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ON SOME PROPERTIES OF AMERICAN FUZZY PUT OPTION MODEL ON FUZZY FUTURE CONTRACTS INVOLVING GENERAL LINEAR OCTAGONAL FUZZY NUMBERS

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Abstract

The existing American put option pricing model studied by several authors (to cite a few, [6], [8], [7]) involving non-overlapping type of fuzzy numbers are not adequate to model the uncertainty of high volatile or low volatile fuzzy stocks. Hence we consider general linear octagonal fuzzy numbers (GLOFN), general in the sense that they are either non-overlapping or overlapping partially or completely to study the properties of fuzzy intrinsic values of American fuzzy put option model on fuzzy future contracts. The same is validated using the real data of Microsoft Corporation shares considered from the website optionistics [10] and the computations are performed using Matlab 2016a software.

1. Introduction

American put and call options models in both discrete and continuous time were proposed by Yoshida [8] in 2003. Later on, in 2008 Muzzioli et al. [6] considered imprecise volatility to study American put option pricing model using non overlapping triangular and trapezoidal fuzzy numbers while Xcaojian Yu [7] considered impreciseness in both risk-free interest rate and volatility of the underlying stock involving non-overlapping trapezoidal fuzzy numbers. A new fuzzy risk-neutral probability measure involving general linear trapezoidal fuzzy numbers defined by us to elucidate the problem pricing American fuzzy put option buyer's model on future contracts in 2019.

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In this study, we discuss the properties of fuzzy intrinsic values of American fuzzy put option model on fuzzy future contracts involving GLOFN (either non-overlapping or overlapping partially or completely). The plan of the paper is as follows: In Section 2, we expound the essential prerequisites for this paper. In Section 3, we prove some of the properties of fuzzy intrinsic values of American fuzzy put option model on fuzzy future contracts involving GLOFN. We use the real time data of Microsoft Corporation shares for the year 2018 to validate the same. In Section 4, we conclude the paper.

2. Preliminaries

For the sake of completeness, we review the required definitions and results. The concept of linear octagonal fuzzy numbers was first introduced by S.U. Malini and Felbin C. Kennedy [3] in 2013.

Definition 1 [3]**.** A fuzzy number $\widetilde{A}$ is a linear octagonal fuzzy number denoted by $\widetilde{A} \approx (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8; k)$ where $a_1 \le a_2 \le a_3 \le a_4 \le a_5 \le a_6 \le a_7 \le a_8$ are real numbers with membership function $\mu_{\widetilde{A}}(x)$ given by

$$\mu_{\widetilde{A}}(x) = \begin{cases} 0, & \text{for } x < a_1 \\ \dfrac{k(x - a_1)}{(a_2 - a_1)}, & \text{for } a_1 \le x \le a_2 \\ k, & \text{for } a_2 \le x \le a_3 \\ k + (1-k)\dfrac{k(x - a_3)}{(a_4 - a_3)}, & \text{for } a_3 \le x \le a_4 \\ 1, & \text{for } a_4 \le x \le a_5 \\ k + (1-k)\dfrac{k(a_6 - x)}{(a_6 - a_5)}, & \text{for } a_5 \le x \le a_6 \\ k, & \text{for } a_6 \le x \le a_7 \\ k\dfrac{k(a_8 - x)}{(a_8 - a_7)}, & \text{for } a_7 \le x \le a_8 \\ 0, & \text{for } x > a_8. \end{cases}$$

where $0 < k < 1$.

Definition 2 [3]**.** The measure of a linear octagonal fuzzy number $\widetilde{A}$ denoted $M^{oct}(\widetilde{A})$ is defined by

$$M^{oct}(\widetilde{A}) = \frac{1}{4}[(a_1 + a_2 + a_7 + a_8)k + (a_3 + a_4 + a_5 + a_6)(1-k)], \quad (1)$$

where $0 < k < 1$.

Using the measure defined in equation (1), any two linear octagonal fuzzy numbers are ranked as follows.

- $\widetilde{A} \succcurlyeq \widetilde{B}$ if $M(\widetilde{A}) \geq M(\widetilde{B})$
- $\widetilde{A} \preccurlyeq \widetilde{B}$ if $M(\widetilde{A}) \leq M(\widetilde{B})$
- $\widetilde{A} \approx \widetilde{B}$ if $M(\widetilde{A}) = M(\widetilde{B})$.

