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OSCILLATORY BEHAVIOR IN A TWO INDUCTIVELY COUPLED VAN DER POL AUTO-GENERATORS MODEL WITH DELAY

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Abstract

In this paper, we discuss a system of two coupled van der Pol auto-generators model with delay. Some sufficient conditions to guarantee the existence of oscillatory solutions for the model are obtained. Oscillation synchronization occurs due to the suitable parameter values. Delay affects the oscillatory frequency but not affects the oscillation synchronization.

1. Introduction

It is very known that coupled van der Pol oscillators models with or without time delays have been extensively studied [1, 3-15]. For two coupled van der Pol oscillators with time delay as follows:

$$\begin{cases} y_1''(t) - \varepsilon(1 - y_1^2(t))y_1'(t) + w_1^2 y_1(t) = \varepsilon\alpha[y_2(t-\tau) - y_2'(t-\tau)], \\ y_2''(t) - \varepsilon(1 - y_2^2(t))y_2'(t) + w_2^2 y_2(t) = \varepsilon\alpha[y_1(t-\tau) - y_1'(t-\tau)]. \end{cases} \tag{1}$$

By the approach of averaging together with truncation of Taylor expansions, Li et al. [7] have investigated the effect of time delay on the nonlinear dynamics of the system. The condition necessary for saddle-node and Hopf bifurcations for symmetric modes were determined. Zhang and Gu also discussed the model of two coupled van der Pol oscillators with two time delays [14]:

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$$\begin{cases} x_1''(t) + \varepsilon(x_1^2(t) - 1)x_1'(t) + x_1(t) = \alpha[y_1(t - \tau_2) - x_1(t)], \\ y_1''(t) + \varepsilon(y_1^2(t) - 1)y_1'(t) + y_1(t) = \alpha[x_1(t - \tau_1) - y_1(t)]. \end{cases} \tag{2}$$

Using the theory of normal form and the center manifold theorem, the authors have concerned the existence of Hopf bifurcations and the stability of the bifurcating periodic solutions. Recently, Beregov and Melkikh have considered the case of inductive coupling of two identical circuits as follows [1]:

$$\begin{cases} x_1''(t) - (\mu_1 - x_1^2(t))x_1'(t) + x_1(t) = -M_{12}\, x_2''(t), \\ x_2''(t) - (\mu_2 - x_2^2(t))x_2'(t) + x_2(t) = -M_{12}\, x_1''(t), \end{cases} \tag{3}$$

where M_{12} is the coefficient of mutual induction. The authors calculated the Lyapunov numbers, checked the sensitivity to different initial conditions, plotted the power spectrum, and established the presence of metastable chaos, a strange non-chaotic attractor and several stable limiting cycles. The areas of parametric dependence of different modes of synchronization also were determined. Motivated by the above models, in this paper we consider the following time delay model:

$$\begin{cases} x_1''(t) - (\mu_1 - x_1^2(t))x_1'(t) + x_1(t) = -M_{12}\, x_2''(t) + \varepsilon[x_2(t - \tau) - x_2'(t - \tau)], \\ x_2''(t) - (\mu_2 - x_2^2(t))x_2'(t) + x_2(t) = -M_{12}\, x_1''(t) + \varepsilon[x_1(t - \tau) - x_1'(t - \tau)], \end{cases} \tag{4}$$

where $M_{12} \neq \pm 1$. By means of mathematical analysis method, we investigate the dynamical behavior of system (4). Due to the existence of time delay, the chaos phenomenon has not been found. According to the chaos theory [2], a dynamical system as chaotic, it must have dense periodic orbits. However, under our restrictive condition, system (4) has only one unstable equilibrium point, and all solutions are bounded. In other words, system (4) has a unique periodic solution, it does not have dense periodic orbits, implying that the chaos phenomenon is not occurred.

