Advances and Applications in Mathematical Sciences
Volume 21, Issue 7, May 2022, Pages 3773-3784

A NOTE ON SOME STRONGER FORM OF SOFT PRE OPEN SETS

M. VIJAY and R. ASOKAN

School of Mathematics
Madurai Kamaraj University
Madurai, Tamil Nadu, India
E-mail: vijaiicm@yahoo.com

School of Mathematics
Madurai Kamaraj University
Madurai, Tamil Nadu, India
E-mail: asokan.maths@mkuniversity.org

Abstract

In this paper, we define a stronger form of soft pre open sets and we prove that the set do not form a soft topology. Also we arrive a decomposition of these sets in terms of soft open sets and soft w dense open sets. Also we define a new kind of continuous functions, occurring between the soft continuous and pre continuous functions. Also, we studied many relationship related to the above defined stronger form of soft pre open sets.

1. Introduction and Preliminaries

In 1999, Molodtsov [3] defined a soft set, which is an ethnic mathematical tool to handle uncertainty. The soft topological spaces was introduced by Shabir and Naz [4] in 2011. Based on this, many topological concepts were modified with inclusion of soft topology. Soft Topology is a research area and many of its applications are placing a major role in current research [1, 2, 5]. Many soft topological concepts continuity, compactness and separation axioms are generalized, which plays an effective role in soft topology structure. Out of these generalizations of open sets, open sets in topological spaces is defined in [6], [7]. In this work, we define a stronger form of soft pre open sets and we prove that the set do not form a soft topology. Also we arrive

2020 Mathematics Subject Classification: 34Bxx, 76-10, 80A30.
Keywords: soft -open, generated soft topology, soft pre-open sets.
Received December 12, 2021 Accepted January 14, 2022.

a decomposition of these sets in terms of soft open sets and soft *w* dense open sets. Also we define a new kind of continuous functions, occurring between the soft continuous and pre continuous functions. Also, we studied many relationship related to the above defined stronger form of soft pre open sets

Definition 1.1 [7] **and** [6]**.** (a) Let (X, τ) be a Topological Space and let $A \subseteq X$. Then A is called as pre open if there exists $B \in \tau$ such that $B \in A \subseteq Cl_{\tau}(B)$. The set of family of all pre open sets of (X, τ) is denoted by $PO(X, \tau)$.

(b) Let (X, τ) be a soft Topological space and let $A \subseteq X$. Then A is called as ω_p-open if for $B \in \tau$, then $B \subseteq A \subseteq Cl_{\tau\omega}(B)$ and the set of family of all ω_p-open sets in (X, τ) is denoted by $\omega_p(X, \tau)$.

Definition 1.2 [10]**.** Let (X, τ, U) be a soft Topological Space and let $A \in SS(X, A)$. Then A is called as soft pre-open if there exists any $B, \in, \tau$ such that $A \subseteq B \subseteq Cl_{\tau}(A)$. The set of family of all soft pre open sets in (X, τ, U) is denoted by $PO(X, \tau, U)$.

Definition 1.3 [1] **and** [11]**.** Let (X, A) be a soft space and V be the set of parameters and $P \in (X, A)$.

(a) If $P(a) = \begin{cases} Zifa = e \\ Qifb = e^1 \end{cases}$

and P is denoted by e_z.

(b) If $P(a) = Z$ for all $b, \in, A$ then P is denoted by e_z.

(c) If $P(a) = \} \begin{matrix} \{y\} \text{ if } a = e \\ q \text{ if } b \neq e^1 \end{matrix}$

And P is denoted by e_y and P is called as a soft point. The set of all soft points in the soft space (X, A) is called as $SP(X, A)$.

Definition 1.4 [11]**.** Let H belongs to soft space (X, A) and a_x belong to the set of soft points of (X, A). Then $a_x \tilde{\in} H$ if $a_x \tilde{\in} H$ if $a_x \tilde{\subseteq} H$ or $a_x \tilde{\in} H$ if $x \in H(a)$.

Definition 1.5 [2] **and** [5]**.** A soft topological space (X, τ, A) will be called as

(a) Soft locally countable if for each soft point b_x There exists $(V, \in, \tau)$ such that $b_x \tilde{\in} U$ and is U countable.

(b) Soft anti locally countable for each $B \in \tau - \{O_A\}$, B is not a countable soft set.

(c) Soft ω-regular, if S is soft closed and $b_x \tilde{\in} l_A - S$, then there exists $(V, \in, \tau)$ and $V, \in \tau_\omega$ such that $b_x \tilde{\in} U, S \subseteq V$ and $U \tilde{\bigcap} V = O_A$.

