Advances and Applications in Mathematical Sciences
Volume 20, Issue 4, February 2021, Pages 491-500

INFORMATION MEASURE ON BIPOLAR INTUITIONISTIC FUZZY SOFT SETS

S. ANITA SHANTHI and PRATHIPA JAYAPALAN

Department of Mathematics
Annamalai University
Chidambaram, 608 002, India
E-mail: shanthi.anita@yahoo.com
prathipajayapalan@gmail.com

Abstract

The aim of this paper is to introduce operators on bipolar intuitionistic fuzzy soft sets (BIFSS). Properties of operators on BIFSS are established. An information measure on BIFSS is proposed. Making use of the proposed information measure a decision making concept is constructed. Finally, this concept is illustrated by an example.

1. Introduction

Zadeh [6] introduced fuzzy sets and Atanassov [2] introduced the IFS. Chiranjibe et.al.,[3] developed the notion of BIFS set. Information measure in fuzzy sets is certainly a measure of fuzziness, while for intuitionistic fuzzy sets information measure measures both the fuzziness and intuitionism. Srinivasan et.al., [5] introduced some operations on intuitionistic fuzzy sets of root type. Anita shanthi et.al., [1] introduced information measure of IVIFSSRT. Mishra [4] developed the intuitionistic fuzzy information measure.

Now, we define operators on BIFSS and study some of its properties. We propose the concept of an information measure of BIFSS. Further decision making concept on information measure of BIFSS is developed.

2010 Mathematics Subject 94D.

Keywords: Bipolar intuitionistic fuzzy soft set, operators on BIFSS, information measure, decision making problem.

Received September 7, 2020; Accepted May 20, 2020

2. Operations on BIFSS

In this section, we define operators on BIFSS. Some properties of these operators are discussed.

Definition 2.1. Let

$$(BF, E) = \{x, ((\mu^{n}_{BF(e)}(x), \mu^{p}_{BF(e)}(x)), ((\nu^{n}_{BF(e)}(x), \nu^{p}_{BF(e)}(x)) : x \in U, e \in E\}$$

be BIFSS. The hesitant degree of BIFSS is denoted by $\pi^{n}_{BF(e)}$ and $\pi^{p}_{BF(e)}$ is defined as

$$\pi^{n}_{BF(e)}(x) = -1 - \mu^{n}_{BF(e)}(x) - \upsilon^{n}_{BF(e)}(x) \text{ and } \pi^{p}_{BF(e)}(x) = 1 - \mu^{p}_{BF(e)}(x) - \upsilon^{p}_{BF(e)}(x)$$

where $\pi^{n}_{BF(e)} \in [-1, 0]$ and $\pi^{p}_{BF(e)} \in [1, 0]$ respectively.

Definition 2.2. Let $\eta \in [0, 1]$ be a fixed number. Given an *BIFSS* (F, E) the operator D_{η} is defined as

$$D_{\eta}(BF, E) = \{x, ((\mu^{n}_{BF(e)}(x) + \eta\pi^{n}_{BF(e)}(x)), ((\mu^{p}_{BF(e)}(x) + \eta\pi^{p}_{BF(e)}(x)),$$

$$((\nu^{n}_{BF(e)}(x) + (1 - \eta)\pi^{n}_{BF(e)}(x)), ((\nu^{p}_{BF(e)}(x) + (1 - \eta)\pi^{p}_{BF(e)}(x))$$

$$: x \in U, e \in E\}.$$

Definition 2.3. For $\eta, \delta \in [0, 1], \eta + \delta \leq 1$ the operator $F_{\eta, \delta}$ for a *BIFSS* (BF, E) is defined as

$$F_{\eta, \delta}(BF, E) = \{x, ((\mu^{n}_{BF(e)}(x) + \eta\pi^{n}_{BF(e)}(x)), ((\mu^{p}_{BF(e)}(x) + \eta\pi^{p}_{BF(e)}(x)),$$

