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PROPERTIES OF OPERATION ON αgs-OPEN SETS

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Abstract

The aim of this paper is to introduce some new kinds of operators using γ operation on αgs-open sets and investigate their properties. Further we define $\alpha gs\gamma g$-closed sets using $\alpha gs\gamma$-open set and study some of its characterizations.

1. Introduction

Rajamani and Viswanathan [9] introduced αgs-closed sets in topological spaces. Kasahara [5] initiated the study of operation approach on topological space and also he introduced the concept of α-closed graphs of functions in topological spaces. Jankovic [4] analysed the functions with α-closed graphs. Ogata [8] renamed the operation α as γ operation and introduced γ-open sets by defining the γ operation on open sets in topological spaces. Sanjay Tahiliani [10] introduced β-γ-open sets using the γ operation on β-open sets. Carpintero et al. [2] studied b-γ-open sets by considering the γ operation on b-open sets. Following this, Ibrahim [3] studied α_γ-open sets by defining γ operation on α-open sets. Asaad [1] defined the operation α on P_S-open sets

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in topological spaces. Mershia Rabuni and Balamani [6] defined the operation γ on $\tau_{\alpha g}$ and introduced αgs-open sets in topological spaces. Narmadha and Balamani [7] introduced the operation γ on αgs-open sets and defined $\alpha gs\gamma$-open sets in topological spaces. In this paper, we introduce new operators namely $\alpha gsInt_{\gamma}(A)$, $\gamma gs_{\gamma}Int(A)$, $\alpha gs_{\gamma}\ker(A)$ using αgs-open sets and discuss their properties. Also, we introduce and study the properties of a new type of generalized closed set called $\alpha gs\gamma$-generalized closed set.

2. Preliminaries

Definition 2.1 [7]**.** Let (X, τ) be a topological space. An operation $\gamma : \tau_{\alpha gs} \to P(X)$ is a mapping from $\tau_{\alpha gs}$ to $P(X) \ni V \subseteq \gamma(V) \forall V \in \tau_{\alpha gs}$, the value of V under the operation γ is denoted by $\gamma(V)$.

Definition 2.2 [7]**.** A non-empty subset A of a space (X, τ) with an operation γ on $\tau_{\alpha gs}$ is called an $\alpha gs\gamma$-open set of (X, τ) if $\forall x \in A, \exists$ an αgs-open set U containing $x \ni \gamma(U) \subseteq A.$ The set of all $\alpha gs\gamma$-open sets is denoted by $\tau_{\alpha gs\gamma}$. The complement of an $\alpha gs\gamma$-open set is called $\alpha gs\gamma$-closed.

Definition 2.3 [7]**.** An operation $\gamma : \tau_{\alpha gs} \to P(X)$ is called αgs-regular if $\forall x \in X$ and $\forall$ pair of αgs-open sets A and B containing $x, \exists$ an αgs-open set C containing $x \ni \gamma(A) \cap \gamma(B) \supseteq \gamma(C)$.

Definition 2.4 [7]**.** An operation γ on $\tau_{\alpha gs}$ is said to be αgs-open if $\forall x \in X$ and $\forall \alpha gs$-open set U containing $x, \exists$ an $\alpha gs\gamma$-open set $V \ni x \in V$ and $V \subseteq \gamma(U)$.

Definition 2.5 [7]**.** Let γ be an operation on $\tau_{\alpha gs\gamma}$. A point $x \in X$ is called an $\alpha gs\gamma$-closure point of a set A if $\gamma(U) \cap A \neq \varphi \forall \alpha gs$-open set U containing x. $\alpha gsCl_{\gamma}(A) = \{x \in X / \gamma(U) \cap A \neq \varphi, \forall U, \alpha gs$-open set containing $x\}$.

Definition 2.6 [7]**.** Let γ be an operation on $\tau_{\alpha gs\gamma}$. Then $\alpha gsCl_{\gamma}(A)$ is defined as the intersection of all $\alpha gs\gamma$-closed sets containing A. $\alpha gsCl_{\gamma}(A) = \bigcap \{F \subseteq X \mid A \subseteq F \text{ and } X \setminus F \in \tau_{\alpha gs\gamma}\}$.

Results 2.7 [7]. (i) Let (Z, τ) be a topological space and $A \subseteq Z$ and γ be an operation on $\tau_{\alpha gs\gamma}$. Then for a given $z \in Z$, $z \in \alpha gs_{\gamma}Cl(A)$ *iff* $M \cap A \neq \varphi \forall M \in \tau_{\alpha gs\gamma}$ containing z.

(ii) Arbitrary union of $\alpha gs\gamma$-open sets is $\alpha gs\gamma$-open, where γ is an operation on $\tau_{\alpha gs}$.

(iii) $\alpha gs_{\gamma}Cl(A)$ is $\alpha gs\gamma$-closed, where A is the subset of Z and γ is an operation on $\tau_{\alpha gs}$.

Theorem 2.8 [7]. *Let* $\gamma : \tau_{\alpha gs} \to P(Z)$ *be an operation on* $\tau_{\alpha gs}$ *and D and B are subsets of Z. Then the results below are true.*

(i) $D \subseteq \alpha gscl_{\gamma}(D)$.

(ii) *D is* $\alpha gs\gamma$*-closed iff* $D = \alpha gscl_{\gamma}(D)$.

(iii) *If* $D \subseteq B$ *then* $\alpha gscl_{\gamma}(D) \subseteq \alpha gscl_{\gamma}(B)$.

3. Some Properties of Operation on αgs-open sets

Definition 3.1. Let (X, τ) be a topological space and γ an operation on $\tau_{\alpha gs}$. A point $a \in A \subseteq X$ is said to be $\alpha gs\gamma$-interior point of A if there exists an αgs-open set N of X containing a such that $\gamma(N) \subseteq A$. We denote the set of all such points by $\alpha gsInt_{\gamma}(A)$.

Thus $\alpha gsInt_{\gamma}(A) = \{x \in A : x \in N \in \tau_{\alpha gs} \text{ and } \gamma(N) \subseteq A\}$.