Definition 3 [3]**.** If $* \in \{+, -\}$ is a binary operation between any two linear octagonal fuzzy numbers $\widetilde{A} \approx (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8; k)$ and $\widetilde{B} \approx (b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8; k)$, then $\widetilde{A} * \widetilde{B}$ are also linear octagonal fuzzy numbers defined by

$$\widetilde{A} + \widetilde{B} \approx (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5, a_6 + b_6, a_7 + b_7, a_8 + b_8; k)$$

$$\widetilde{A} - \widetilde{B} \approx (a_1 - b_8, a_2 - b_7, a_3 - b_6, a_4 - b_5, a_5 - b_4, a_6 - b_3, a_7 - b_2, a_8 - b_1, k)$$

Remark 4 [3]**.** If $\widetilde{C} \approx (c, c, c, c, c, c, c, c; k)$ is a constant linear octagonal fuzzy number, where $c \in \mathbb{R}$, then $\widetilde{C}\widetilde{A} \approx (ca_1, ca_2, ca_3, ca_4, ca_5, ca_6, ca_7, ca_8; k)$.

Definition 5 [3]**.** For any two non-negative linear octagonal fuzzy numbers $\widetilde{A}$ and $\widetilde{B}$ with $a_1 \geq 0$ and $b_1 \geq 0$, then the multiplication and division are also non-negative linear octagonal fuzzy numbers defined by $\widetilde{A} \times \widetilde{B} \approx (a_1b_1, a_2b_2, a_3b_3, a_4b_4, a_5b_5, a_6b_6, a_7b_7, a_8b_8; k)$ and $\widetilde{A} / \widetilde{B} \approx (a_1 / b_8, a_2 / b_7, a_3 / b_6, a_4 / b_5, a_5 / b_4, a_6 / b_3, a_7 / b_2, a_8 / b_1; k)$, where $b_1 \geq 0$.

Definition 6 [2]**.** If $\widetilde{A}$ and $\widetilde{B}$ are two non-negative linear octagonal fuzzy numbers, then

$$\max(\widetilde{A}, \widetilde{B}) = \{(\max(a_1, b_1), \max(a_2, b_2), \max(a_3, b_3), \max(a_4, b_4), \max(a_5, b_5), \max(a_6, b_6), \max(a_7, b_7), \max(a_8, b_8); k)\}$$

If $\widetilde{C} = (c, c, c, c, c, c, c, c; k)$ is a constant linear octagonal fuzzy number, where $c \in \mathbb{R}$ then

$$\max(\widetilde{A}, \widetilde{C}) = \{(\max(a_1, c), \max(a_2, c), \max(a_3, c), \max(a_4, c), \max(a_5, c), \max(a_6, c), \max(a_7, c), \max(a_8, c); k)\}$$

Definition 7 [4]**.** Let $\widetilde{F}_{n,\,i}$, $\widetilde{S}_{n,\,i}$ denoted the fuzzy future price and the fuzzy stock price at time n respectively and if the contract expired after a total period, N, then the fuzzy future price at time n can be computed to be

$$\widetilde{F}_{n,\,i} \approx (\widetilde{1} + \widetilde{R})^{N-n}\widetilde{S}_{n,\,i},\ i = 0, 1, \ldots, n \text{ and } n = 0, 1, 2, \ldots, N \tag{2}$$

where $\widetilde{R}$ is the discrete fuzzy risk-free interest rate. On expiration date $n = N$, equation (2) yielding $\widetilde{F}_{N,\,i} \approx \widetilde{S}_{N,\,i}$.

Definition 8 [5]**.** Let $\widetilde{U} \approx (u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8; k)$, $\widetilde{D} \approx (d_1, d_2, d_3, d_4, d_5, d_6, d_7, d_8; k)$ and $\widetilde{R} = (r_1, r_2, r_3, r_4, r_5, r_6, r_7, r_8; k)$ denoted the up and down fuzzy jump factors and the discrete fuzzy risk-free interest rate of the fuzzy stock.