$$((\nu^{n}_{BF(e)}(x) + \delta\pi^{n}_{BF(e)}(x)), ((\nu^{p}_{BF(e)}(x)$$

$$+ \delta\pi^{p}_{BF(e)}(x)) : x \in U, e \in E\}.$$

Definition 2.4. Let $\eta, \delta \in [0, 1]$. Given a *BIFSS* (BF, E), the operator $G_{\eta, \delta}$ is defined as

$$G_{\eta,\delta}(BF, E) = \{x, ((\eta\mu^{n}_{BF(e)}(x), \eta\mu^{n}_{BF(e)}(x)), ((\delta\nu^{n}_{BF(e)}(x),$$

$$\delta\nu^{p}_{BF(e)}(x)) : x \in U, e \in E\}.$$

Obviously $G_{1,1}(BF, E) = (BF, E)$ and $G_{0,0}(BF, E) = \Phi$.

Definition 2.5. (BF, E) and (BG, E) are two *BIFSS*. The operation @ on (BF, E) and (BG, E) is defined as

$$(BF, E)@(BG, E) = \left\{x, \left(\frac{\mu^{n}_{BF(e)}(x) + \mu^{n}_{BG(e)}(x)}{2}, \frac{\mu^{p}_{BF(e)}(x) + \mu^{p}_{BG(e)}(x)}{2}\right),\right.$$

$$\left.\left(\frac{\nu^{n}_{BF(e)}(x) + \nu^{n}_{BG(e)}(x)}{2}, \frac{\nu^{p}_{BF(e)}(x) + \nu^{p}_{BG(e)}(x)}{2}\right) x \in U, e \in E\right\}.$$

Theorem 2.6. *Let* (BF, E) *and* (BG, E) *are two BIFSS and* $\eta, \delta \in [0, 1]$, *then the following conditions holds*:

(i) $D_{\eta}((BF, E)@(BG, E)) = D_{\eta}(BF, E)@ D_{\eta}(BG, E)$

(ii) $F_{\eta,\delta}((BF, E)@(BG, E)) = F_{\eta,\delta}(BF, E)@ F_{\eta,\delta}(BG, E)$

(ii) $G_{\eta,\delta}((BF, E)@(BG, E)) = G_{\eta,\delta}(BF, E)@ G_{\eta,\delta}(BG, E)$.

Proof.

(i) $$(BF, E)@(BG, E) = \left\{x, \left(\frac{1}{2}, (\mu^{n}_{BF(e)}(x) + \mu^{n}_{BG(e)}(x), \mu^{p}_{BF(e)}(x) + \mu^{p}_{BG(e)}(x))\right),\right.$$

$$\left.\left(\frac{1}{2}, (\nu^{n}_{BF(e)}(x) + \nu^{n}_{BG(e)}(x), \nu^{p}_{BF(e)}(x) + \nu^{p}_{BG(e)}(x))\right)\right\}$$

$$D_{\eta}((BF, E)@(BG, E)) = \left\{x, \left(\frac{1}{2}(\mu^{n}_{BF(e)}(x) + \eta\pi^{n}_{BF(e)}(x) + \mu^{n}_{BG(e)}(x) + \eta\pi^{n}_{BG(e)}(x),\right.\right.$$

$$\left.\mu^{p}_{BF(e)}(x) + \eta\pi^{p}_{BF(e)}(x) + \mu^{p}_{BG(e)}(x) + \eta\pi^{p}_{BG(e)}(x))\right)$$

$$\left(\frac{1}{2}(\nu^{n}_{BF(e)}(x) + (1-\eta)\pi^{n}_{BF(e)}(x) + \nu^{n}_{BG(e)}(x) + (1-\eta)\pi^{n}_{BG(e)}(x), \nu^{p}_{BF(e)}(x)\right.$$

$$+ (1-\eta)\pi^{p}_{BF(e)}(x) + \nu^{p}_{BG(e)}(x) + (1-\eta)\pi^{p}_{BG(e)}(x)) \big)\big\}$$