Theorem 1.6 [4]**.** *Let* (X, τ, A) *be a soft Topological space and the collection* $\{H(a), H \in \tau\}$ *be a topology on X for each* $a, \in, A$ *then this topology is called as* τ_a.

Theorem 1.7. *Let* (X, τ_1) *be a topological space and let A be the set of parameter*, *then the soft topology on X, relative to A is denoted by* $\{H \in SS(Y, A), H(a) \in \tau$ *for each* $a, \in, A\}$ *and this topology is denoted by* $\tau(\tau_1)$.

Definition 1.8 [13]**.** Let (X, τ_1) and (Y, τ_2) be two topological space and the function $f : (X, \tau_1) \to (Y, \tau_2)$ is said to be the pre continuuous if for every $A \in \tau_2 . f^{-1}(A) \in PO(X, \tau)$.

Definition 1.9 [6]**.** Let (X, τ_1) and (Y, τ_2) be two topological space and the function $f : (X, \tau_1) \to (Y, \tau_2)$ is said to be the ω_p-continuous if for every $A \in \tau_2 f^{-1}(A) \in \omega_p(X, \tau_1)$.

2. Some Stronger form of Soft Pre-open Sets ($S\delta$-Open Sets)

Definition 2.1. In a soft topological space (X, σ, U) and a soft space (X, U), we define a stronger form of soft pre-open sets ($S\delta$-Open Sets) as follows. Let $G \in (X, U)$. Then G called as-$S\delta$-Open set in (X, σ, U) if we can find $F \in \tau$ with $\tilde{\subseteq} F \tilde{\subseteq} Cl_{\tau\omega}(G)$.

The family of all $S\delta$-Open Sets in (X, σ, U) is $SP(X, \sigma, U)$.

Theorem 2.2. *In any soft topological space* (X, σ, U), $\sigma \subseteq SP(X, \sigma, U) \subseteq PO(X, \sigma, U)$.

Proof. To prove $\sigma \subseteq SP(X, \sigma, U)$ let $G \in \sigma$. Take $F = G$. Then $F \in \sigma$ and $G \tilde{\subseteq} F \tilde{\subseteq} Cl_{\sigma\omega}$ Therefore $G \in SP(X, \sigma, U)$. To prove $SP(X, \sigma, U) \subseteq PO(X, \sigma, U)$, let $G \in SP(X, \sigma, U)$, then $F \in \tau$ such that $G \tilde{\subseteq} F \tilde{\subseteq} Cl_{\sigma\omega}(G) \tilde{\subseteq} Cl_{\sigma}(G)$ then $G \in PO(X, \sigma, U)$. Hence we have the result.

Theorem 2.3. *In any soft topological space* (X, σ, U), $CSS(X, U) \bigcap SP(X, \sigma, U) \subseteq \sigma$.

Proof. Let $M \in CSS(X, U) \bigcap SP(X, \sigma, U)$, which implies $M \in CSS(X, U)$ and $M \in SP(X, \sigma, U)$. As $M \in CSS(X, U)$, as per Cor. 5 from [2], $Cl_{\sigma\omega}(M) = M$. And $M \in SP(X, \sigma, U)$, we have some $N \in \sigma$ with $M \tilde{\subseteq} N \tilde{\subseteq} Cl_{\sigma\omega}(M) = M$. So $M = N$ then we have $G \in \sigma$.

Theorem 2.4. *Any soft topological space* (X, σ, U) *which is locally countable* (*soft*), *we have* $SP(X, \sigma, U) = \sigma$.

Proof. Let (X, σ, U) is locally countable, which is soft, for $SP(X, \sigma, U) = \sigma$, take $M \in SP(X, \sigma, U)$, then if any $N \in \sigma$ we have $M \tilde{\subseteq} N \tilde{\subseteq} Cl_{\sigma\omega}(M)$. As (X, σ, U) is locally countable which is soft, as per cor. 5 from [2], $Cl_{\sigma\omega}(N) = N$, hence we have $N \in \sigma$. Then by Theorem 2.3 $\sigma \subseteq SP(X, \sigma, U)$.

Theorem 2.5. *In any soft topological space* (X, σ, U), *let* $\{A_\mu : \mu \in M\} \subseteq SP(X, \sigma, U)$ *then* $\tilde{\bigcup}_{\mu \in M} A_\mu \in SP(X, \sigma, U)$.

Proof. Take $\{A_\mu : \mu \in \Gamma\} \subseteq SP(X, \sigma, U)$, to any $\mu \in M$, we have $B_\mu \in \sigma$ with $A_\mu \tilde{\subseteq} B_\mu \tilde{\subseteq} Cl_{\sigma\omega}(A_\mu)$.