The no arbitrage condition satisfied by the up and down fuzzy jump factors and the fuzzy risk-free interest rate is given by,

$$d_1 \le d_2 \le d_3 \le d_4 \le u_1 \le u_2 \le u_3 \le u_4 \le (1 + r_1) \le (1 + r_2) \le (1 + r_3) \le (1 + r_4) \le (1 + r_5) \le (1 + r_6) \le (1 + r_7) \le (1 + r_8) \le u_5 \le d_5 \le u_6 \le d_6 \le d_7 \le u_7 \le u_8 \le d_8.$$

The assumption of possible no arbitrage opportunities are

$$d_i < (1 + r_k) < u_j \tag{3}$$

for $i = 1, 2, 3, 4$, $k = 1, 2, 3, 4, 5, 6, 7, 8$ and $j = 5, 6, 7, 8$.

From equation (3), we have chosen few no arbitrage assumptions

$$d_4 < (1 + r_1) < u_5 \tag{4}$$

$$d_4 < (1 + r_2) < u_5 \tag{5}$$

$$d_4 < (1 + r_3) < u_5 \tag{6}$$

$$d_4 < (1 + r_4) < u_5 \tag{7}$$

$$d_4 < (1 + r_5) < u_5 \tag{8}$$

$$d_4 < (1 + r_6) < u_5 \tag{9}$$

$$d_4 < (1 + r_7) < u_5 \tag{10}$$

$$d_4 < (1 + r_8) < u_5 \tag{11}$$

Using the assumptions given in equations (4) to (11), the up and down fuzzy risk-neutral probability measures $Q(\widetilde{P}_u, \widetilde{P}_d)$ are defined by

$$\widetilde{P}_u \approx \left(\frac{1 + r_1 - d_4}{u_5 - d_4}, \frac{1 + r_2 - d_4}{u_5 - d_4}, \frac{1 + r_3 - d_4}{u_5 - d_4}, \frac{1 + r_4 - d_4}{u_5 - d_4}, \frac{1 + r_5 - d_4}{u_5 - d_4}, \frac{1 + r_6 - d_4}{u_5 - d_4}, \frac{1 + r_7 - d_4}{u_5 - d_4}, \frac{1 + r_8 - d_4}{u_5 - d_4}; k\right) \tag{12}$$

$$\widetilde{P}_d \approx \left(\frac{u_5 - 1 - r_8}{u_5 - d_4}, \frac{u_5 - 1 - r_7}{u_5 - d_4}, \frac{u_5 - 1 - r_6}{u_5 - d_4}, \frac{u_5 - 1 - r_5}{u_5 - d_4}, \frac{u_5 - 1 - r_4}{u_5 - d_4}, \frac{u_5 - 1 - r_3}{u_5 - d_4}, \frac{u_5 - 1 - r_2}{u_5 - d_4}, \frac{u_5 - 1 - r_1}{u_5 - d_4}; k\right) \tag{13}$$

For different choices of i, j and k in inequality (3), the fuzzy jump factors and the fuzzy risk-free interest rate involving GLOFN satisfying the no-arbitrage conditions yield up and down fuzzy risk neutral probability measures.

Definition 9 [4]**.** The fuzzy intrinsic values of American fuzzy put option model on future contracts is defined by

$$\widetilde{V}_N^{\mathcal{AP}}(\widetilde{F}_{N,i}) \approx \max\{\widetilde{F}_{N,i} - \widetilde{K}, \widetilde{0}\}, \text{ for } i = 0, 1, \ldots, N$$

and

$$\widetilde{V}_n^{\mathcal{AP}}(\widetilde{F}_{n,i}) \approx \max\left\{\widetilde{F}_{n,i} - \widetilde{K}, \frac{\widetilde{1}}{\widetilde{1} + \widetilde{R}} \widetilde{E}_n^{Q}(\widetilde{V}_{n+1}^{\mathcal{AP}}(\widetilde{F}_{n+1,i}))\right\}$$

where $\widetilde{E}_n^Q(\widetilde{V}_{n+1}^{AP}(\widetilde{F}_{n,\,1})) \approx (\widetilde{P}_u \widetilde{V}_{n+1}^{AP}(\widetilde{F}_{n+1,\,i}) + \widetilde{P}_d \widetilde{V}_{n+1}^{AP}(\widetilde{F}_{n+1,\,i}))$ for $n = N-1,$ $N-2, \ldots, 0$ and $i = 0, 1, \ldots, n,$ and $\widetilde{E}_n^Q$ denotes the expectation with respect to the fuzzy risk-neutral probability measure Q.