Theorem 3.2. *Let* (X, τ) *be a topological space and* γ *an operation on* $\tau_{\alpha gs}$. *If A and B are two subsets of X, then the following statements are true*

(i) $\alpha gsInt_{\gamma}(A) \subseteq A$

(ii) *A is* $\alpha gs\gamma$*-open iff* $A = \alpha gsInt_{\gamma}(A)$

(iii) *If* $A \subseteq B$, *then* $\alpha gsInt_{\gamma}(A) \subseteq \alpha gsInt_{\gamma}(B)$

(iv) $\alpha gsInt_{\gamma}(\alpha gsInt_{\gamma}(A)) \subseteq \alpha gsInt_{\gamma}(A)$

(v) $\alpha gsInt_{\gamma}(A) \cup \alpha gsInt_{\gamma}(B) \subseteq \alpha gsInt_{\gamma}(A \cup B)$

(vi) $\alpha gsInt_{\gamma}(A \cap B) \subseteq \alpha gsInt_{\gamma}(A) \cap \alpha gsInt_{\gamma}(B)$

(vii) *If* γ *is* αgs*-regular*, *then* $\alpha gsInt_{\gamma}(A) \cap \alpha gsInt_{\gamma}(B) = \alpha gsInt_{\gamma}(A \cap B)$

Proof.

(i) From Definition 3.1, $\alpha gsInt_{\gamma}(A) \subseteq A$.

(ii) If $A = \alpha gsInt_{\gamma}(A)$, then by Definition 3.1, for every $x \in \alpha gsInt_{\gamma}(A)$, there exists an αgs-open set N of X containing x such that $\gamma(N) \subseteq A$. Hence, A is $\alpha gs\gamma$-open. Conversely, let A be an $\alpha gs\gamma$-open. Then to prove $A = \alpha gsInt_{\gamma}(A)$. By (i), $\alpha gsInt_{\gamma}(A) \subseteq A$, so it is enough to prove that $A \subseteq \alpha gsInt_{\gamma}(A)$. Let $x \in A$. Since A is $\alpha gs\gamma$-open, $\forall x \in A$, $\exists$ an αgs-open set U containing $x \ni \gamma(U) \subseteq A$ which implies that x is an $\alpha gs\gamma$-interior point of A. i.e., $x \in \alpha gsInt_{\gamma}(A)$. Therefore, $A \subseteq \alpha gsInt_{\gamma}(A)$. Hence, $A = \alpha gsInt_{\gamma}(A)$.

(iii) Let $A \subseteq B \subseteq X$. Let $x \in \alpha gsInt_{\gamma}(A)$, then there exists an αgs-open set U of X containing x such that $\gamma(U) \subseteq A$. Since $A \subseteq B$, the same αgs-open set U of X containing x such that $\gamma(U) \subseteq B$. This implies $x \in \alpha gsInt_{\gamma}(B)$. Hence, $\alpha gsInt_{\gamma}(A) \subseteq \alpha gsInt_{\gamma}(B)$.

(iv) By Definition 3.1, $x \in N \subseteq \gamma(N) \subseteq A$ if x is an $\alpha gs\gamma$-interior point of A. Hence, the collection implies that $\alpha gsInt_{\gamma}(A) \subseteq A$. Hence, $\alpha gsInt_{\gamma}$ $(\alpha gsInt_{\gamma}(A)) \subseteq \alpha gsInt_{\gamma}(A)$ by (iii).

(v) Since $A \subseteq A \cup B, B \subseteq A \cup B$ and by (iii), $\alpha gsInt_{\gamma}(A) \subseteq \alpha gsInt_{\gamma}$ $(A \cup B)$ and $\alpha gsInt_{\gamma}(B) \subseteq \alpha gsInt(A \cup B)$. Therefore, $\alpha gsInt_{\gamma}(A) \cup \alpha gsInt_{\gamma}(B)$

$\subseteq \alpha gsInt_{\gamma}(A \cup B)$.

(vi) Since $\cap B \subseteq A, A \cap B \subseteq B$ and by (iii), $\alpha gsInt_{\gamma}(A \cap B) \subseteq \alpha gsInt_{\gamma}(A)$ and $\alpha gsInt_{\gamma}(A \cap B) \subseteq \alpha gsInt_{\gamma}(B)$. Therefore, $\alpha gsInt_{\gamma}(A \cap B) \subseteq \alpha gsInt_{\gamma}(A) \cap \alpha gsInt_{\gamma}(B)$.

(vii) By (vi), $\alpha gsInt_{\gamma}(A \cap B) \subseteq \alpha gsInt_{\gamma}(A) \cap \alpha gsInt_{\gamma}(B)$. Let $x \in \alpha gsInt_{\gamma}(A) \cap \alpha gsInt_{\gamma}(B)$. This implies $x \in \alpha gsInt_{\gamma}(A)$ and $x \in \alpha gsInt_{\gamma}(B)$. Therefore, there exists an αgs -open sets U, V containing x such that $\gamma(U) \subseteq A$ and $\gamma(V) \subseteq B$. Implies that $\gamma(U) \cap \gamma(V) \subseteq A \cap B$. Since γ is αgs -regular, there exists an αgs -open set W containing x such that $\gamma(U) \cap \gamma(V) \supseteq \gamma(W)$. Implies that $\gamma(W) \subseteq A \cap B$. Therefore, $x \in \alpha gsInt_{\gamma}(A \cap B)$. Hence, $\alpha gsInt_{\gamma}(A) \cap \alpha gsInt_{\gamma}(B) = \alpha gsInt_{\gamma}(A \cap B)$.

Remark 3.3. The reverse inclusion of (iv) in Theorem 3.2 need not be true as observed from the following example.

Example 3.4. Let $X = \{a, b, c\}$ and $\tau = \{\varphi, \{a, b\}, X\}$. Then $\tau_{\alpha gs} = \{\varphi, \{a\}, \{b\}, \{a, b\}, X\}$. Let $\gamma : \tau_{\alpha gs} \to P(X)$ be an operation on $\tau_{\alpha gs}$ defined by

$$\gamma(A) = \begin{cases} A & \text{if } A = \{a\} \text{ or } \{a, b\} \\ \{b, c\} & \text{if } A = \{b\} \end{cases} \forall A \in \tau_{\alpha gs}$$

Here for $A = \{b, c\}, \alpha gsInt_{\gamma}(A) = \{b\}$ and $\alpha gsInt_{\gamma}(\alpha gsInt_{\gamma}(A)) = \varphi$. Therefore $\alpha gsInt_{\gamma}(A) \nsubseteq \alpha gsInt_{\gamma}(\alpha gsInt_{\gamma}(A))$.