$$D_{\eta}(BF, E) = \{x, (\mu^{n}_{BF(e)}(x) + \eta\pi^{n}_{BF(e)}(x) + \mu^{p}_{BF(e)}(x) + \eta\pi^{p}_{BF(e)}(x))$$

$$(\nu^{p}_{BF(e)}(x) + (1-\eta)\pi^{p}_{BF(e)}(x), \nu^{p}_{BF(e)}(x) + (1-\eta)\pi^{p}_{BF(e)}(x))\}$$

$$D_{\eta}(BG, E) = \{x, (\mu^{n}_{BG(e)}(x) + \eta\pi^{n}_{BG(e)}(x), \mu^{p}_{BG(e)}(x) + \eta\pi^{p}_{BG(e)}(x)),$$

$$(\nu^{p}_{BG(e)}(x) + (1-\eta)\pi^{n}_{BG(e)}(x), \nu^{p}_{BG(e)}(x) + (1-\eta)\pi^{p}_{BG(e)}(x))\}$$

$$D_{\eta}(BF, E) @ D_{\eta}(BG, E)$$

$$= \{x, \frac{1}{2}(\mu^{n}_{BF(e)}(x) + \eta\pi^{n}_{BF(e)}(x) + \mu^{n}_{BG(e)}(x) + \eta\pi^{n}_{BG(e)}(x))$$

$$\mu^{p}_{BF(e)}(x) + \eta\pi^{p}_{BF(e)}(x) + \mu^{p}_{BG(e)}(x) + \eta\pi^{p}_{BG(e)}(x))),$$

$$(\frac{1}{2}(\nu^{n}_{BF(e)}(x) + (1-\eta)\pi^{n}_{BF(e)}(x) + \nu^{n}_{BG(e)}(x) + (1-\eta)\pi^{n}_{BG(e)}(x)),$$

$$\nu^{p}_{BF(e)}(x) + (1-\eta)\pi^{p}_{BF(e)}(x) + \nu^{p}_{BG(e)}(x) + (1-\eta)\pi^{p}_{BG(e)}(x)))\}.$$

Therefore $D_{\eta}((BF, E) @ (BG, E)) = D_{\eta}(BF, E) @ D_{\eta}(BG, E)$.

(ii) $$F_{\eta,\delta}((BF, E) @ (BG, E)) = \{x, \frac{1}{2}(\mu^{n}_{BF(e)}(x) + \eta\pi^{n}_{BF(e)}(x) + \mu^{n}_{BG(e)}(x)$$

$$+ \eta\pi^{n}_{BG(e)}(x)), \mu^{p}_{BF(e)}(x) + \eta\pi^{p}_{BF(e)}(x) + \mu^{p}_{BG(e)}(x) + \eta\pi^{p}_{BG(e)}(x))),$$

$$(\frac{1}{2}(\nu^{n}_{BF(e)}(x) + \delta\pi^{n}_{BF(e)}(x) + \nu^{n}_{BG(e)}(x) + \delta\pi^{n}_{BG(e)}(x)),$$

$$\upsilon^{p}_{BF(e)}(x) + \delta\pi^{p}_{BF(e)}(x) + \nu^{p}_{BG(e)}(x) + \delta\pi^{p}_{BG(e)}(x)))\}$$

$$F_{\eta,\delta}(BF, E) = \{x, (\mu^{n}_{BF(e)}(x) + \eta\pi^{n}_{BF(e)}(x) + \mu^{p}_{BF(e)}(x) + \eta\pi^{p}_{BF(e)}(x)),$$

$$(\nu^{n}_{BF(e)}(x) + \delta\pi^{n}_{BF(e)}(x) + \nu^{p}_{BF(e)}(x) + \delta\pi^{p}_{BF(e)}(x))\}$$