Hence $\tilde{\bigcup}_{\mu \in M} A_\mu \in \sigma$ and $\tilde{\bigcup}_{\mu \in M} A_\mu \tilde{\subseteq} \tilde{\bigcup}_{\mu \in M} B_\mu \tilde{\subseteq} \tilde{\bigcup}_{\mu \in M} Cl_{\sigma\omega}(A_\mu)$ $\tilde{\subseteq} Cl_{\sigma\omega}(\tilde{\bigcup}_{\mu \in M} A_\mu)$. Therefore we have the result $\tilde{\bigcup}_{\lambda \in M} A_\mu \in SP(X, \sigma, U)$.

Theorem 2.6. *In any Soft Topological Space* (X, σ, U), $PO(X, \sigma_\omega, U)$ $SP(X, \sigma_\omega, U)$.

Proof. Let (X, σ, U) be Soft Topological Space. By Theorem 2.3 $SP(X, \sigma_\omega, U) \subseteq PO(X, \sigma_\omega, U)$. To prove that $PO(X, \sigma_\omega, U) \subseteq SP(X, \sigma_\omega, U)$, take $M \in PO(X, \sigma_\omega, U)$, we can find $N \in \sigma_\omega$ with $M \tilde{\subseteq} N \tilde{\subseteq} Cl_{(\sigma_\omega)\omega}(M)$. As per Theorem 5 (from [2]) we have $(\sigma_\omega)_\omega = \sigma_\omega$, hence $Cl_{(\sigma_\omega)\omega}(N) = Cl_{\sigma\omega}(N)$, which shows that $M \in SP(X, \sigma_\omega, U)$.

Theorem 2.7. *Let* (X, σ, U) *be a soft topological space which is anti locally soft countable, then*

$$\sigma_\omega \cap PO(X, \sigma, U) \subseteq SP(X, \sigma, U).$$

Proof. Let (X, σ, U) be anti-locally countable, which is soft and take $M \in \sigma_\omega \cap PO(X, \sigma, U)$. As (X, σ, U) is anti-locally countable (soft) and $M \in \sigma_\omega$ then $Cl_{\sigma\omega}(M) = Cl_{\sigma\omega}(M)$. As $M \in PO(X, \sigma, U)$, we have some $N \in \tau$ such that $M \tilde{\subseteq} N \tilde{\subseteq} Cl_\sigma(M) = Cl_{\sigma\omega}(M)$. Hence we have $N \in SP(X, \sigma, U)$.

Theorem 2.8. *Let* (X, σ, U) *Soft Topological space and M belongs to the soft space* (X, U). *Then M belongs to* $SP(X, \sigma, U)$ *iff* $M \tilde{\subseteq} \operatorname{int}_\sigma(Cl_{\sigma\omega}(M))$.

Proof. Assume that $M \in SP(X, \sigma, U)$. Here we can find $N \in \tau$ with $M \tilde{\subseteq} N \tilde{\subseteq} Cl_{\sigma\omega}(M)$. So $G \tilde{\subseteq} \operatorname{int}_\sigma(Cl_{\sigma\omega}(G))$. Conversely, if $M \subseteq \operatorname{int}_\sigma(\widetilde{Cl_{\sigma\omega}}(M))$, $N = \operatorname{int}_\sigma(Cl_{\sigma\omega}(M))$. Then $N \in \tau$ and $M \tilde{\subseteq} N \tilde{\subseteq} \operatorname{int}_\sigma(Cl_{\sigma\omega}(M)) \tilde{\subseteq} Cl_{\sigma\omega}(M)$. So we have $N \in SP(X, \sigma, U)$.

Definition 2.9. Let (X, σ, U) be a Soft Topological Space and G belongs to the soft space (X, U). Then G known as ω-dense soft set if $Cl_{\sigma\omega}(G) = 1_U$.

Theorem 2.10. *Let* (X, σ, U) *be a soft topological space and let G belong to the soft space* (X, A). *Then* $M \in SP(X, \sigma, U)$ *if and only if is M may be expressed as intersection* (*soft*) *of open set and a* ω-*dense*, *both are soft sets*.

Proof. Assume $M \in SP(X, \sigma, U)$, from 2.9 $M \tilde{\subseteq} \text{int}_\sigma(Cl_{\sigma\omega}(M))$. Assign $N = \text{int}_\sigma(Cl_{\sigma\omega}(M))$ and $V = (1_U - M) \tilde{\bigcup} N$. So N will be soft open and $M = N \tilde{\bigcap} V$. Also

$$Cl_{\sigma\omega}(V) = Cl_{\sigma\omega}((1_U - \text{int}_\sigma(Cl_{\sigma\omega}(M))) \tilde{\bigcup} M)$$

$$= Cl_{\sigma\omega}((1_U - \text{int}_\sigma Cl_{\sigma\omega}((M))) \tilde{\bigcup} Cl_{\sigma\omega}(M))$$

$$= Cl_{\sigma\omega}(Cl_{\sigma\omega}(1_U - Cl_{\sigma\omega}(M))) \tilde{\bigcup} Cl_{\sigma\omega}(M)$$

$$= Cl_{\sigma\omega}(Cl_{\sigma\omega} - \text{int}_{\tau\omega}(1_U - (M)) \tilde{\bigcup} Cl_{\sigma\omega}(M)$$

$$\tilde{\supseteq} \text{int}_{\sigma\omega}(1_U - M) \tilde{\bigcup} Cl_{\sigma\omega}(M) = 1_U.$$