Remark 3.5. $\alpha gsInt_{\gamma}(A)$ need not be $\alpha gs\gamma$ -open as observed from the following example.

Example 3.6. Let $X = \{a, b, c\}$ and $\tau = \{\varphi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, X\}$. Then $\tau_{\alpha gs} = \{\varphi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, X\}$. Let $\gamma : \tau_{\alpha gs} \to P(X)$ be an operation on $\tau_{\alpha gs}$ defined by

$$\gamma(A) = \begin{cases} A \cup \{c\} & \text{if } A = \{a\} \text{ or } \{b\} \\ A & \text{if } A \neq \{a\}, \{b\}, \{c\} \end{cases} \forall A \in \tau_{\alpha gs}$$

Then $\tau_{\alpha gs\gamma} = \{\varphi, \{a, b\}, \{a, c\}, X\}$. Here for $A = \{b, c\}, \alpha gsInt_\gamma(A) = \{b\}$ which is not $\alpha gs\gamma$-open in (X, τ).

Theorem 3.7. *Let (X, τ) be a topological space and γ be an operation on $\tau_{\alpha gs}$. If A is a subset of X, then*

(i) $\alpha gsInt_\gamma(X \setminus A) = X \setminus \alpha gsCl_\gamma(A)$.

(ii) $\alpha gsCl_\gamma(X \setminus A) = X \setminus \alpha gsInt_\gamma(A)$.

(iii) $\alpha gsInt_\gamma(A) = X \setminus \alpha gsCl_\gamma(X \setminus A)$.

(iv) $\alpha gsCl_\gamma(A) = X \setminus \alpha gsInt_\gamma(X \setminus A)$.

Proof. (i) Let $x \in \alpha gsInt_\gamma(X \setminus A)$, then there exists an αgs-open sets U containing x such that $\gamma(U) \subseteq X \setminus A$. This implies that $\gamma(U) \cap A = \varphi$. This gives that $x \notin \alpha gsCl_\gamma(A)$ and so $x \in X \setminus \alpha gsCl_\gamma(A)$. Hence, $\alpha gsInt_\gamma(X \setminus A) \subseteq X \setminus \alpha gsCl_\gamma(A)$.

Conversely, let $x \in X \setminus \alpha gsCl_\gamma(A)$ implies that $x \notin \alpha gsCl_\gamma(A)$, then there exists an αgs-open sets V containing x such that $\gamma(V) \cap A = \varphi$ implies that $x \in V \subseteq \gamma(V) \subseteq X \setminus A$. It follows that $x \in \alpha gsInt_\gamma(X \setminus A)$. Hence, $X \setminus \alpha gsCl_\gamma(A) \subseteq \alpha gsInt_\gamma(X \setminus A)$. Therefore, $\alpha gsInt_\gamma(X \setminus A) = X \setminus \alpha gsCl_\gamma(A)$.

(ii) Let $x \notin \alpha gsCl_\gamma(X \setminus A)$ implies there exists an αgs-open set U containing x such that $\gamma(U) \cap (X \setminus A) = \varphi$. Implies $\gamma(U) \subseteq A$. Thus, $x \in \alpha gsInt_\gamma(A)$. $x \notin X \setminus \alpha gsInt_\gamma(A)$. Hence $X \setminus \alpha gsInt_\gamma(A) \subseteq \alpha gsCl_\gamma(X \setminus A)$.

Reversing the steps we get $\alpha gsCl_\gamma(X \setminus A) \subseteq X \setminus \alpha gsInt_\gamma(A)$. Therefore, $\alpha gsCl_\gamma(X \setminus A) = X \setminus \alpha gsInt_\gamma(A)$.

The proof of (iii) and (iv) follows from (i) and (ii).

Remark 3.8. Let (X, τ) be a topological space and $x \in X$. If $\{x\} \in \tau_{\alpha gs\gamma}$, then $\gamma(\{x\}) = \{x\}$.

Definition 3.9. Let A be a subset of a topological space (X, τ) and γ be an operation on $\tau_{\alpha gs}$. The union of all $\alpha gs\gamma$-open sets contained in A is called the $\alpha gs\gamma$-interior of A and denoted by $\alpha gs_{\gamma}Int(A)$.

Theorem 3.10. *Let (X, τ) be a topological space and γ be an operation on $\tau_{\alpha gs}$. For any subsets A, B of X we have the following.*

(i) $\alpha gs_{\gamma}Int(A)$ *is an* $\alpha gs\gamma$*-open set in X.*

(ii) $\alpha gs_{\gamma}Int(\varphi) = \varphi$ *and* $\alpha gs_{\gamma}Int(X) = X$.

(iii) *A is* $\alpha gs\gamma$*-open if and only if* $A = \alpha gs_{\gamma}Int(A)$.

(iv) $\alpha gs_{\gamma}Int(A) \subseteq A$.

(v) *If* $A \subseteq B$, *then* $\alpha gs_{\gamma}Int(A) \subseteq \alpha gs_{\gamma}Int(B)$.

(vi) $\alpha gs_{\gamma}Int(\alpha gs_{\gamma}Int(A)) = \alpha gs_{\gamma}Int(A)$.

(vii) $\alpha gs_{\gamma}Int(A \cup B) \supseteq \alpha gs_{\gamma}Int(A) \cup \alpha gs_{\gamma}Int(B)$.

(viii) $\alpha gs_{\gamma}Int(A \cap B) \subseteq \alpha gs_{\gamma}Int(A) \cap \alpha gs_{\gamma}Int(B)$.

Proof.

(i) By Definition 3.9 and by Result 2.7 (ii), we have, $\alpha gs\gamma Int(A)$ is an $\alpha gs\gamma$-open set.

(ii) Obvious from the Definition 3.9.

(iii) Necessity. Since A is $\alpha gs\gamma$-open, $\alpha gs_{\gamma}Int(A) =$ Union of all $\alpha gs\gamma$-open sets contained in $A = A$.

Sufficiency. Since $A = \alpha gs_{\gamma}Int(A)$ and from (i), $\alpha gs_{\gamma}Int(A)$ is $\alpha gs\gamma$-open. We get A is $\alpha gs\gamma$-open.

(iv) It is obvious from the Definition 3.9.

(v) Suppose $A \subseteq B$. Let $x \in \alpha gs_{\gamma}Int(A)$ and F be an $\alpha gs\gamma$-open set contained in A which is contained B. Therefore, $x \in F$. Hence,

$x \in \alpha gs_{\gamma} Int(B)$. Therefore, $x \in \alpha gs_{\gamma} Int(A) \subseteq \alpha gs_{\gamma} Int(B)$.

