

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