3. Properties of Fuzzy Intrinsic Values of American Fuzzy Put Option Model

We prove some of the properties of the fuzzy analogue of the intrinsic values of American fuzzy put option model in which (i), (ii), (iii) are direct extensions while property (iv) is different from the crisp case.

Property (i). If $\widetilde{V}_{K_1}^{AP}, \widetilde{V}_{K_2}^{AP}$ are the fuzzy intrinsic values of American fuzzy future contract with respect to the constant fuzzy strike prices $\widetilde{K}_1$ and $\widetilde{K}_2$ satisfying $\widetilde{K}_1 \succcurlyeq \widetilde{K}_2$ for the same underlying fuzzy stock and having the same expiration date N, then

$$\widetilde{V}_{(K_1)}^{AP} \succcurlyeq \widetilde{V}_{(K_2)}^{AP}. \tag{14}$$

Proof. We prove equation (14) by backward inductions on n. First we prove for $n = N$.

$$\widetilde{V}_N^{AP}(\widetilde{F}_N^{uu\ldots u})_{K_1} \approx \max\{\widetilde{K}_1 - \widetilde{F}_N^{uu\ldots u},\, \widetilde{0}\} \text{ (since } \widetilde{K}_1 \succcurlyeq \widetilde{K}_2 \Rightarrow \widetilde{K}_1 - \widetilde{F}_N^{uu\ldots u}$$

$$\succcurlyeq \widetilde{K}_2 - \widetilde{F}_N^{uu\ldots u}) \Rightarrow \widetilde{V}_N^{AP}(\widetilde{F}_N^{uu\ldots u})_{\widetilde{K}_1} \succcurlyeq \max\{\widetilde{K}_2 - \widetilde{F}_N^{uu\ldots u},\, \widetilde{0}\}$$

$$\Rightarrow \widetilde{V}_N^{AP}(\widetilde{F}_N^{uu\ldots u})_{\widetilde{K}_1} \succcurlyeq \widetilde{V}_N^{AP}(\widetilde{F}_N^{uu\ldots u})_{\widetilde{K}_2}.$$

Similarly, we can prove it for other states.

Next assume that equation (14) is true for some n between 0 and $N-1$, we have

$$\widetilde{V}_{n+1}^{AP}(\widetilde{F}_{n+1}^{uu\ldots u})_{\widetilde{K}_1} \preceq \widetilde{V}_{n+1}^{AP}(\widetilde{F}_{n+1}^{uu\ldots u})_{\widetilde{K}_2} \tag{15}$$

and true for other states. Now we prove equation (14) for $n-1$.

$$\widetilde{V}_n^{AP}(\widetilde{F}_n^{uu\ldots u})_{\widetilde{K}_1} \approx \max\{\widetilde{K}_1 - \widetilde{F}_n^{uu\ldots u}, \frac{\widetilde{1}}{(\widetilde{1}+\widetilde{R})}(\widetilde{P}_u \widetilde{V}_{n+1}^{AP}(\widetilde{F}_{n+1}^{uu\ldots u})_{\widetilde{K}_1} + \widetilde{P}_d \widetilde{V}_{n+1}^{AP}(\widetilde{F}_{n+1}^{uu\ldots d})_{\widetilde{K}_1})\}.$$

Using equation (15), we have

$$\widetilde{V}_n^{AP}(\widetilde{F}_n^{uu\ldots u})_{\widetilde{K}_1} \preceq \max\{\widetilde{K}_2 - \widetilde{F}_n^{uu\ldots u}, \frac{\widetilde{1}}{(\widetilde{1}+\widetilde{R})}(\widetilde{P}_u \widetilde{V}_{n+1}^{AP}(\widetilde{F}_{n+1}^{uu\ldots u})_{\widetilde{K}_2} + \widetilde{P}_d \widetilde{V}_{n+1}^{AP}(\widetilde{F}_n^{uu\ldots d})_{\widetilde{K}_2})\} \Rightarrow \widetilde{V}_n^{AP}(\widetilde{F}_n^{uu\ldots u})_{\widetilde{K}_1} \succcurlyeq \widetilde{V}_n^{AP}(\widetilde{F}_n^{uu\ldots u})_{\widetilde{K}_2}.$$

Similarly, we can prove it is not true for other states i.e., a contradiction. for other states.