(vi) From (i) and (iii), we have $\alpha gs_{\gamma} Int(\alpha gs_{\gamma} Int(A)) = \alpha gs_{\gamma} Int(A)$.

(vii) Since $A \subseteq A \cup B, B \subseteq A \cup B$ and by (v) $\alpha gs_{\gamma} Int(A) \subseteq \alpha gs_{\gamma} Int(A \cup B)$ and $\alpha gs_{\gamma} Int(B) \subseteq \alpha gs_{\gamma} Int(A \cup B)$. Hence, $\alpha gs_{\gamma} Int(A) \cup \alpha gs_{\gamma} Int(B) \subseteq \alpha gs_{\gamma} Int(A \cup B)$.

(viii) As $A \cap B \subseteq A$, $A \cap B \subseteq B$ and by (v), $\alpha gs_{\gamma} Int(A \cap B) \subseteq \alpha gs_{\gamma} Int(A)$ and $\alpha gs_{\gamma} Int(A \cap B) \subseteq \alpha gs_{\gamma} Int(B)$.

$\therefore \alpha gs_{\gamma} Int(A \cap B) \subseteq \alpha gs_{\gamma} Int(A) \cap \alpha gs_{\gamma} Int(B)$.

Remark 3.11. The reverse inclusion of (vii) in Theorem 3.10 need not be true as observed from the following example.

Example 3.12. Let $X = \{a, b, c\}$ and $\tau = \{\varphi, \{a\}, X\}$. Then $\tau_{\alpha gs} = \{\varphi, \{a\}, \{a, b\}, \{a, c\}, X\}$. Let $\gamma : \tau_{\alpha gs} \to P(X)$ be an operation on $\tau_{\alpha gs}$ defined by

$$\gamma(A) = \begin{cases} A & \text{if } b \in A \\ cl(A) & \text{if } b \neq A \end{cases} \forall A \in \tau_{\alpha gs}$$

Then $\tau_{\alpha gs} = \{\varphi, \{a, b\}, X\}$. Let $A = \{a\}$ and $B = \{b\}$. Then $A \cup B = \{a, b\}$. Therefore $\alpha gs_{\gamma} Int(A \cup B) = \{a, b\}$ and $\alpha gs_{\gamma} Int(A) \cup \alpha gs_{\gamma} Int(B) = \varphi$. Hence $\alpha gs_{\gamma} Int(A \cup B) \nsubseteq \alpha gs_{\gamma} Int(A) \cup \alpha gs_{\gamma} Int(B)$.

Remark 3.13. The reverse inclusion of (viii) in Theorem 3.10 need not be true as observed from the following example.

Example 3.14. Let $X = \{a, b, c\}$ and $\tau = \{\varphi, \{a\}, \{b, c\}, X\}$. Then $\tau_{\alpha gs} = P(X)$. Let $\gamma : \tau_{\alpha gs} \to P(X)$ be an operation on $\tau_{\alpha gs}$ defined by

$$\gamma(A) = \begin{cases} A & \text{if } a \in A \\ cl(A) & \text{if } a \neq A \end{cases} \forall A \in \tau_{\alpha gs}$$

Then $\tau_{\alpha gs} = \{\varphi, \{a\}, \{a, b\}, \{b, c\}, \{a, c\}, X\}$. Let $A = \{b, c\}$ and $B = \{a, c\}$.

Then $A \cap B = \{c\}$. Therefore $\alpha gs_{\gamma}(A \cap B) = \varphi$ and $\alpha gs_{\gamma} Int(A) \cap \alpha gs_{\gamma} Int(B)$ $= \{c\}$. Hence $\alpha gs Int(A) \cap \alpha gs Int(B) \nsubseteq \alpha gs_{\gamma} Int(A \cap B)$.

Theorem 3.15. *If* γ *is* αgs *-regular then* $\alpha gs_{\gamma} Int(A \cap B) = \alpha gs_{\gamma} Int(A)$ $\cap\ \alpha gs_{\gamma} Int(B)$.

Proof. Let $y \in \alpha gs_{\gamma} Int(A) \cap \alpha gs Int(B)$. Then $y \in \alpha gs_{\gamma} Int(A)$ and $y \in \alpha gs_{\gamma} Int(B)$. Then $y \in \bigcup\{O \subseteq X : O \subseteq A$ where $O \in \tau_{\alpha gs\gamma}\}$. and $y \in \bigcup\{O \subseteq X : O \subseteq B$ where $O \in \tau_{\alpha gs\gamma}\}$. Then γ belongs to at least one $\alpha gs\gamma$ -open set, say U, contained in A and y belongs to at least one $\alpha gs\gamma$ -open set, say V, contained in B. Since γ is αgs -regular, $U \cap V$ is an $\alpha gs\gamma$ -open set. Therefore, $y \in U \cap V \subseteq A \cap B$. Hence, $y \in \alpha gs_{\gamma} Int(A \cap B)$ and $\alpha gs_{\gamma} Int(A)$ $\cap\ \alpha gs_{\gamma} Int(B) \subseteq \alpha gs_{\gamma} Int(A \cap B)$. From Theorem 3.10 (viii), $\alpha gs_{\gamma} Int(A \cap B)$ $\subseteq \alpha gs_{\gamma} Int(A) \cap \alpha gs_{\gamma} Int(B)$. Hence, $\alpha gs_{\gamma} Int(A \cap B) = \alpha gs_{\gamma} Int(A) \cap \alpha gs_{\gamma} Int(B)$.

Proposition 3.16. *Let* (X, τ) *be a topological space and* γ *be an operation on* $\tau_{\alpha gs}$. *Let A be a subset of X. Then* $\alpha gs_{\gamma} Int(A) \subseteq \alpha gs Int_{\gamma}(A)$.

Proof. Let $x \in \alpha gs_{\gamma} Int(A)$. Then $x \in \bigcup\{O \subseteq X : O \subseteq A$ where $O \in \tau_{\alpha gs\gamma}\}$. Then x belongs to at least one $\alpha gs\gamma$ -open set O contained in A. Since $x \in O$, there exists an αgs -open set U of X containing x such that $\gamma(U) \subseteq O \subseteq A$. Therefore, $x \in \alpha gs Int_{\gamma}(A)$. Hence, $\alpha gs_{\gamma} Int(A) \subseteq \alpha gs Int_{\gamma}(A)$.