Hence we have the result

Conversely, assume $V = M \tilde{\bigcap} N$ with $M \in \sigma$ and N is ω-dense soft set. To prove that $M \tilde{\subseteq} Cl_{\sigma\omega}(V)$, let us take $c_z \in M - Cl_{\sigma\omega}(V)$. As $c_z \tilde{\in} 1_U - Cl_{\sigma\omega}(M)$, we can find $W \in \sigma_\omega$ with $c_z \tilde{\in} W$ and $W \tilde{\bigcap} V = 0_U$. As $c_z \tilde{\in} W \tilde{\bigcap} M \in \sigma_\omega$ and H is ω-dense soft set, then $W \tilde{\bigcap} M \tilde{\bigcap} N = W \tilde{\bigcap} V$ $\neq 0_U$, which is a contradiction. Therefore $V \in SP(X, \sigma, U)$.

Proposition 2.11. *Let* (X, σ, U) *be a soft topological space and N belongs to the soft space* (X, U). *The result* $Cl_\sigma(M \tilde{\bigcap} N) = Cl_\sigma(M \tilde{\bigcap} Cl_\sigma(N))$ *holds for each* $M \in \sigma$.

Proof. Take N belong to the soft space (X, A) and $M \in \sigma$. As $N \tilde{\subseteq} Cl_\sigma(N)$, we have $M \tilde{\bigcap} N \tilde{\subseteq} M \tilde{\bigcap} Cl_\sigma(N)$. Hence $Cl_\sigma(M \tilde{\bigcap} N) \tilde{\subseteq} Cl_\sigma(M \tilde{\bigcap} Cl_\sigma(N))$. Then to prove that $Cl_\sigma(M \tilde{\bigcap} Cl_\sigma(N))$ $\tilde{\subseteq} Cl_\sigma(M \tilde{\bigcap} N)$, take $b_y \tilde{\in} Cl_\sigma(M \tilde{\bigcap} Cl_\sigma(N))$ and take $V \in \tau$ such that

$b_y \tilde{\in} V$, then $(N \tilde{\cap} Cl_\sigma(M)) \tilde{\cap} V \neq 0_V$. Take $a_x \tilde{\in} (M \tilde{\cap} \widetilde{Cl_\sigma}(N)) \tilde{\cap} V$, we have $a_x \tilde{\in} Cl_\sigma(N)$ and $a_x \tilde{\in} M \tilde{\cap} V \in \sigma$ and so $(M \tilde{\cap} V) \tilde{\cap} N = (M \tilde{\cap} N) \tilde{\cap} M \neq 0_U$, which implies that $b_y \tilde{\in} Cl_\sigma(M \tilde{\cap} N)$.

Theorem 2.12. *Let* (X, σ, U) *be a Soft Topological Space and let* M *belong to the soft space* (X, U). *Then* $M \in SP(X, \sigma, U)$ *ifff or each* $N \in \sigma$, $M \tilde{\cap} N \in SP(X, \sigma, U)$.

Proof. Assume that $M \in SP(X, \sigma, U)$ and take $N \in \sigma$. As $M \in SP(X, \sigma, U)$, we can find such that $M \tilde{\subseteq} U \tilde{\subseteq} Cl_{\sigma\omega}(M)$. Hence $M \tilde{\cap} N \tilde{\subseteq} U \tilde{\cap} N \tilde{\subseteq} N \tilde{\cap} Cl_{\sigma\omega}(M)$. As $M \in \sigma \subseteq \sigma_\omega$, we have by 2.11, $Cl_{\sigma\omega}(N \tilde{\cap} Cl_{\sigma\omega}(M)) = Cl_{\sigma\omega}(N \tilde{\cap} M)$. Hence, $(N \tilde{\cap} Cl_{\sigma\omega}(M)) \tilde{\subseteq} Cl_{\sigma\omega}(N \tilde{\cap} M)$. So $U \tilde{\cap} M \in \tau$ with $M \tilde{\cap} N \tilde{\subseteq} U \tilde{\cap} N \tilde{\subseteq} N \tilde{\cap} Cl_{\sigma\omega}(M) \tilde{\subseteq} Cl_{\sigma\omega}(N \tilde{\cap} M)$. So $M \tilde{\cap} N \in SP(X, \sigma, U)$. Conversely, for every $N \in \sigma$, $M \tilde{\cap} N \in SP(X, \sigma, U)$. As $1_U \in \sigma$, $M \tilde{\cap} 1_U = N \in SP(X, \sigma, U)$.