$$F_{\eta,\delta}(BG, E) = \{x, (\mu^{n}_{BG(e)}(x) + \eta\pi^{n}_{BG(e)}(x) + \mu^{p}_{BG(e)}(x) + \eta\pi^{p}_{BG(e)}(x)),$$

$$(\nu^{n}_{BG(e)}(x) + \delta\pi^{n}_{BG(e)}(x), \nu^{p}_{BG(e)}(x) + \delta\pi^{p}_{BG(e)}(x))\}$$

$$F_{\eta,\delta}(BF, E) @ F_{\eta,\delta}(BG, E) = \{x, (\frac{1}{2}(\mu^n_{BF(e)}(x) + \eta\pi^n_{BF(e)}(x) + \mu^n_{BG(e)}(x)$$

$$+ \eta\pi^n_{BG(e)}(x)), \mu^p_{BF(e)}(x) + \eta\pi^p_{BF(e)}(x) + \mu^p_{BG(e)}(x) + \eta\pi^p_{BG(e)}(x))),$$

$$(\frac{1}{2}(\nu^n_{BF(e)}(x) + \delta\pi^n_{BF(e)}(x) + \nu^n_{BG(e)}(x) + \delta\pi^n_{BG(e)}(x)),$$

$$\upsilon^p_{BF(e)}(x) + \delta\pi^p_{BF(e)}(x) + \nu^p_{BG(e)}(x) + \delta\pi^p_{BG(e)}(x)))\}.$$

Therefore $F_{\gamma,\delta}((BF, E) @ (BG, E)) = F_{\gamma,\delta}(BF, E) @ F_{\gamma,\delta}(BG, E)$

(iii) $G_{\eta,\delta}((BF, E) @ (BG, E))$

$$= \{x, \frac{1}{2}(\eta\mu^n_{BF(e)}(x) + \eta\mu^n_{BG(e)}(x) + \eta\mu^p_{BF(e)}(x) + \eta\pi^p_{BG(e)}(x))),$$

$$(\frac{1}{2}(\delta\nu^n_{BF(e)}(x) + \delta\nu^n_{BG(e)}(x) + \delta\nu^p_{BF(e)}(x) + \delta\nu^n_{BG(e)}(x))\}$$

$$G_{\eta,\delta}(BF, E) = \{x, (\eta\mu^p_{BF(e)}(x) + \eta\pi^p_{BF(e)}(x)), (\delta\mu^p_{BF(e)}(x) + \delta\nu^p_{BF(e)}(x))\}$$

$$G_{\eta,\delta}(BG, E) = \{x, (\eta\mu^p_{BG(e)}(x) + \eta\mu^p_{BG(e)}(x)), (\delta\mu^p_{BG(e)}(x) + \delta\nu^p_{BG(e)}(x))\}$$

$$G_{\eta,\delta}(BF, E) @ G_{\eta,\delta}(BG, E)$$

$$= \{x, \frac{1}{2}(\eta\mu^n_{BF(e)}(x) + \eta\mu^n_{BG(e)}(x), \eta\mu^p_{BF(e)}(x) + \eta\pi^p_{BG(e)}(x))),$$

$$(\frac{1}{2}(\delta\nu^n_{BF(e)}(x) + \delta\nu^n_{BG(e)}(x) + \delta\nu^p_{BF(e)}(x) + \delta\nu^n_{BG(e)}(x))\}.$$

Therefore $G_{\eta,\delta}((BF, E) @ (BG, E)) = G_{\eta,\delta}(BF, E) @ G_{\eta,\delta}(BG, E)$.

3. Information measure on *BIFSS*

In this section, we define information measure on BIFSS and prove that it is an entropy.