Remark 3.17. $\alpha gs Int_{\gamma}(A) \nsubseteq \alpha gs_{\gamma} Int(A)$ as observed from the following example.

Example 3.18. From the Example 3.4, $\tau_{\alpha gs\gamma} = \{\varphi, \{a\}, \{a, b\}, X\}$. Then $\alpha gs Int_{\gamma}(A) = \varphi$ and $\alpha gs_{\gamma} Int_{\gamma}(A) = \{b\}$. Hence $\alpha gs Int_{\gamma}(A) \nsubseteq \alpha gs_{\gamma} Int(A)$.

Proposition 3.19. *If* $\gamma : \tau_{\alpha gs} \to P(X)$ *is an* αgs *-open operation on* $\tau_{\alpha gs}$ *and* $A \subseteq X$. *Then*

(a) $\alpha gs Int_{\gamma}(A) = \alpha gs_{\gamma} Int(A)$ *and* $\alpha gs Int_{\gamma}(\alpha gs Int_{\gamma}(A)) = \alpha gs Int_{\gamma}(A)$.

(b) $\alpha gsInt_{\gamma}(A)$ *is* $\alpha gs\gamma$*-open in X.*

Proof. (a) Let γ be an αgs-open operation on $\tau_{\alpha gs}$. We have to prove that $\alpha gsInt_{\gamma}(A) \subseteq \alpha gs_{\gamma}Int(A)$. Let $x \in \alpha gsInt_{\gamma}(A)$. Then there exists an αgs-open set U of X containing x such that $\gamma(U) \subseteq A$. Since γ is an αgs-open operation, for all αgs-open set U containing x, there exists an $\alpha gs\gamma$-open set V containing x such that $V \subseteq \gamma(U)$. Therefore, $x \in V \subseteq \gamma(U) \subseteq A$. Hence, V is an $\alpha gs\gamma$-open set contained in A and $x \in \bigcup \{V \subseteq X : V \subseteq A$ where $V \in \tau_{\alpha gs\gamma}\}$. Therefore, $x \in \alpha gs_{\gamma}Int(A)$. Thus, $\alpha gsInt_{\gamma}(A) \subseteq \alpha gs_{\gamma}Int(A)$. Hence, by Proposition 3.16, $\alpha gsInt_{\gamma}(A) = \alpha gs_{\gamma}Int(A)$.

Now, $\alpha gsInt_{\gamma}(\alpha gsInt_{\gamma}(A)) = \alpha gs_{\gamma}Int(\alpha gs_{\gamma}Int(A)) = \alpha gs_{\gamma}Int(A)$

$$= \alpha gsInt_{\gamma}(A).$$

(b) Follows from part (a) and Theorem 3.10.

Theorem 3.20. *Let* (X, τ) *be a topological space and* γ *be an operation on* $\tau_{\alpha gs}$. *Then for* $A \subseteq X$,

(i) $\alpha gs_{\gamma}Cl(X \setminus A) = X \setminus \alpha gs_{\gamma}Int(A)$.

(ii) $\alpha gs_{\gamma}Int(X \setminus A) = X \setminus \alpha gs_{\gamma}Cl(A)$.

(iii) $\alpha gs_{\gamma}Int(A) = X \setminus \alpha gs_{\gamma}Cl(X \setminus A)$.

(iv) $\alpha gs_{\gamma}Cl(A) = X \setminus \alpha gs_{\gamma}Int(X \setminus A)$.

Proof. Obvious.

Theorem 3.21. *Let* (X, τ) *be a topological space and* γ *be an* αgs*-regular operation on* $\tau_{\alpha gs}$. *Then for every* $A \subseteq X$ *the following holds.*

(a) $\alpha gs_{\gamma}Cl(A) \cap V \subseteq \alpha gs_{\gamma}Cl(A \cap V)$ *for every* $\alpha gs\gamma$*-open set V.*

(b) $\alpha gs_{\gamma}Int(A \cup E) \subseteq \alpha gs_{\gamma}Int(A) \cup E$ *for every* $\alpha gs\gamma$*-closed set E.*

Proof. (a) Let $x \in \alpha gs_{\gamma}Cl(A) \cap V$ for every $\alpha gs\gamma$-open set V. Then

$x \in \alpha gs_{\gamma}Cl(A)$ and $x \in V$. Let U be any $\alpha gs\gamma$-open set of X containing x. Since γ is αgs-regular, $V \cap U$ is $\alpha gs\gamma$-open in X. By Result 2.7 (i), $A \cap (V \cap U) \neq \varphi$. This implies that $(A \cap V) \cap U \neq \varphi$. Again by Result 2.7 (i), $x \in \alpha gs_{\gamma}Cl(A \cap V)$. Hence, $\alpha gs_{\gamma}Cl(A) \cap V \subseteq \alpha gs_{\gamma}Cl(A \cap V)$ for every $\alpha gs\gamma$-open set V.

(b) From (a), $\alpha gs_{\gamma}Cl(A) \cap V \subseteq \alpha gs_{\gamma}Cl(A \cap V)$ for every $\alpha gs\gamma$-open set V. Then $X \setminus \alpha gs_{\gamma}Cl(A \cap V) \subseteq (X \setminus \alpha gs_{\gamma}Cl(A)) \cup (X \setminus V)$. By Theorem 3.20, $\alpha gs_{\gamma}Int(X \setminus (A \cap V)) = \alpha gs_{\gamma}Int((X \setminus A) \cup (X \setminus V)) \subseteq \alpha gs_{\gamma}Int(X \setminus A) \cup (X \setminus V)$. Hence, $\alpha gs_{\gamma}Int((X \setminus A) \cup (X \setminus V)) \subseteq \alpha gs_{\gamma}Int(X \setminus A) \cup (X \setminus V)$ for every $\alpha gs\gamma$-closed set $X \setminus V$.

Remark 3.22. If γ is not an αgs-regular operation on $\tau_{\alpha gs}$, then Theorem 3.21 fails as observed from the following example.