Theorem 2.13. *Let* $\{(X, \mathcal{S}_u) : u \in U\}$ *be a family of topological spaces with* $\sigma = \oplus \mathcal{S}_u$. *And let* G *belongs to soft space* (X, U). *Then* $M \in SP(X, \sigma, U)$ *iff* $M(a) \in SP(X, \sigma_u)$ *for every* $u \in U$.

Proof. Assume that $M \in SP(X, \sigma, U)$ and let $u \in U$. Then we have $N \in \sigma$ such that $M \tilde{\subseteq} N \tilde{\subseteq} Cl_{\sigma\omega}(N)$. Hence $M(u) \subseteq N(u) \subseteq (Cl_{\sigma\omega}(M)(u))$. As $(a) \in \sigma_u$, $(Cl_{\sigma\omega}(M)(u) = cl_{(\sigma u)\omega}(M)(u) = cl_{(\sigma u)\omega}(N(a))$, we have $N(u) \in SP(X, \sigma_u,)$ Conversely, if $N(u) \in SP(X, \sigma_u) \forall u \in U$. For each $u \in U$, there exists $W_u \in \sigma_u = \mathcal{S}_a$ such that $N(u) \subseteq W_u \subseteq cl_{(\sigma a)\omega}(M(u))$. Let F belongs to the soft space (X, U), $F_u = W_u \in \mathcal{S}_u \forall u \in U$. Then $N \in \oplus \mathcal{S}_u = \sigma$. Also $(Cl_{\sigma\omega}(M)(u) = cl_{(\sigma u)\omega}(N(a)) \forall u \in U$. Hence $M \tilde{\subseteq} N \tilde{\subseteq} Cl_{\sigma\omega}(M)$. So $M \in SP(X, \sigma, U)$.

3. Continuity on some Stronger form of Soft Pre-open Sets (Sδ - Open Sets)

Definition 3.1. A function $g_{up} : (X, \sigma, U) \to (Y, \tau, V)$ is said to be soft pre-continuous if $g_{up}^{-1}(F) \in PO(X, \sigma, U)$ for every $F \in \tau$.

Theorem 3.2. *Let* (X, σ, U) *and* (Y, τ, V) *be two soft topological spaces and the soft function is defined as* $g_{up} : (X, \sigma, U) \to (Y, \tau, V)$.

(1) g_{up} *is soft pre-continuous*

(2) $g_{up}(cl_{\sigma}(\operatorname{int}_{\sigma}(N))) \widetilde{\subseteq} cl_{\tau}(g_{up}(N))$ *for every N belongs to the soft space* (X, U).

(3) $g_{up}(cl_{\sigma}(\operatorname{int}_{\sigma}(M))) \widetilde{\subseteq} cl_{\tau}(g_{up}(M))$ *for every belongs to the soft topological space* (X, σ, U).

(4) $g_{up}(cl_{\sigma}(F)) \widetilde{\subseteq} cl_{\tau}(g_{up}(F))$ *for each* $F \in \tau$.

Proof. (a) (1) implies (2) Let us assume that g_{up} is soft pre-continuous and let N belong to the soft space (X, U). Let $a_x \widetilde{\in} g_{up}(cl_{\sigma}(\operatorname{int}_{\sigma}(N)))$ and let $M \in \tau$ such that $a_x \widetilde{\in} M$. Now let us prove that $g_{up}(N) \widetilde{\cap} M \neq 0_V$. Choose $b_y \widetilde{\in} cl_{\sigma}(\operatorname{int}_{\sigma}(N))$ such that $a_x = g_{up}(b_y)$. As the function g_{up} is soft pre-continuous, then $g_{up}^{-1}(M) \in PO(X, \sigma, U)$ and hence $g_{up}^{-1}(M) \widetilde{\subseteq} \operatorname{int}_{\sigma}(Cl_{\sigma}(Cl_{\sigma}(g_{up}^{-1}(M))))$. As $b_y \widetilde{\in} g_{up}^{-1}(M)$ then $b_y \widetilde{\in} \operatorname{int}_{\sigma}(Cl_{\sigma}(g_{up}^{-1}(M))) \in \tau$. As $b_y \widetilde{\in} Cl_{\sigma}(\operatorname{int}_{\sigma} N))$, then $\operatorname{int}_{\sigma}(N) \widetilde{\cap} \operatorname{int}_{\sigma}(Cl_{\sigma}(g_{up}^{-1}(M))) \neq 0_U$. It follows that $\operatorname{int}_{\sigma}(N) \widetilde{\cap} Cl_{\sigma}(g_{up}^{-1}(M)) \neq 0_U$ and $N \widetilde{\cap} g_{up}^{-1}(M) \neq 0_U$. Choose $c_z \widetilde{\in} N$ such that $g_{up}(c_z) \widetilde{\in} M$. Hence, $g_{up}(c) \widetilde{\in} g_{up}(N) \widetilde{\cap} M$ and therefore we have $g_{up}(N) \widetilde{\cap} M \neq 0_V$.