2. Preliminaries

It is convenient to write (4) as an equivalent four dimensional first-order system

$$\begin{cases} u_1'(t) = u_2(t), \\ u_2'(t) = (\mu_1 - u_1^2(t)u_2(t) - u_1(t) - M_{12}u_4'(t) + \varepsilon u_3(t - \tau) - \varepsilon u_4(t - \tau), \\ u_3'(t) = u_4(t), \\ u_4'(t) = (\mu_2 - u_3^2(t))u_4(t) - u_3(t) - M_{12}u_2'(t) + \varepsilon u_1(t - \tau) - \varepsilon u_2(t - \tau). \end{cases} \tag{5}$$

Substitute $u_4'(t)$ into the second equation and $u_2'(t)$ into the fourth equation we have

$$\begin{cases} u_1'(t) = u_2(t), \\ u_2'(t) = \alpha_1 u_1(t) + \alpha_2 u_2(t) + \alpha_3 u_3(t) + \alpha_4 u_4(t) + \varepsilon_1 u_1(t-\tau) + \varepsilon_2 u_2(t-\tau) \\ + \varepsilon_3 u_3(t-\tau) + \varepsilon_4 u_4(t-\tau) + N_1 u_1^2(t) u_2(t) + N_2 u_3^2(t) u_4(t), \\ u_3'(t) = u_4(t), \\ u_4'(t) = \beta_1 u_1(t) + \beta_2 u_2(t) + \beta_3 u_3(t) + \beta_4 u_4(t) + \varepsilon_1 u_1(t-\tau) + \varepsilon_2 u_2(t-\tau) \\ + \varepsilon_3 u_3(t-\tau) + \varepsilon_4 u_4(t-\tau) + N_2 u_1^2(t) u_2(t) + N_1 u_3^2(t) u_4(t), \end{cases} \tag{6}$$

where $\alpha_1 = -\dfrac{1}{1-M_{12}^2}$, $\alpha_2 = \dfrac{\mu_1}{1-M_{12}^2}$, $\alpha_3 = \dfrac{M_{12}}{1-M_{12}^2}$, $\alpha_4 = -\dfrac{\mu_1 M_{12}}{1-M_{12}^2}$; $\varepsilon_1 = -\dfrac{\varepsilon M_{12}}{1-M_{12}^2}$, $\varepsilon_2 = \dfrac{\varepsilon M_{12}}{1-M_{12}^2}$, $\varepsilon_3 = \dfrac{\varepsilon}{1-M_{12}^2}$, $\varepsilon_4 = \dfrac{\varepsilon}{1-M_{12}^2}$; $\beta_1 = \dfrac{M_{12}}{1-M_{12}^2}$, $\beta_2 = -\dfrac{\mu_1 M_{12}}{1-M_{12}^2}$, $\beta_3 = -\dfrac{1}{1-M_{12}^2}$; $\beta_4 = \dfrac{\mu_2}{1-M_{12}^2}$; $N_1 = -\dfrac{1}{1-M_{12}^2}$, $N_2 = \dfrac{M_{12}}{1-M_{12}^2}$.

The linearized system of (6) is the follows:

$$\begin{cases} u_1'(t) = u_2(t), \\ u_2'(t) = \alpha_1 u_1(t) + \alpha_2 u_2(t) + \alpha_3 u_3(t) + \alpha_4 u_4(t) + \varepsilon_1 u_1(t-\tau) + \varepsilon_2 u_2(t-\tau) \\ + \varepsilon_3 u_3(t-\tau) + \varepsilon_4 u_4(t-\tau), \\ u_3'(t) = u_4(t), \\ u_4'(t) = \beta_1 u_1(t) + \beta_2 u_2(t) + \beta_3 u_3(t) + \beta_4 u_4(t) + \varepsilon_1 u_1(t-\tau) + \varepsilon_2 u_2(t-\tau) \\ + \varepsilon_3 u_3(t-\tau) + \varepsilon_4 u_4(t-\tau). \end{cases} \tag{7}$$