Example 3.23. From the Example 3.6, $\tau^{c}_{\alpha gs} = \{\varphi, \{b\}, \{c\}, X\}$ and γ is not an αgs-regular operation on $\tau_{\alpha gs}$. Here $A = \{b, c\}$ and $V = \{a, b\}$. Then $\alpha gs_{\gamma}Cl(A) \cap V = \{a, b\}$ and $\alpha gs_{\gamma}Cl(A \cap V) = \{b\}$. Hence $\alpha gs_{\gamma}Cl(A) \cap V \nsubseteq \alpha gs_{\gamma}Cl(A \cap V)$.

Definition 3.24. Let A be a subset of a topological space (X, τ) and γ an operation on $\tau_{\alpha gs}$. The $\alpha gs\gamma$-kernel of A, denoted by $\alpha gs_{\gamma}\ker(A)$ is defined to be the set $\alpha gs_{\gamma}\ker(A) = \cap \{U : A \subseteq U, U \in \tau_{\alpha gs\gamma}\}$.

Proposition 3.25 *Let (X, τ) be a topological space with an operation γ on $\tau_{\alpha gs}$ and $x \in X$. Then $y \in \alpha gs_{\gamma}\ker(\{x\})$ if and only if $x \in \alpha gs_{\gamma}\ker(\{y\})$.*

Proof. If $y \in \alpha gs_{\gamma}\ker(\{x\})$, then $y = \cap \{U : \{x\} \subseteq U, U \in \tau_{\alpha gs\gamma}\}$. i.e., y belongs to every $\alpha gs\gamma$-open set containing $\{x\}$. Thus, $U \cap \{y\} \neq \varphi$ for every $\alpha gs\gamma$-open set U containing x. Then by Result 2.7 (i), $x \in \alpha gs_{\gamma}Cl(\{y\})$. Now, let $x \in \alpha gs_{\gamma}Cl(\{y\})$. By Result 2.7 (i), $U \cap \{y\} \neq \varphi$ for every $\alpha gs\gamma$-open set U containing x. From this y belongs to every $\alpha gs\gamma$-open set containing $\{x\}$.

Hence, $y \in \alpha gs_\gamma \ker(\{x\})$.

Proposition 3.26. *Let* (X, τ) *be a topological space and* γ *be an operation on* $\tau_{\alpha gs}$. *Let A and B be subsets of X. Then*

(i) $A \subseteq \alpha gs_\gamma \ker(A)$.

(ii) If $A \subseteq B$ *then* $\alpha gs_\gamma \ker(A) \subseteq \alpha gs_\gamma \ker(B)$.

(iii) $\alpha gs_\gamma \ker(\varphi) = \varphi$ *and* $\alpha gs_\gamma \ker(X) = X$.

Proof. (i) From Definition 3.24, we have $A \subseteq \alpha gs_\gamma \ker(A)$.

(ii) Let $A \subseteq B$ and $x \in \alpha gs_\gamma \ker(A)$. From Definition 3.24, $\alpha gs_\gamma \ker(A)$ is the intersection of all $\alpha gs\gamma$-open sets containing A which is contained in B. Thus, $x \in \alpha gs_\gamma \ker(B)$. Therefore, $\alpha gs_\gamma \ker(A) \subseteq \alpha gs_\gamma \ker(B)$.

(iii) It is Obvious.

Proposition 3.27. *Let* (X, τ) *be a topological space and* γ *be an* αgs-*regular operation on* $\tau_{\alpha gs}$. *Then A is a subset of X and is an* $\alpha gs\gamma$-*open set if and only if* $A = \alpha gs_\gamma \ker(A)$.

Proof. Let A be an $\alpha gs\gamma$-open set and $x \in \alpha gs_\gamma \ker(A)$. Then x belongs to every $\alpha gs\gamma$-open set containing A. Now A is $\alpha gs\gamma$-open, $x \in A$. Hence, $\alpha gs_\gamma \ker(A) \subseteq A$. From Proposition 3.26 (i), we have $A \subseteq \alpha gs_\gamma \ker(A)$. Thus, $A = \alpha gs_\gamma \ker(A)$. Conversely, let $A = \alpha gs_\gamma \ker(A)$. Thus, A is the intersection of all $\alpha gs\gamma$-open sets containing A and here γ be an αgs-regular operation on $\tau_{\alpha gs}$. Therefore, A is an $\alpha gs\gamma$-open set.

Remark 3.28. The following example shows that if $A = \alpha gs_\gamma \ker(A)$. then A is not an $\alpha gs\gamma$-open set when γ is not an αgs-regular operation on $\tau_{\alpha gs}$.

Example 3.29. Let $X = \{a, b, c\}$, $\tau = \{\varphi, \{a\}, \{a, b\}, X\}$ and $\tau_{\alpha gs} = \{\varphi,$

$\{a\}, \{a, b\}, \{a, c\}, X\}$. Let $\gamma : \tau_{\alpha gs} \to P(X)$ be an operation on $\tau_{\alpha gs}$ defined by

$$\gamma(A) = \begin{cases} A & \text{if } b \in A \\ cl(A) & \text{if } b \notin A \end{cases} \forall A \in \tau_{\alpha gs}$$

Then $\tau_{\alpha gs} = \{\varphi, \{a\}, \{a, b\}, \{a, c\}, X\}$. and γ is not an αgs-regular operation on $\tau_{\alpha gs}$. Take $A = \{a\}$. Then $A = \alpha gs_\gamma \ker(A)$ but A is not an $\alpha gs\gamma$-open set.

Proposition 3.30. *Let* (X, τ) *be a topological space and* γ *be an operation on* $\tau_{\alpha gs}$. *Then* $\alpha gs_\gamma \ker(\alpha gs_\gamma \ker(A)) = \alpha gs_\gamma \ker(A)$ *where A is a subset of X.*

Proof. Let $x \notin \alpha gs_\gamma \ker(A)$. Then there exists an $\alpha gs\gamma$-open set V containing A such that $x \notin V$. Now V is an $\alpha gs\gamma$-open set. By Proposition 3.27, we have $V = \alpha gs_\gamma \ker(V)$. Since $A \subseteq V$ and by Proposition 3.26 (ii), $\alpha gs_\gamma \ker(A) \subseteq \alpha gs_\gamma \ker(V) = V$. Again applying Proposition 3.26 (ii), we have $\alpha gs_\gamma \ker(\alpha gs_\gamma \ker(A)) \subseteq V$. Thus, $x \notin \alpha gs_\gamma \ker(\alpha gs_\gamma \ker(A))V$. Hence, $\alpha gs_\gamma \ker(\alpha gs_\gamma \ker(A)) \subseteq \alpha gs_\gamma \ker(A)$. By Proposition 3.26 (i) and (ii), we have $\alpha gs_\gamma \ker(A) \subseteq \alpha gs_\gamma \ker(\alpha gs_\gamma \ker(A))$. Thus, $\alpha gs_\gamma \ker(\alpha gs_\gamma \ker(A)) = \alpha gs_\gamma \ker(A)$.