Hence equation (14) is true for n which implies equation (15) is false for $n-1$.

Remark 10. We elucidate Property (i) using the data obtained from a website optionistics [10] that includes Microsoft Corporation shares recorded on 21/06/2018 with the following specifications.

Table 1. Quotes of American style MSFT (2018) shares.

Symbol	Initial stock price S_0	Strike Price K	Premium	Expiry Date	Option style	Risk free interest rate R
MSFT	\$102.075	\$104.80	\$2.70	13/07/2018	C	0.88

- Constant initial fuzzy stock price $\widetilde{S}_0 \approx$ (102.075, 102.075, 102.075, 102.075, 102.075, 102.075, 102.075, 102.075; 0.3)
- Expiration date $T = 22/360$ days
- Fuzzy risk-free interest rate
- $\widetilde{R} \approx$ (0.84, 0.85, 0855, 0.86, 0.88, 0.89, 0.90, 0.91; 0.3)
- *Up* fuzzy jump factor $\widetilde{U} \approx$ (0.9703, 0.9708, 0.9718, 0.9736, 1.003, 1.005, 1.0285, 1.0301; 0.3)

- Down fuzzy jump factor $\widetilde{D} \approx (0.9467, 0.9472, 0.9473, 0.9474,$ $1.004, 1.006, 1.0236, 1.0285, 1.0301; 0.3)$ along with five different constant fuzzy strike prices
- $\widetilde{K}_1 \approx (103.20, 103.20, 103.20, 103.20, 103.20, 103.20, 103.20; 0.3)$
- $\widetilde{K}_2 \approx (104.50, 104.50, 104.50, 104.50, 104.50, 104.50, 104.50; 0.3)$
- $\widetilde{K}_3 \approx (105.32, 105.32, 105.32, 105.32, 105.32, 105.32, 105.32; 0.3)$
- $\widetilde{K}_4 \approx (106.00, 106.00, 106.00, 106.00, 106.00, 106.00, 106.00; 0.3)$
- $\widetilde{K}_5 \approx (107.44, 107.44, 107.44, 107.44, 107.44, 107.44, 107.44,$ $107.44; 0.3)$

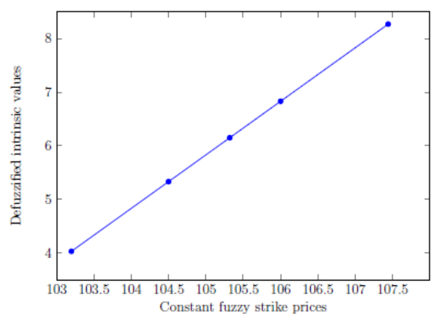

Figure 1. Constant fuzzy strike prices vs defuzzified fuzzy intrinsic values.

Assuming that the same fuzzy put option pricing parameters are utilized for the five different constant fuzzy strike prices respectively. We obtain the following fuzzy intrinsic values of American fuzzy put option model involving GLOFN (see Definition 9) using the up and down fuzzy risk-neutral probability measures (see equations (12) and (13)). We utilize Matlab program to estimate the same.

- (1.0682, 1.0689, 1.0695, 1.0701, 6.6975, 7.0391, 7.2297, 7.3305; 0.3)
- (2.3682, 2.3689, 2.3695, 2.3701, 7.9971, 8.3389, 8.5297, 8.6309; 0.3)
- (3.1882, 3.1889, 3.1895, 3.1901, 8.8169, 9.1588, 9.3497, 9.4511; 0.3)
- (3.8682, 3.8689, 3.8695, 3.8701, 9.4967, 9.8387, 10.0298, 10.1312; 0.3)

- (5.3082, 5.3089, 5.3095, 5.3101, 10.9362, 11.2785, 11.4698, 11.5716; 0.3)

Defuzzify the fuzzy intrinsic values using Definition 2 by choosing $k = 0.3$ and the values are 4.0306, 5.3306, 6.1505, 6.8305, 8.2704 represented diagrammatically against the constant initial fuzzy strike prices shown in Figure 1. The above property reveals that whenever the constant fuzzy strike price increases, the fuzzy intrinsic values of the American fuzzy put option model with respect to the same underlying fuzzy stock will also increase.