System (7) can be written as a matrix form:

$$U'(t) = AU(t) + BU(t-\tau), \tag{8}$$

where $U(t) = [u_1(t), u_2(t), u_3(t), u_4(t)]^T$, $U(t-\tau) = [u_1(t-\tau), u_2(t-\tau), u_3(t-\tau), u_4(t-\tau)]^T$,

$$A = (a_{ij})_{4\times 4} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 \\ 0 & 0 & 0 & 1 \\ \beta_1 & \beta_2 & \beta_3 & \beta_4 \end{pmatrix},$$

$$B = (b_{ij})_{8\times 8} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ \varepsilon_1 & \varepsilon_2 & \varepsilon_3 & \varepsilon_4 \\ 0 & 0 & 0 & 0 \\ \varepsilon_1 & \varepsilon_2 & \varepsilon_3 & \varepsilon_4 \end{pmatrix}.$$

Lemma 1. *Suppose that the following condition holds*

$$(\alpha_1 + \varepsilon_1)(\beta_3 + \varepsilon_3) \neq (\alpha_3 + \varepsilon_3)(\beta_1 + \varepsilon_1) \tag{9}$$

then system (6) *has a unique equilibrium point.*

Proof. An equilibrium point $u^* = (u_1^*, u_2^*, u_3^*, u_4^*)^T$ of system (6) is a constant solution of the following algebraic equation:

$$\begin{cases} u_2^* = 0, \\ \alpha_1 u_1^* + \alpha_2 u_2^* + \alpha_3 u_3^* + \alpha_4 u_4^* + \varepsilon_1 u_1^* + \varepsilon_2 u_2^* + \varepsilon_3 u_3^* + \varepsilon_4 u_4^* \\ + N_1 (u_1^2)^* u_2^* + N_2 (u_3^2)^* u_4^* = 0, \\ u_4^* = 0, \\ \beta_1 u_1^* + \beta_2 u_2^* + \beta_3 u_3^* + \beta_4 u_4^* + \varepsilon_1 u_1^* + \varepsilon_2 u_2^* + \varepsilon_3 u_3^* + \varepsilon_4 u_4^* \\ + N_2 (u_1^2)^* u_2^* + N_1 (u_3^2)^* u_4^* = 0. \end{cases} \tag{10}$$

Since $u_2^* = 0$ and $u_4^* = 0$, system (10) is the following:

$$\begin{cases} (\alpha_1 + \varepsilon_1) u_1^* + (\alpha_3 + \varepsilon_3) u_3^* = 0, \\ (\beta_1 + \varepsilon_1) u_1^* + (\beta_3 + \varepsilon_3) u_3^* = 0. \end{cases} \tag{11}$$

System (11) can be written as a matrix form:

$$PU_1 = 0, \tag{12}$$

where $U_1 = (u_1^*, u_3^*)^T$, and the coefficient matrix of system (12) is

$$P = (p_{ij})_{2\times 2} = \begin{pmatrix} \alpha_1 + \varepsilon_1 & \alpha_3 + \varepsilon_3 \\ \beta_1 + \varepsilon_1 & \beta_3 + \varepsilon_3 \end{pmatrix}.$$

Under the restrictive condition (9), the coefficient matrix P of system (12) is a nonsingular matrix. Based on the principle theorem of linear algebra, system (12) has a unique trivial solution, implying that there exists a unique equilibrium point of system (6), which is exactly the zero point.

Lemma 2. *Assume that* $N_1 < 0$ *and* $N_2 < 0$, *both* $u_2 > 0$ *and* $u_4 > 0$ (*or* $u_2 < 0$ *and* $u_4 < 0$) *hold, then the solutions of system* (6) *are bounded.*

Proof. To prove the boundedness of the solutions in system (6), we construct a Lyapunov function $V(t) = \sum_{i=1}^{4} \frac{1}{2} u_i^2(t)$. Calculating the derivative of $V(t)$ through system (6) one get:

$$
\begin{aligned}
V'(t)|_{(6)} = {} & \sum_{i=1}^{4} u_i(t) u_i'(t) \\
= {} & u_1(t)u_2(t) + u_2(t)\{\alpha_1 u_1(t) + \alpha_2 u_2(t) + \alpha_3 u_3(t) + \alpha_4 u_4(t) + \varepsilon_1 u_1(t-\tau) \\
& + \varepsilon_2 u_2(t-\tau) + \varepsilon_3 u_3(t-\tau) + \varepsilon_4 u_4(t-\tau) + N_1 u_1^2(t) u_2(t) + N_2 u_3^2(t) u_4(t)\} \\
& + u_3(t)u_4(t) + u_4(t)\{\beta_1 u_1(t) + \beta_2 u_2(t) + \beta_3 u_3(t) + \beta_4 u_4(t) + \varepsilon_1 u_1(t-\tau) \\
& \varepsilon_2 u_2(t-\tau) + \varepsilon_3 u_3(t-\tau) + \varepsilon_4 u_4(t-\tau) + N_2 u_1^2(t) u_2(t) + N_1 u_3^2(t) u_4(t)\} \\
\le {} & (1+\alpha_1) u_1(t) u_2(t) + (1+\beta_3) u_3(t) u_4(t) + \sum_{i=2}^{4} |\alpha_i| |u_i(t)| |u_2(t)| \\
& + \sum_{i=2}^{4} |\beta_i| |u_i(t)| |u_4(t)| \\
& + \sum_{i=1}^{4} |\varepsilon_1| |u_i(t-\tau)| |u_2(t)| + \sum_{i=1}^{4} |\varepsilon_1| |u_i(t-\tau)| |u_4(t)| \\
& + N_1 u_1^2(t) u_2^2(t) + N_2 u_3^2(t) u_2(t) u_4(t) + N_2 u_1^2(t) u_2(t) u_4(t) \\
& + N_1 u_3^2(t) u_4^2(t). \qquad (13)
\end{aligned}
$$

Noting that as $u_{i-1}(t)u_i(t)\,(i = 2, 3, 4)$ tend to infinity, $u_2^2(t)u_4^2(t)$, $u_3^2(t)u_4^2(t)$, $u_3^2(t)u_2(t)u_4(t)$, and $u_1^2(t)u_2(t)u_4(t)$ are higher order infinity than

$u_{i-1}(t)u_i(t)(i = 2, 3, 4)$. Noting that $N_i < 0(i = 1, 2)$, both $u_2(t) > 0$ and $u_4(t) > 0$ (or $u_2(t) < 0$ and $u_4(t) < 0$) hold. Therefore, there exists suitably large $M > 0$ such that $V'(t)|_{(6)} < 0$ as $|u_i(t)| \geq M(i = 1, 2, 3, 4)$. This means that all solutions of system (6) are bounded.

3. Main Result

Theorem 1. *Suppose that system* (6) *has a unique equilibrium point and all solutions are bounded. If matrix A has a positive* (*or a positive real part*) *eigenvalue, then the unique equilibrium point* $(0, 0, 0, 0)^T$ *of system* (6) *is unstable. In other words, system* (6) *generates an oscillatory solution.*

Proof. Obviously, system (6) has a unique unstable equilibrium point if and only if system (7) has a unique unstable equilibrium point. Therefore, in the following we consider the instability of the unique equilibrium point of system (7). Considering the matrix form (8) of system (7), let $\rho_1, \rho_2, \rho_3, \rho_4$ and $\theta_1, \theta_2, \theta_3, \theta_4$ are eigenvalues of matrix A and B, respectively, the characteristic equation of (8) is

$$\det[\lambda I_{ij} - a_{ij} - b_{ij}e^{-\lambda\tau}] = 0, \tag{13}$$

namely,

$$\prod_{i=1}^{4}[\lambda - \rho_i - \theta_i e^{-\lambda\tau}] = 0. \tag{14}$$

Noting that B has a zero eigenvalue. From (4) we consider the following equation for some j,

$$\lambda - \rho_j - \theta_j e^{-\lambda\tau} = 0. \tag{15}$$

Specially, we select $\rho_j > 0$, and $\theta_j = 0$. Then $\lambda = \rho_j$. Since $\rho_j > 0$ or ρ_j has a positive real part, this means that the characteristic equation of (15) has a positive real root (or a positive real part). Therefore, the trivial solution of (7) is unstable, implying that system (6) generates an oscillatory solution. The proof is completed.