Theorem 3.31. *Let* (X, τ) *be a topological space and* γ *be an operation on* $\tau_{\alpha gs}$. *Then for any points x and y the following are equivalent*:

(i) $\alpha gs_\gamma \ker(\{x\}) \neq \alpha gs_\gamma \ker(\{y\})$

(ii) $\alpha gs_\gamma Cl(\{x\}) \neq \alpha gs_\gamma Cl(\{y\})$.

Proof. (i) $\Rightarrow$ (ii) Let $\alpha gs_\gamma \ker(\{x\}) \neq \alpha gs_\gamma \ker(\{y\})$. Then there exist a point z in X such that $z \in \alpha gs_\gamma \ker(\{y\})$ and $z \notin \alpha gs_\gamma \ker(\{x\})$. By Proposition 3.25, $y \in \alpha gs_\gamma \ker(\{z\})$ and $x \notin \alpha gs_\gamma \ker(\{z\})$. Since $y \in \alpha gs_\gamma Cl(\{z\}), \alpha gs_\gamma Cl(\{y\}) \subseteq \alpha gs_\gamma Cl(\{z\})$. Thus $\alpha gs_\gamma Cl(\{y\}) \cap \{x\} = \varphi$. Hence, $\alpha gs_\gamma Cl(\{x\}) \neq \alpha gs_\gamma Cl(\{y\})$.

(ii) $\Rightarrow$ (i) Let $\alpha gs_\gamma Cl(\{x\}) \neq \alpha gs_\gamma Cl(\{y\})$. Then there exist a point z in X such that $z \in \alpha gs_\gamma Cl(\{y\})$ and $z \notin \alpha gs_\gamma Cl(\{x\})$. By Result 2.7 (i), for every

$\alpha gs\gamma$-open set V containing z, $V \cap \{y\} \neq \varphi$ and for some $\alpha gs\gamma$-open set V containing z, $V \cap \{x\} \neq \varphi$. Thus, an $\alpha gs\gamma$-open set V containing z contains y but not x. Therefore, $x \notin \alpha gs_{\gamma} \ker(\{y\})$. Hence, $\alpha gs_{\gamma} \ker(\{x\}) \neq \alpha gs_{\gamma} \ker(\{y\})$.

Proposition 3.32. *Let* (X, τ) *be a topological space with an operation* γ *on* $\tau_{\alpha gs}$ *and* A *be a subset of* X. *Then* $\alpha gs_{\gamma} \ker(A) = \{x \in X : \alpha gs_{\gamma} Cl(\{x\}) \cap A \neq \varphi\}$.

Proof. Let $x \in \alpha gs_{\gamma} \ker(A)$ and suppose $\alpha gs_{\gamma} Cl(\{x\}) \cap A = \varphi$. Then $x \notin X \setminus \alpha gs_{\gamma} Cl(\{x\})$. Since $\alpha gs_{\gamma} Cl(\{x\})$ is an $\alpha gs\gamma$-closed set, $X \setminus \alpha gs_{\gamma} Cl(\{x\})$ is an $\alpha gs\gamma$-open set containing A. This is a contradiction. Hence, $\alpha gs_{\gamma} Cl(\{x\}) \cap A \neq \varphi$. Conversely, let $x \in X$ such that $\alpha gs_{\gamma} Cl(\{x\}) \cap A \neq \varphi$ and suppose that $x \notin \alpha gs_{\gamma} \ker(A)$. Then there exists an $\alpha gs\gamma$-open set V containing A and $x \notin V$. This is a contradiction. Hence, $x \in \alpha gs_{\gamma} \ker(A)$.

4. $\alpha gs\gamma$-Generalized Closed Sets

Definition 4.1. A subset A of a topological space (X, τ) with an operation γ on $\tau_{\alpha gs}$ is said to be $\alpha gs\gamma$-generalized closed set (briefly $\alpha gs\gamma g$-closed) if $\alpha gsCl_{\gamma}(A) \subseteq U$ whenever $A \subseteq U$ and U is an $\alpha gs\gamma$-open set in X.

Proposition 4.2. *Every* $\alpha gs\gamma$*-closed set is* $\alpha gs\gamma g$*-closed.*

Proof. Let A be any $\alpha gs\gamma$-closed set. Consider V be any $\alpha gs\gamma$-open set containing A. Since A is an $\alpha gs\gamma$-closed set. By Theorem 2.8 (ii), $A = \alpha gsCl_{\gamma}(A)$. Thus, $\alpha gsCl_{\gamma}(A) = A \subseteq V$. Hence, A is an $\alpha gs\gamma g$-closed set in X.

Example 4.3. Let $X = \{a, b, c\}$ and $\tau = \{\varphi, a, X\}$. Then $\tau_{\alpha gs} = \{\varphi, \{a\}, \{a, b\}, \{a, c\}, X\}$. Let $\gamma : \tau_{\alpha gs} \to P(X)$ be an operation on $\tau_{\alpha gs}$ defined by

$$\gamma(A) = \begin{cases} A & \text{if } b \in A \\ cl(A) & \text{if } b \notin A \end{cases} \forall A \in \tau_{\alpha gs}$$

Then $\alpha gs\gamma$-closed sets are $\varphi, \{a\}, \{c\}, X$ and $\alpha gs\gamma g$-closed sets are $\varphi, \{b\}, \{c\}, \{b, c\}, X$. Thus $\{b, c\}$ is an $\alpha gs\gamma g$-closed set but not $\alpha gs\gamma$-closed set in X.

Theorem 4.4. *Let* (X, τ) *be a topological space and* γ *be an operation on* $\tau_{\alpha gs}$. *Then the following statements are equivalent for any subset A in X*

(i) *A is an* $\alpha gs\gamma g$ *-closed set in* (X, τ)

(ii) $\alpha gs_{\gamma}Cl(\{x\}) \cap A \neq \varphi$ *for every* $x \in \alpha gsCl_{\gamma}(A)$

(iii) $\alpha gsCl_{\gamma}(A) \subseteq \alpha gs_{\gamma} \ker(A)$.