Definition 3.1. Let the universal set be $U = \{A_1, A_2, \ldots, A_m\}$ and the set of parameters be $E = \{e_1, e_2, \ldots, e_n\}$. For any *BIFSS* (BF, E) an information measure to indicate the degree of fuzziness of (BF, E) is defined as

$$BI_m(BF, E_p) = \frac{1}{n}\sum_{p=1}^{n} \frac{\wedge\{\mu^n_{BF(e)}(x_p), \nu^n_{BF(e)}(x_p)\} - \wedge\{\mu^p_{BF(e)}(x_p), \nu^p_{BF(e)}(x_p)\}}{\vee\{\mu^n_{BF(e)}(x_p), \nu^n_{BF(e)}(x_p)\} - \vee\{\mu^p_{BF(e)}(x_p), \nu^p_{BF(e)}(x_p)\}}$$

where, $\wedge, \vee$ denotes the minimum and maximum, respectively.

Theorem 3.2. *The information measure* $BI_m(BF, E)$ *for BIFSS is an entropy.*

Proof. If $(BF, E) \in P(U)$, then for each $x_p \in A_p$ and $e \in E$ we have:

Case (i). If $(\mu^n_{BF(e)}(x_p), \mu^p_{BF(e)}(x_p)) = (\nu^n_{BF(e)}(x_p), \nu^p_{BF(e)}(x_p))$, then

$$\mu^n_{BF(e)}(x_p) = \nu^n_{BF(e)}(x_p), \mu^p_{BF(e)}(x_p) = \nu^p_{BF(e)}(x_p)$$

$$\wedge\{\mu^n_{BF(e)}(x_p), \nu^n_{BF(e)}(x_p)\} - \wedge\{\mu^p_{BF(e)}(x_p), \nu^p_{BF(e)}(x_p)\}$$

$$= \vee\{\mu^n_{BF(e)}(x_p), \nu^n_{BF(e)}(x_p)\} - \vee\{\mu^p_{BF(e)}(x_p), \nu^p_{BF(e)}(x_p)\}.$$

So, $BI_m(BF, E) = 1$.

Conversely, suppose that $BI_m(BF, E) = 1$.

This implies that

$$\wedge\{\mu^n_{BF(e)}(x_p), \nu^n_{BF(e)}(x_p)\} = \wedge\{\mu^n_{BF(e)}(x_p), \nu^n_{BF(e)}(x_p)\}$$

and

$$\wedge\{\mu^p_{BF(e)}(x_p), \nu^p_{BF(e)}(x_p)\} - \wedge\{\mu^p_{BF(e)}(x_p), \nu^p_{BF(e)}(x_p)\}.$$

Hence $\mu^n_{F(e)}(x_p) = \nu^n_{F(e)}(x_p)$ and $\mu^p_{F(e)}(x_p) = \nu^p_{F(e)}(x_p)$. Therefore, we have

$$(\mu^n_{BF(e)}(x_p), \mu^p_{BF(e)}(x_p)) = (\nu^n_{BF(e)}(x_p), \nu^p_{BF(e)}(x_p)).$$

Case (ii). Suppose that (BF, E) is less than (BG, E) then,

$$(\mu^{n}_{BF(e)}(x_p),\ \mu^{p}_{BF(e)}(x_p)) \le (\mu^{n}_{BG(e)}(x_p),\ \mu^{p}_{BG(e)}(x_p)),$$

$$(\nu^{n}_{BF(e)}(x_p),\ \nu^{p}_{BF(e)}(x_p)) \ge (\nu^{n}_{BG(e)}(x_p),\ \nu^{p}_{BG(e)}(x_p)).$$

For $(\mu^{n}_{BG(e)}(x_p),\ \mu^{p}_{BG(e)}(x_p)) \le (\nu^{n}_{BG(e)}(x_p),\ \nu^{p}_{BG(e)}(x_p)),$

$$\mu^{n}_{BF(e)}(x_p) \le \mu^{n}_{BG(e)}(x_p)) \le \nu^{n}_{BG(e)}(x_p) \le \nu^{n}_{BF(e)}(x_p)$$

and

$$\mu^{p}_{BF(e)}(x_p) \le \mu^{p}_{BG(e)}(x_p) \le \nu^{p}_{BG(e)}(x_p) \le \nu^{p}_{BF(e)}(x_p).$$