Theorem 2. *Let* $L = \max\{|\alpha_1| + |\beta_1|, |\alpha_3| + |\beta_3|, |\alpha_2| + 1 + |\beta_2|, |\alpha_4| + 1 + |\beta_4|\}$, *and* $K = \max\{|\varepsilon_1|, |\varepsilon_2|, |\varepsilon_3|, |\varepsilon_4|\}$. *If the following condition holds*:

$$2K\tau e \exp(-L\tau) > 1. \tag{16}$$

Then the trivial solution of (7) *is unstable*, *implying that system* (6) *generates an oscillatory solution*.

Proof. Let $y(t) = \sum_{i=1}^{4} u_i(t)$, from system (7) we have

$$y'(t) \le Ly(t) + 2Ky(t-\tau), \qquad t \ge \tau. \tag{17}$$

Consider the scalar delay differential equation

$$z'(t) = Lz(t) + 2Kz(t-\tau),\ t \ge \tau \tag{18}$$

with $y(t) = z(t),\ t \in (0, \tau)$. According to the Comparison Theorem of differential equation, we have $y(t) \le z(t)$ for $t \ge \tau$. We claim that the trivial solution of (18) is unstable. Suppose that this is not the case, then the characteristic equation associated with (18) given by

$$\lambda = L + 2Ke^{-\lambda\tau} \tag{19}$$

will have a real nonpositive root, say $\lambda^* < 0$. Noting that $|\lambda^*| = -\lambda^*$, and

$$|\lambda^*| = |L + 2Ke^{-\lambda^*\tau}| \ge 2Ke^{|\lambda^*|\tau} - L. \tag{20}$$

Using the formula $e^x \ge ex$, from (20) we get

$$\begin{aligned} 1 &\ge \frac{2Ke^{|\lambda^*|\tau}}{|\lambda^*| + L} = \frac{2K\tau e^{(|\lambda^*|+L)\tau} \cdot e^{-L\tau}}{(|\lambda^*| + L)\tau} \\ &\ge \frac{2K\tau e \cdot (|\lambda^*| + L)\tau \cdot e^{-L\tau}}{(|\lambda^*| + L)\tau} = (2K\tau e)e^{-L\tau}. \end{aligned} \tag{21}$$

The last inequality contradicts (16). Hence, our claim regarding the instability of the trivial solution of (18) is valid. It follows that the trivial solution of (17) is also unstable. Since $y(t) = \sum_{i=1}^{4} u_i(t)$, the instability of $y(t)$

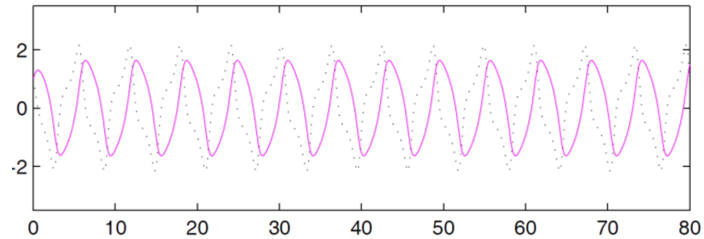
means that the trivial solution of (7) is unstable, implying that system (6) generates an oscillatory solution.