Proof. (i) $\Rightarrow$ (ii) Let A be an $\alpha gs\gamma g$-closed set. Suppose that $\alpha gs_{\gamma}Cl(\{x\}) \cap A \neq \varphi$ for some $x \in \alpha gsCl_{\gamma}(A)$. Therefore, $A \subseteq X \setminus \alpha gs_{\gamma}Cl(\{x\})$. By Result 2.7 (iii), $\alpha gs_{\gamma}Cl(\{x\})$ is an $\alpha gs\gamma$-closed set in X. Thus, $X \setminus \alpha gs_{\gamma}Cl(\{x\})$ is an $\alpha gs\gamma$-open set containing A in X. Since A is an $\alpha gs\gamma g$-closed set, $\alpha gs_{\gamma}Cl(A) \cap X \setminus \alpha gs_{\gamma}Cl(\{X\})$. This implies that $x \notin \alpha gsCl_{\gamma}(A)$ which is a contradiction to $x \in \alpha gsCl_{\gamma}(A)$. Hence, $\alpha gs_{\gamma}Cl(\{x\}) \cap A \neq \varphi$ for every $x \in \alpha gsCl_{\gamma}(A)$.

(ii) $\Rightarrow$ (iii) Follows from the Proposition 3.32.

(iii) $\Rightarrow$ (i) Let $x \in \alpha gsCl_{\gamma}(A) \subseteq \alpha gs_{\gamma} \ker(A)$. Consider V be an $\alpha gs\gamma$-open set such that $A \subseteq V$. It is enough to prove that $\alpha gsCl_{\gamma}(A) \subseteq V$. Since $x \in \alpha gs_{\gamma} \ker(A), x \in V$. Thus $\alpha gsCl_{\gamma}(A) \subseteq V$. Hence, A is an $\alpha gs\gamma g$-closed set in (X, τ).

Theorem 4.5. *If A is* $\alpha gs\gamma$*-open and* $\alpha gs\gamma g$*-closed then A is an* $\alpha gs\gamma$*-closed.*

Proof. Let A is $\alpha gs\gamma$-open and $\alpha gs\gamma g$-closed. Now, $A \subseteq A$. By Definition 4.1, $\alpha gsCl_{\gamma}A \subseteq A$. By the Theorem 2.8 (i), $A \subseteq \alpha gsCl_{\gamma}(A)$. Thus, $A = \alpha gsCl_{\gamma}(A)$. By the Theorem 2.8 (ii), A is an $\alpha gs\gamma$-closed.

Theorem 4.6. *Let* (X, τ) *be a topological space and* γ *be an operation on* $\tau_{\alpha gs}$. *If a subset A of X is* $\alpha gs\gamma g$ *-closed then* $\alpha gsCl_{\gamma}(A) \setminus A$ *does not contain any non-empty* $\alpha gs\gamma$ *-closed.*

Proof. Let A be an $\alpha gs\gamma g$ -closed set in X. Suppose that there exist an non-empty $\alpha gs\gamma$ -closed set E such that $E \subseteq \alpha gsCl_{\gamma}(A) \setminus A$. Therefore, $X \setminus E$ is an $\alpha gs\gamma$ -open set. Let $A \subseteq X \setminus E$. Since A is $\alpha gs\gamma g$ -closed, $\alpha gsCl_{\gamma}(A) \subseteq X \setminus E$. From this, $X \setminus \alpha gsCl_{\gamma}(A) \supseteq E$. Thus, $E \subseteq (\alpha gsCl_{\gamma}(A) \setminus A) \bigcap (X \setminus \alpha gsCl_{\gamma}(A))$. Hence, $E = \varphi$. This is a contradiction. Therefore, $\alpha gsCl_{\gamma}(A) \setminus A$ does not contain any non-empty $\alpha gs\gamma$ -closed.

Theorem 4.7. *Let* (X, τ) *be a topological space and* γ *be an operation on* $\tau_{\alpha gs}$. *Then for each* $x \in X$, $\{x\}$ *is an* $\alpha gs\gamma$ *-closed set or* $X \setminus \{x\}$ *is an* $\alpha gs\gamma g$ *-closed set in* (X, τ).

Proof. Suppose that $\{x\}$ is not an $\alpha gs\gamma$ -closed set. Then $X \setminus \{x\}$ is not an $\alpha gs\gamma$ -open set. Hence, X is the only $\alpha gs\gamma$ -open set containing $X \setminus \{x\}$. Also, $\alpha gsCl_{\gamma}(X \setminus \{x\}) \subseteq X$. Thus $X \setminus \{x\}$ is an $\alpha gs\gamma g$ -closed set in (X, τ).

Theorem 4.8. *Let* (X, τ) *be a topological space and* γ *be an operation on* $\tau_{\alpha gs}$. *Then* $\tau_{\alpha gs\gamma} = \tau^{c}_{\alpha gs\gamma}$ *if and only if every subsets of X is an* $\alpha gs\gamma g$ *-closed set in* (X, τ).

Proof. Let $\tau_{\alpha gs\gamma} = \tau^{c}_{\alpha gs\gamma}$. Let V be an $\alpha gs\gamma$ -open set and A be any subset of X such that $A \subseteq V$. By Theorem 2.8 (iii), $\alpha gs\gamma Cl_{\gamma}(A) \subseteq \alpha gsCl_{\gamma}(V)$. Since $\tau_{\alpha gs\gamma} = \tau^{c}_{\alpha gs\gamma}$. Thus, $\alpha gsCl_{\gamma}(V) = V$ and $\alpha gsCl_{\gamma}(A) = V$. Hence, A is an $\alpha gs\gamma g$ -closed set. Conversely, let every subsets of X is an $\alpha gs\gamma g$ -closed set. Consider V be an $\alpha gs\gamma$ -open set. By the assumption, V is an $\alpha gs\gamma g$ -closed set. From this, $\alpha gsCl_{\gamma}(V) = V$ whenever $V \subseteq V$. By Theorem 2.8 (i), $V \subseteq \alpha gsCl_{\gamma}(V)$. Thus, $\alpha gsCl_{\gamma}(V) = V$. Hence, V is an $\alpha gs\gamma$ -closed set in (X, τ). Therefore, $\tau_{\alpha gs\gamma} = \tau^{c}_{\alpha gs\gamma}$.

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