Property (ii). Suppose that $\widetilde{V}_{K_1}^{\mathcal{A}P}, \widetilde{V}_{K_2}^{\mathcal{A}P}$ are the fuzzy intrinsic values of American fuzzy put option model on fuzzy contract with $\widetilde{K}_1 \preccurlyeq \widetilde{K}_2$ then $\widetilde{V}_{K_1}^{\mathcal{A}P}, \widetilde{V}_{K_2}^{\mathcal{A}P} \preccurlyeq \widetilde{K}_2 - \widetilde{K}_1$.

Proof. By Property (i), if $\widetilde{K}_1 \preccurlyeq \widetilde{K}_2$ then we have $\widetilde{V}_{K_1}^{\mathcal{A}P} \preccurlyeq \widetilde{V}_{K_2}^{\mathcal{A}P}$ i.e., $\widetilde{V}_{K_1}^{\mathcal{A}P} - \widetilde{V}_{K_2}^{\mathcal{A}P} \preccurlyeq \widetilde{0}$. Since $\widetilde{K}_1 \preccurlyeq \widetilde{K}_2$, $\widetilde{K}_2 - \widetilde{K}_1 \succcurlyeq \widetilde{0}$ i.e., $\widetilde{V}_{K_1}^{\mathcal{A}P} - \widetilde{V}_{K_2}^{\mathcal{A}P} \preccurlyeq \widetilde{K}_2 - \widetilde{K}_1$. □

Property (iii). The upper and lower bounds of the fuzzy intrinsic values of American fuzzy put option model are $\max\{\widetilde{K} - \widetilde{F}_{n,i}, \widetilde{0}\} \preccurlyeq \widetilde{V}^{\mathcal{A}P} \preccurlyeq \widetilde{K}$.

Proof. Since $\widetilde{V}^{\mathcal{A}P} \succcurlyeq \max\{\widetilde{K} - \widetilde{F}_{n,i}, \widetilde{0}\}$ and $\max\{\widetilde{K} - \widetilde{F}_{n,i}, \widetilde{0}\} \preccurlyeq \widetilde{K}$, we have $\max\{\widetilde{K} - \widetilde{F}_{n,i}, \widetilde{0}\} \preccurlyeq \widetilde{V}^{\mathcal{A}P} \preccurlyeq \widetilde{K}$. □

Property (iv). If $\widetilde{S}_0^1$ and $\widetilde{S}_0^2$ are the different constant initial fuzzy stock prices with $\widetilde{S}_0^1 \succcurlyeq \widetilde{S}_0^2$ and that $\widetilde{V}^{\mathcal{A}P_1}, V^{\mathcal{A}P_2}$ are the fuzzy intrinsic values of the American fuzzy put option model on fuzzy future contract, then

$$\widetilde{V}^{\mathcal{A}P_1} \preceq \widetilde{V}^{\mathcal{A}P_2} \tag{16}$$

Proof. We prove equation (16) by backward inductions on n. Obiviously, equation (16) holds for $n = N$. $\widetilde{V}_N^{\mathcal{A}P_1}(\widetilde{F}_N^1)^{uu\ldots u} \approx \max\{\widetilde{K} - (\widetilde{F}_N^1)^{uu\ldots u}, \widetilde{0}\}$

$$\Rightarrow \widetilde{V}_N^{\mathcal{A}P_1}(F_N^1)^{uu\ldots u} \preceq \max\{\widetilde{K} - (F_N^1)^{uu\ldots u}, \widetilde{0}\} \text{ (since } \widetilde{F}_N^1 \preceq \widetilde{F}_N^2)$$

$$\Rightarrow \widetilde{V}_N^{\mathcal{A}P_1}((\widetilde{F}_N^1)^{uu\ldots u}) \preceq \widetilde{V}_N^{\mathcal{A}P_2}(\widetilde{F}_N^2)^{uu\ldots u}.$$

Similarly, we can prove it for other states.