So,

$$\frac{\wedge\{\mu^{n}_{BF(e)}(x_p),\ \nu^{n}_{BF(e)}(x_p)\} - \wedge\{\mu^{p}_{BF(e)}(x_p),\ \nu^{p}_{BF(e)}(x_p)\}}{\vee\{\mu^{n}_{BF(e)}(x_p),\ \nu^{n}_{BF(e)}(x_p)\} - \vee\{\mu^{p}_{BF(e)}(x_p),\ \nu^{p}_{BF(e)}(x_p)\}},$$

$$= \frac{\mu^{n}_{BF(e)}(x_p) + \mu^{p}_{BF(e)}(x_p)}{\nu\mu^{n}_{BF(e)}(x_p) - \nu^{n}_{BF(e)}(x_p)} \le \frac{\mu^{n}_{BG(e)}(x_p) + \mu^{p}_{BG(e)}(x_p)}{\nu\mu^{n}_{BG(e)}(x_p) - \nu^{n}_{BG(e)}(x_p)},$$

$$= \frac{\wedge\{\mu^{n}_{BG(e)}(x_p),\ \nu^{n}_{BG(e)}(x_p)\} - \wedge\{\mu^{p}_{BG(e)}(x_p),\ \nu^{p}_{BG(e)}(x_p)\}}{\vee\{\mu^{n}_{BG(e)}(x_p),\ \nu^{n}_{BG(e)}(x_p)\} - \vee\{\mu^{p}_{BG(e)}(x_p),\ \nu^{p}_{BG(e)}(x_p)\}}.$$

By taking summation on both sides and dividing it by n, we get

$$\frac{1}{n}\sum_{p=1}^{n} \frac{\wedge\{\mu^{n}_{BF(e)}(x_p),\ \nu^{n}_{BF(e)}(x_p)\} - \wedge\{\mu^{p}_{BF(e)}(x_p),\ \nu^{p}_{BF(e)}(x_p)\}}{\vee\{\mu^{n}_{BF(e)}(x_p),\ \nu^{n}_{BF(e)}(x_p)\} - \vee\{\mu^{p}_{BF(e)}(x_p),\ \nu^{p}_{BF(e)}(x_p)\}}$$

$$\le \frac{1}{n}\sum_{p=1}^{n} \frac{\wedge\{\mu^{n}_{BG(e)}(x_p),\ \nu^{n}_{BG(e)}(x_p)\} - \wedge\{\mu^{p}_{BG(e)}(x_p),\ \nu^{p}_{BG(e)}(x_p)\}}{\vee\{\mu^{n}_{BG(e)}(x_p),\ \nu^{n}_{BG(e)}(x_p)\} - \vee\{\mu^{p}_{BG(e)}(x_p),\ \nu^{p}_{BG(e)}(x_p)\}}$$

i.e., $BI_m(BF,\ E) \le BI_m(BG,\ E)$.

Case (iii). Suppose that $(BF,\ E)$ is greater than $(BG,\ E)$ then,

$$(\mu^{n}_{BF(e)}(x_p),\ \mu^{p}_{BF(e)}(x_p)) \ge (\mu^{n}_{BG(e)}(x_p),\ \mu^{p}_{BG(e)}(x_p)),$$

$$(\nu^{n}_{BF(e)}(x_p),\ \nu^{p}_{BF(e)}(x_p)) \geq (\nu^{n}_{BG(e)}(x_p),\ \nu^{p}_{BG(e)}(x_p)).$$

For $(\mu^{n}_{BF(e)}(x_p),\ \mu^{p}_{BF(e)}(x_p)) \geq (\nu^{n}_{BG(e)}(x_p),\ \nu^{p}_{BG(e)}(x_p))$,

$$\mu^{n}_{BF(e)}(x_p) \geq \mu^{n}_{BF(e)}(x_p)) \geq \nu^{n}_{BG(e)}(x_p),\ \nu^{n}_{BG(e)}(x_p)$$

and

$$\mu^{p}_{BF(e)}(x_p) \geq \mu^{p}_{BG(e)}(x_p)) \geq \nu^{p}_{BG(e)}(x_p) \geq \nu^{p}_{BF(e)}(x_p).$$