4. Simulation Results

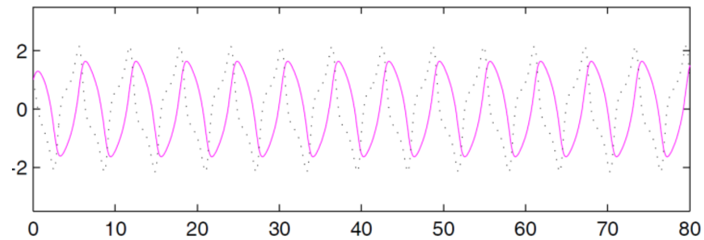
The simulation is based on system (6). We first select the parameter values: $\varepsilon = 0.1$, $\mu_1 = 1$, $\mu_2 = 0.5$, $M_{12} = -0.2$, then $\alpha_1 = -1.04$, $\alpha_2 = 1.04$, $\alpha_3 = -0.21$, $\alpha_4 = 0.11$, $\beta_1 = -0.21$, $\beta_2 = 0.21$, $\beta_3 = -1.04$, $\beta_4 = 1.04$, $\varepsilon_1 = 0.21$, $\varepsilon_2 = -0.21$, $\varepsilon_3 = 0.11$, $\varepsilon_4 = -0.11$; $N_1 = -1.04$, $N_2 = -0.208$, and time delays $\tau = 0.5, 1.2$ respectively. The condition of Lemma 1 is satisfied. The eigenvalues of matrix A are $0.6009 \pm 0.9476i$, $0.4391 \pm 0.7945i$. Noting that there is an eigenvalue of A which has a positive real part. Based on Theorem 1, there exists an oscillatory solution. From figure 1 and figure 2 we know that the solution is oscillation synchronization, and time delay affects the oscillatory frequency (see Figure 1 and Figure 2). Then we select the parameter values: $\varepsilon = 0.2, \mu_1 = 0.5, \mu_2 = 1, M_{12} = -0.5$, thus $\alpha_1 = -1.333$, $\alpha_2 = 0.667, \alpha_3 = -0.667, \alpha_4 = 0.667, \beta_1 = -0.667, \beta_2 = 0.333, \beta_3 = -1.333, \beta_4 = 1.333$, $\varepsilon_1 = 0.013$, $\varepsilon_2 = -0.013$, $\varepsilon_3 = 0.267$, $\varepsilon_4 = -0.267$; $N_1 = -1.333$, $N_2 = -0.667$. The eigenvalues of matrix A are $0.7517 \pm 1.1452i$, $0.2483 \pm 0.8051i$. From Theorem 1, there exists a solution which is oscillation but not oscillation synchronization (see Figure 3).

5. Conclusion

This paper investigates the oscillatory property of a coupled van der Pol auto-generators model with delay. In our assumptions, system only has a unique unstable equilibrium point, and all solutions of the system are bounded. This specific instability of the solutions forces system to generate a permanent periodic oscillation. Oscillation synchronization phenomenon is appeared in which the selection of parameters is important. Time delay affects the oscillatory frequency but not affects the oscillation synchronization. Simulation also indicates that our assumptions are only sufficient conditions.

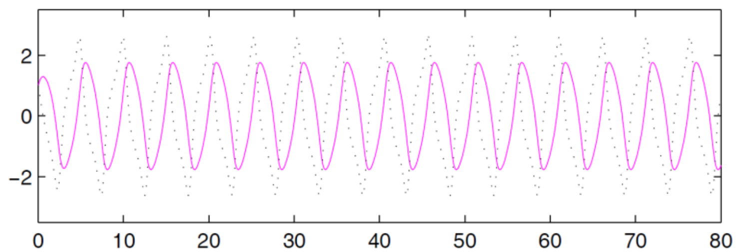


(a) Solid line: $u_1(t)$, dotted line: $u_2(t)$.

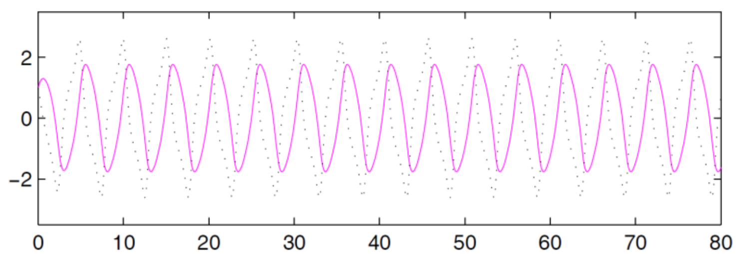


(b) Solid line: $u_3(t)$, dotted line: $u_4(t)$.

Figure 1. Oscillation synchronization $(u_1(t) = u_3(t),\ u_2(t) = u_4(t))$, delay: 0.5.



(a) Solid line: $u_1(t)$, dotted line: $u_2(t)$.



(b) Solid line: $u_3(t)$, dotted line: $u_4(t)$.

Figure 2. Oscillation synchronization $(u_1(t) = u_3(t),\ u_2(t) = u_4(t))$, delay: 1.2.