(b) (2) implies (3) Assume that $g_{up}(cl_{\sigma}(\operatorname{int}_{\sigma}(N))) \widetilde{\subseteq} cl_{\tau}(g_{up}(N))$ for M belongs to the soft space (X, U) and let $M \in SO(X, \sigma, U)$. Then $N \widetilde{\subseteq} cl_{\sigma}(\operatorname{int}_{\sigma}(M))$ and hence $cl_{\sigma}(M) \widetilde{\subseteq} cl_{\sigma}(\operatorname{int}_{\tau}(M))$. Thus by assumption,

$g_{up}(cl_\sigma(\text{int}_\sigma(N)) \,\tilde{\subseteq}\, g_{up}((cl_\sigma(\text{int}_\sigma(M)))) \,\tilde{\subseteq}\, cl_\tau(g_{up}(M))$.

(c) (3) implies (4) Straightforward.

(d) (4) implies (1) Assume that $g_{up}(cl_\sigma(H)) \,\tilde{\subseteq}\, cl_\tau(g_{up}(H))$ for each $H \in \sigma$ and let $M \in \tau$. To prove that $g_{up}^{-1}(M) \,\tilde{\subseteq}\, \text{int}_\sigma((cl_\sigma(g_{up}^{-1}(M))))$, let $p_x \,\tilde{\in}\, g_{up}^{-1}(M)$. As $1_U - cl_\sigma(g_{up}^{-1}(M)) \in \sigma$, then by assumption, $g_{up}(cl_\sigma(1_U - cl_\sigma(g_{up}^{-1}(M))) \,\tilde{\subseteq}\, cl_\tau(g_{up}(1_U - cl_\sigma(g_{up}^{-1}(M)))$ and hence $cl_\sigma(1_U - cl_\sigma(g_{up}^{-1}(M))) \,\tilde{\subseteq}\, g_{up}^{-1}(cl_\tau(g_{up}(1_U - cl_\sigma(g_{up}^{-1}(M)))))$ So we have, $1_U - g_{up}^{-1}(Cl_\tau(g_{up}(1_U - cl_\sigma(g_{up}^{-1}(M))))) = \text{int}_\sigma(cl_\sigma(g_{up}^{-1}(N)))$. We will prove that $p_x \,\tilde{\in}\, 1_U - g_{up}^{-1}(Cl_\tau(g_{up}(1_U - cl_\sigma(g_{up}^{-1}(M)))))$. For this, let us assume the contrary, that $p \,\tilde{\in}\, g_{up}^{-1}(Cl_\tau(g_{up}(1_U - cl_\sigma(g_{up}^{-1}(M)))))$. Then $g_{up}(p_x) \,\tilde{\in}\, (Cl_\sigma(g_{up}(1_U - cl_\sigma(g_{up}^{-1}(M)))))$. Since $g_{up}(p_x) \,\tilde{\in}\, M \in \tau$, we have $g_{up}(1_U - Cl_\sigma(g_{up}^{-1}(M)) \,\tilde{\cap}\, M \neq 0_V$. Take $q_y \,\tilde{\in}\, 1_U - Cl_\sigma(g_{up}^{-1}(M))$ such that $g_{up}(q_y) \,\tilde{\in}\, M \in \tau$. Since $q_y \,\tilde{\in}\, 1_U - Cl_\sigma(g_{up}^{-1}(M))$, then there exists such that and $g_{up}^{-1}(M) \,\tilde{\cap}\, H = 0_U$. But $q_y \,\tilde{\in}\, g_{up}^{-1}(M) \,\tilde{\cap}\, H$, which is a contradiction to the assumption. Hence we have the result.

Definition 3.3. A $g_{up} : (X, \sigma, U) \to (Y, \tau, V)$. Soft function is called as ω_p soft continuous if for each $g_{up}^{-1}(F) \in SP(X, \sigma, U)$.

Theorem 3.4. *Let* (X, σ, U) *and* (Y, τ, V) *be two soft topological spaces and the soft function is defined as* $g_{up} : (X, \sigma, U) \to (Y, \tau, V)$.