Suppose (iv) is true for some n between 0 and $N-1$, we have

$$\widetilde{V}_{n+1}^{\mathcal{A}P_1}(\widetilde{F}_{n+1}^1)^{uu\ldots u} \preccurlyeq \widetilde{V}_{n+1}^{\mathcal{A}P_2}(\widetilde{F}_{n+1}^2)^{uu\ldots u} \tag{17}$$

and it is true for other states. We now prove equation (16) is false for n.

$$\widetilde{V}_n^{\mathcal{A}P_1}(\widetilde{F}_n^1)^{uu\ldots u} \approx \max\{\widetilde{K} - (\widetilde{F}_{n+1}^1)^{uu\ldots u}, \frac{\widetilde{1}}{(\widetilde{1} + \widetilde{R})}(\widetilde{P}_u \widetilde{V}_{n+1}^{\mathcal{A}P_1}(\widetilde{F}_{n+1}^1)^{uu\ldots u})$$

$$+ \widetilde{P}_d \widetilde{V}_{n+1}^{\mathcal{A}P_1}(\widetilde{F}_{n+1}^1)^{uu\ldots d})\} \text{ (by equation 17)}$$

$$\Rightarrow \widetilde{V}_n^{\mathcal{A}P_1}(\widetilde{F}_n^1) \succeq \max\{\widetilde{K} - (\widetilde{F}_{n+1}^2)^{uu\ldots u}, \frac{\widetilde{1}}{(\widetilde{1} + \widetilde{R})}(\widetilde{P}_u \widetilde{V}_{n+1}^{\mathcal{A}P_2}(\widetilde{F}_{n+1}^2)^{uu\ldots u})$$

$$+ \widetilde{P}_d \widetilde{V}_{n+1}^{\mathcal{A}P_2}(\widetilde{F}_{n+1}^2)^{uu\ldots d})\} \text{ (by equation (17))}$$

$$\Rightarrow \widetilde{V}_n^{\mathcal{A}P_1}(\widetilde{F}_n^1) \succeq \widetilde{V}_n^{\mathcal{A}P_2}(\widetilde{F}_n^2).$$

Similarly, we can prove it is false other states and this is a contradiction. Since equation (16) is true for n this implies equation (17) is false for $n+1$.

Remark 11. We substantiate Property (iv) by considering the stock market data of 5 different constant initial fuzzy stock prices 102.101, 102.325, 102.528, 102.699 and 102.760 shown in Figure 2. Assuming that except the constant initial fuzzy stock prices all the other fuzzy put option pricing parameters remains the same as discussed in Property (i). Using Matlab program, we obtain the following fuzzy intrinsic values of American fuzzy put option model involving GLOFN respectively.

- (2.6422, 2.6428, 2.6435, 2.6441, 8.2725, 8.6144, 8.8053, 8.9065; 0.3)
- (2.4181, 2.4187, 2.4193, 2.4200, 8.0608, 8.4034, 8.5947, 8.6961; 0.3)
- (2.2150, 2.2156, 2.2162, 2.2169, 7.8689, 8.2122, 8.4038, 8.5054; 0.3)
- (2.0439, 2.0445, 2.0451, 2.0458, 7.7073, 8.0511, 8.2430, 8.3447; 0.3)
- (1.9828, 1.9835, 1.9841, 1.9847, 7.6497, 7.9937, 8.1857, 8.2874; 0.3)

We defuzzify the same using Definition 2 by taking $k = 0.3$ and the values are 5.6053, 5.3877, 5.1905, 5.0243, 4.9651 depicted graphically against the five different constant initial fuzzy stock prices (see Figure 2).

4. Conclusion

In this chapter, we have discussed the impacts of constant fuzzy strike prices and constant initial fuzzy stock prices on defuzzified fuzzy intrinsic values of American fuzzy put option model and validated the same involving GLOFN using the real time data. It is used to analyse the upside and downside changes in the fuzzy intrinsic values corresponding to the constant fuzzy strike prices and constant initial fuzzy stock prices.

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