So,

$$\frac{\wedge\{\mu^{n}_{BF(e)}(x_p),\ \nu^{n}_{BF(e)}(x_p)\} - \wedge\{\mu^{p}_{BF(e)}(x_p),\ \nu^{p}_{BF(e)}(x_p)\}}{\vee\{\mu^{n}_{BF(e)}(x_p),\ \nu^{n}_{BF(e)}(x_p)\} - \vee\{\mu^{p}_{BF(e)}(x_p),\ \nu^{p}_{BF(e)}(x_p)\}},$$

$$= \frac{\mu^{n}_{BF(e)}(x_p) + \mu^{p}_{BF(e)}(x_p)}{\nu\mu^{n}_{BF(e)}(x_p) - \nu^{p}_{BF(e)}(x_p)} \geq \frac{\mu^{n}_{BG(e)}(x_p) - \mu^{p}_{BG(e)}(x_p)}{\nu\mu^{n}_{BG(e)}(x_p) - \nu^{p}_{BG(e)}(x_p)},$$

$$= \frac{\wedge\{\mu^{n}_{BG(e)}(x_p),\ \nu^{n}_{BG(e)}(x_p)\} - \wedge\{\mu^{p}_{BG(e)}(x_p),\ \nu^{p}_{BG(e)}(x_p)\}}{\vee\{\mu^{n}_{BG(e)}(x_p),\ \nu^{n}_{BG(e)}(x_p)\} - \vee\{\mu^{p}_{BG(e)}(x_p),\ \nu^{p}_{BG(e)}(x_p)\}}.$$

By taking summation on both sides and dividing it by n, we get

$$\frac{1}{n}\sum_{p=1}^{n}\frac{\wedge\{\mu^{n}_{BF(e)}(x_p),\ \nu^{n}_{BF(e)}(x_p)\} - \wedge\{\mu^{p}_{BF(e)}(x_p),\ \nu^{p}_{BF(e)}(x_p)\}}{\vee\{\mu^{n}_{BF(e)}(x_p),\ \nu^{n}_{BF(e)}(x_p)\} - \vee\{\mu^{p}_{BF(e)}(x_p),\ \nu^{p}_{BF(e)}(x_p)\}}$$

$$\leq \frac{1}{n}\sum_{p=1}^{n}\frac{\wedge\{\mu^{n}_{BG(e)}(x_p),\ \nu^{n}_{BG(e)}(x_p)\} - \wedge\{\mu^{p}_{BG(e)}(x_p),\ \nu^{p}_{BG(e)}(x_p)\}}{\vee\{\mu^{n}_{BG(e)}(x_p),\ \nu^{n}_{BG(e)}(x_p)\} - \vee\{\mu^{p}_{BG(e)}(x_p),\ \nu^{p}_{BG(e)}(x_p)\}}$$

i.e., $BI_m(BF, E) \geq BI_m(BG, E)$.

Case (iv).

$$(BF, E)^c = \{x, (\nu^{n}_{BF(e)}(x_p),\ \nu^{p}_{BF(e)}(x_p))$$

$$(\mu^{n}_{BF(e)}(x_p),\ \mu^{n}_{BF(e)}(x_p)) : x_p \in A_p,\ e \in E\}.$$

Therefore,

$$BI_m(BF, E)^C = \frac{1}{n}\sum_{p=1}^{n}\frac{\wedge\{\nu^n_{BF(e)}(x_p), \mu^n_{BF(e)}(x_p)\} - \wedge\{\nu^p_{BF(e)}(x_p), \mu^p_{BF(e)}(x_p)\}}{\vee\{\nu^n_{BF(e)}(x_p), \mu^n_{BF(e)}(x_p)\} - \vee\{\nu^p_{BF(e)}(x_p), \mu^p_{BF(e)}(x_p)\}}$$

$$= BI_m(BF, E).$$

Hence an information measure is an entropy.