(1) g_{up} *is* ω_p *soft continuous*

(2) $g_{up}(cl_\sigma(F)) \,\tilde{\subseteq}\, cl_\tau(g_{up}(F))$ *for each* $F \in \sigma_\omega$.

Proof. (a). (1) implies (2): Assume that g_{up} is ω_p soft continuous and let $F \in \sigma_\omega$. To prove that $g_{up}(cl_\sigma(F)) \,\tilde{\subseteq}\, cl_\tau(g_{up}(F))$, take $a_x \,\tilde{\in}\, g_{up}(cl_\sigma(F))$ and $M \in \tau$ let such that $a_x \,\tilde{\in}\, M$. Now let us prove that $g_{up}(F) \,\tilde{\cap}\, M = 0_y$.

Choose $b_y \,\tilde{\in}\, cl_\sigma(F)$ such that $a_x = g_{up}(b_y)$. As g_{up} is ω_p soft continuous, we have $g_{up}^{-1}(M) \in SP(X, \sigma, U)$ and so $g_{up}^{-1}(M) \,\tilde{\subseteq}\, \text{int}_\sigma(Cl_{\sigma\omega}(g_{up}^{-1}(M)))$. As $b_y \,\tilde{\in}\, g_{up}^{-1}(M)$, then $b_y \,\tilde{\in}\, \text{int}_\sigma(Cl_{\sigma\omega}(g_{up}^{-1}(M))) \in \sigma$. As $b_y \,\tilde{\in}\, Cl_\sigma(F)$, we have $F \,\tilde{\bigcap}\, \text{int}_\sigma(Cl_{\sigma\omega}(g_{up}^{-1}(M))) \neq 0_U$ and hence $F \,\tilde{\bigcap}\, Cl_{\sigma\omega}(g_{up}^{-1}(M)) \neq 0_U$. Take $c_z \,\tilde{\in}\, F \,\tilde{\bigcap}\, Cl_{\sigma\omega}(g_{up}^{-1}(M))$. As we have $c_z \,\tilde{\in}\, F \in \tau_\sigma$ and $c_z \,\tilde{\in}\, Cl_{\sigma\omega}(g_{up}^{-1}(M))$ then $F \,\tilde{\bigcap}\, g_{up}^{-1}(M) \neq 0_U$. Take $d_w \,\tilde{\in}\, F$ such that $g_{up}(d_w) \,\tilde{\in}\, M$. Then $g_{up}(d_w) \,\tilde{\in}\, g_{up}(F) \,\tilde{\bigcap}\, M$ and so $g_{up}(F) \,\tilde{\bigcap}\, M \neq 0_V$.

(b). (2) implies (1): Assume that $g_{up}(cl_\sigma(F)) \,\tilde{\subseteq}\, cl_\tau(g_{up}(F))$ for each $F \in \sigma_\omega$ and let $M \in \tau$. To Prove that $g_{up}^{-1}(M) \,\tilde{\subseteq}\, \text{int}_\sigma((Cl_{\sigma\omega}(g_{up}^{-1}(M))))$, take $b_y \,\tilde{\in}\, g_{up}^{-1}(M)$. As $1_U - (Cl_{\sigma\omega}(g_{up}^{-1}(M)) \in \sigma$, then by assumption, $g_{up}(Cl_\sigma(1_U - (Cl_{\sigma\omega}(g_{up}^{-1}(M))))) \,\tilde{\subseteq}\, Cl_\tau(g_{up}(1_U - Cl_{\sigma\omega}(g_{up}^{-1}(M)))$ and hence $Cl_\sigma(1_U - Cl_{\sigma\omega}(g_{up}^{-1}(M)))$. Hence $1_U - g_{up}^{-1}(Cl_\tau(g_{up}(1_U - Cl_{\sigma\omega}(g_{up}^{-1}(M))))) \,\tilde{\subseteq}\, 1_U - Cl_\sigma(Cl_\sigma(1_U - Cl_{\sigma\omega}(g_{up}^{-1}(M))) = \text{int}_\sigma(Cl_{\sigma\omega}(g_{up}^{-1}(M)))$. Now we can prove that $b_y \,\tilde{\in}\, 1_U - g_{up}^{-1}(Cl_\tau(g_{up}(1_U - Cl_{\sigma\omega}(g_{up}^{-1}(M)))))$. For this let us assume on the contrary that $b_y \,\tilde{\in}\, g_{up}^{-1}(Cl_\tau(g_{up}(1_U - Cl_{\sigma\omega}(g_{up}^{-1}(M)))))$. Then $g_{up}(b_y) \,\tilde{\in}\, Cl_\tau(g_{up}(1_U - Cl_{\sigma\omega}(g_{up}^{-1}(M)))))$. As $g_{up}(b_y) \,\tilde{\in}\, M \in \tau$, we have $g_{up}(1_U - Cl_{\sigma\omega}(g_{up}^{-1}(M))) \,\tilde{\bigcap}\, M \neq 0_V$. Choose $a_x \,\tilde{\in}\, (1_U - Cl_{\sigma\omega}(g_{up}^{-1}(M))$. such that $g_{up}(a_x) \in M$. As $a_x \,\tilde{\in}\, 1_U - Cl_{\sigma\omega}(g_{up}^{-1}(M))$, then we have $H \in \sigma_\omega$ such that $a_x \,\tilde{\in}\, H$ and $g_{up}^{-1}(M) \,\tilde{\bigcap}\, H = 0_U$, which is a contradiction for $a_x \,\tilde{\in}\, g_{up}^{-1}(M) \,\tilde{\bigcap}\, H$. Hence we have the result.