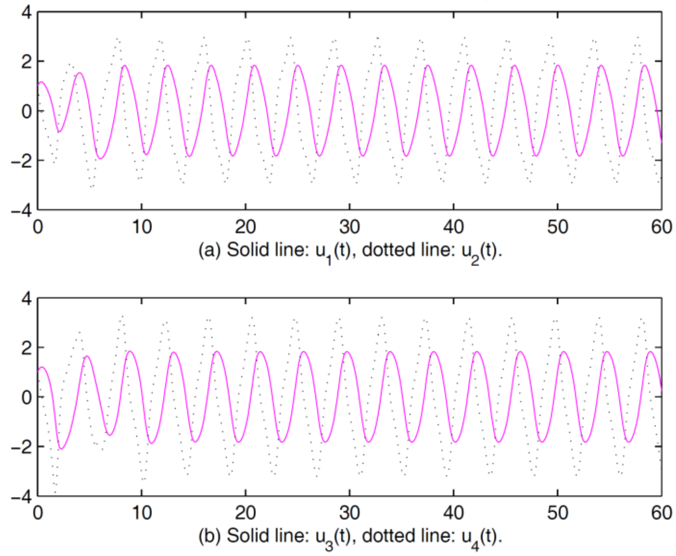


Figure 3. Oscillation of the solutions, delay: 1.5.

References

[1] R. Y. Beregov and A. V. Melkikh, De-synchronization and chaos in two inductively coupled Van der Pol autogenerators, Chaos, Solitons and Fractals 73 (2015), 17-28.

[2] H. Boris and A. Katok, A First Course in Dynamics: With a Panorama of Recent Development, Cambridge University Press, 2003.

[3] D. Ghosh, A. R. Chowdhury and P. Saha, On the various kinds of synchronization in delayed Duffing-Van der Pol system, Commun. Nonlinear Sci. Numer. Simul. 13(4) (2008), 790-803.

[4] T. A. Giresse and K. T. Crepin, Chaos generalized synchronization of coupled Mathieu-Van der Pol and couple Duffing-Van der Pol systems using fractional order-derivative, Chaos, Solitons and Fractals 98 (2017), 88-100.

[5] J. C. Ji and N. Zhang, Additive resonances of a controlled van der Pol-Duffing oscillator, J. Sound and Vibration 315(1-2) (2008), 22-33.

[6] P. Kumar, S. Narayanan and S. Gupta, Investigations on the bifurcation of a noisy Duffing-Van der Pol oscillator, Probabilistic Engineering Mechanics 45 (2016), 70-86.

[7] X. Y. Li, J. C. Ji and C. H. Hansen, Dynamics of two delay coupled van der Pol oscillators, Mech. Res. Commun. 33 (2006), 614-627.

[8] A. Maccari, Vibration amplitude control for a van der Pol-Duffing oscillator with delay, J. Sound and Vibration 317(2) (2008), 993-1007.

[9] A. Maccari, Response control for the externally excited van der Pol oscillator with non-local feedback, J. Sound and Vibration 331(5) (2012), 997-995.

[10] L. Makouo and P. Woafo, Experimental observation of bersting patterns in Van der Pol oscillators, Chaos, Solitons and Fractals 94 (2017), 95-101.

[11] Sandro E. de Pinto, Sergio R. Lopez and Ricardo L. Viana, Collective behavior in a chain of Van der Pol oscillators with power-law coupling, Physica A, 303 (2002), 339-356.

[12] K. Rompala, R. Rand and H. Howland, Dynamics of three coupled van der Pol oscillators with application to circadian rhythms, Commun. Nonlinear Sci. Numer. Simul. 12 (2007), 794-803.

[13] V. Wiggers and P.C. Rech, Multistability and organization of periodicity in a Van der Pol Duffing oscillator, Chaos, Solitons and Fractals 103 (2017), 632-637.

[14] J. M. Zhang and X. S. Gu, Stability and bifurcation analysis in the delay-coupled van der Pol oscillators, Appl. Math. Model 34 (2010), 2291-2299.

[15] J. Zhou, X. H. Cheng, L. Xiang and Y. C. Zhang, Impulsive control and synchronization of chaotic systems consisting of Van der Pol oscillators coupled to linear oscillators, Chaos, Solitons and Fractals 33(2) (2007), 607-616.