4. Information measure on *BIFSS*

In this section, we develop a decision making concept based on information measure of *BIFSS*. A procedure for decision making method is framed.

4.1. Procedure:

The following are the steps to be followed for this method:

Step 1: Construct the *BIFS* sets (BF, E_q) over *U*.

Step 2. Determine the information measure of (BF, E_q) by using

Definition 3.1.

Step 3. Compare the values of $BI_m(BF, E_q)$ and conclude. Thus the better choice is the alternative for which the information measure is the least.

Example 4.2. A customer decides to buy a television. Television (alternatives) of three different companies (BF, E_1), (BF, E_2), (BF, E_3) are evaluated over six factors $\{t_1, t_2, t_3, t_4, t_5, t_6\}$, where $t_1 =$ price range, $t_2 LED$ or $OLED$, $t_3 =$ cables and accessories, $t_4 =$ audio upgrades, $t_5 =$ screen size and $t_6 =$ smart television. Depending on the six factors, the best alternative is determined.

Step 1. BIFSS over *U* to assess the alternatives is as follows:

U	(BF, E_1)	(BF, E_2)	(BF, E_3)
t_1	((-0.13,0.47)(-0.25,0.1))	((-0.2,0.68),(-0.32,0.15))	((0.18,0.42),(0.28,0.12))
t_2	((-0.34,0.62),(-0.16,0.28))	((-0.18,0.83),(-0.61,0.1))	((0.17,0.22),(0.18,0.06))
t_3	((-0.25,0.73),(-0.4,0.14))	((-0.32,0.45),(-0.27,0.13))	((-0.14,0.73),(0.3,0.12))
t_4	((-0.1,0.51),(-0.38,0.08))	((-0.41,0.64),(-0.5,0.3))	((-0.24,0.6),(0.04,0.11))
t_5	((-0.12,0.38),(-0.26,0.04))	((-0.24,0.76),(-0.61,0.15))	((0.31,0.52),(0.17,0.28))
t_6	((-0.27,0.47), (-0.3,0.13))	((-0.3,0.54),(-0.43,0.28))	((0.16,0.72),(0.22,0.12))

Step 2. The $BI_m(BF, E_q)$ are estimated as follows:

$$BI_m(BF, E_q) = 0.6987$$

$$BI_m(BF, E_2) = 0.6969$$

$$BI_m(BF, E_3) = 0.5922 .$$

Step 3. The best alternative is the one which has least information measure with respect to six factors. We have $BI_m(BF, E_3) < BI_m(BF, E_2) < BI_m(BF, E_1)$. Therefore, the information measure $BI_m(BF, E_3)$ has the least value and its corresponding alternative is the best alternative. So, the television (BF, E_3) is the best one among the others.

References

[1] S. Anita Shanthi and J.Vadivel Naidu, A decision making method based on information measure of interval valued intuitionistic fuzzy soft sets of root type, The Journal of Fuzzy Mathematics 24(2) (2016), 343-358.

[2] K. Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20(1) (1986), 87-96.

[3] Chiranjibe Jana and Madhumangal Pal, Application of bipolar intuitionistic fuzzy soft sets in decision making problem, International Journal of Fuzzy System Applications 7(3) (2018), 33-55.

[4] A. R. Mishra, Intuitionistic fuzzy information measures with application in rating of township development, Iranian Journal of Fuzzy Systems 13(3) (2016), 49-70.

[5] R. Srinivasan and N. Palaniappan, Some operators on intuitionistic fuzzy sets of root type, Annals of Fuzzy Mathematics and Informatics 4(2) (2012), 377-383.

[6] L. A. Zadeh, Information and Control 8 (1965), 338-353.