Theorem 15. *Let $f : (A, M) \to (B, N)$ be a function defined from the topological space A to B and let $h : P \to Q$ be defined from one set of parameters to another set. Then $g_{up} : (A, \sigma(M), P) \to (B, \tau(N), Q)$ is ω_p soft continuous iff f is ω_p soft continuous.*

Proof. Assume $g_{up} : (A, \sigma(M), P) \to (B, \tau(N), Q)$ is ω_p soft continuous.

Take $H \in N$ and $x \in P$, then $(h(x)_H) \in \sigma(M)$. As g_{up} is ω_p soft continuous, $g_{up}^{-1}(h(x)_H) \in SP(A, \sigma(M), P)$. So, we have $g_{up}^{-1}(h(x)_H)(x) \in \omega_p(A, M)$. But $g_{up}^{-1}(h(x)_H)(x) = f^{-1}((h(x)_H)(h(x)) = f^{-1}(H)$. Hence f is ω_p continuous.

Conversely, assume that f is ω_p continuous. Take $H \in \sigma(N)$. Then it is enough to prove that $g_{up}^{-1}(H)(x) \in \omega_p(A, M)\ \forall\ y \in P$. Take $y \in P$, $H(h(y)) \in N$. As f is ω_p continuous, we have $f^{-1}(H(h(y))) \in \omega_p(A, M)$. As $f^{-1}(H(h(y))) = (g_{pu}^{-1}(H))(y)$, we have the result.

4. Conclusions

In continuation of the above work, concepts ω_p soft open functions, separation axioms through ω_p soft sets and compactness and may be developed and the relationships between them may be studied.

References

[1] Al Ghour and A. Bin-Saadon, On some generated soft topological spaces and soft homogeneity, Heliyon 5(7) (2019), e02061 [Cross Ref.].

[2] S. Al Ghour and W. Hamed, On two classes of soft topological spaces, Symmetry 12(2) (2020), 265 [Cross Ref.].

[3] D. Molodtsov, Soft set theory-First results, Comput. Math. Appl. 37(4-5) (1999), 19-31 [Cross Ref.].

[4] M. Shabir and M. Naz, On sosft topological spaces, Comput. Math. Appl. 61 (2011), 1786-1799 [Cross Ref.].

[5] S. Al Ghour, Weaker forms of soft regular and T_2-soft topological spaces, Mathematics 9(7) (2021), 2153 [Corss Ref.]

[6] H. M. Darwesh, Between preopen and open sets in topological spaces, Thai J. Math. 11(1) (2013), 143-155.

[7] Z. Xiao, L. Chen, B. Zhong and S. Ye, Recognition for soft information based on the theory of soft sets, In Proceedings of the International Conference on Services Systems and Services Management, Chongquing, China 13-15 June (2005), 1104-1106.

[8] D. Pei and D. Miao, From soft sets to information systems, IEEE Int. Granul. Comput. 2 (2005), 617-621.

[9] P. K. Maji, A. R. Ray and R. Biswas, An application of soft sets in a decision making problem, Comput. Math. Appl. 44 (2002), 1077-1083 [Cross Ref.].

[10] I. Arockiarani and A. Arokia lancy, Generalized soft $S\beta$-closed sets in soft topological spaces, Int. J. Math. Arch. 4 (2013), 1-7.

[11] S. Das and S. K. Samanta, Soft metric, Ann. Fuzzy Math. Inform. 6 (2013), 77-94.

[12] S. Nazmul and S. K. Samanta, Neighbourhood properties of soft topological spaces, Ann. Fuzzy Math. Inform. 6 (2013), 1-15.

[13] A. Mashhour, M. Abd El-Monsef and S. El-Deep, On precontinuous and weak precontinuous mappings, Proc. Math. Phys. Soc. Egypt 53 (1982), 47-53.